TI in Focus: AP® Calculus

2018 AP® Calculus Exam: AB-5
Technology Solutions and Problem Extensions

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions using technology

(4) Problem Extensions

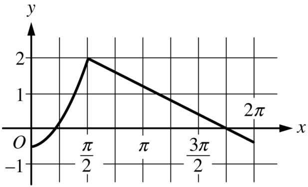
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\dfrac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\displaystyle\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^{\pi} - 1}{\pi}.$$

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

$$2 : \begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$$

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}} e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\frac{1}{\sqrt{2}} e^{5\pi/4}$.

$$3 : \begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \quad \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$$

(d) $\lim_{x \to \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim_{x \to \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

$$\lim_{x \to \pi/2} \frac{f(x)}{g(x)} = \lim_{x \to \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

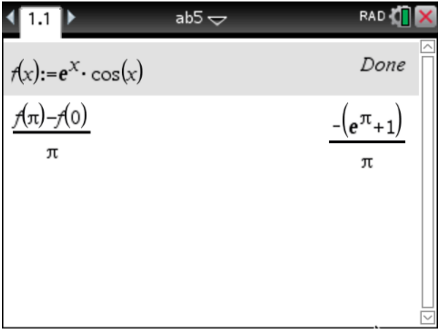
$$3 : \begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \quad \text{and limits equal } 0 \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$$

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

Part (a)

The average rate of change of f on the interval $[0, \pi]$:

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^{\pi} - 1}{\pi}$$

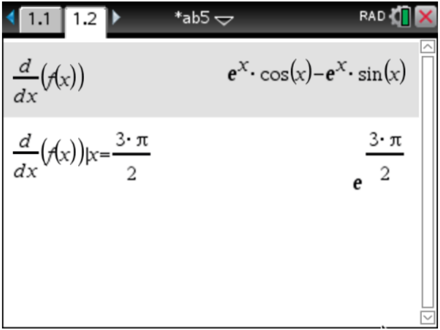


Part (b)

Slope of the tangent line at $x = \frac{3\pi}{2}$.

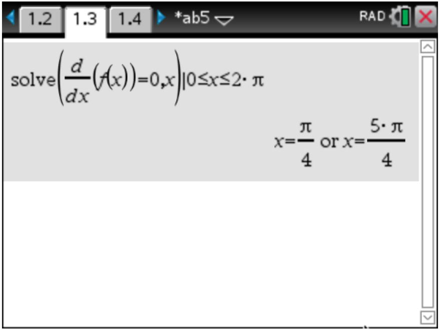
$$f'(x) = e^x \cos x - e^x \sin x$$

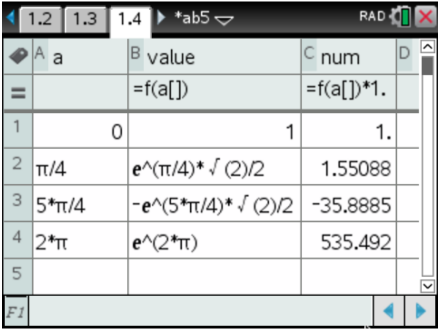
$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$



Part (c)

Find the absolute minimum value of f on the interval $[0, 2\pi]$.

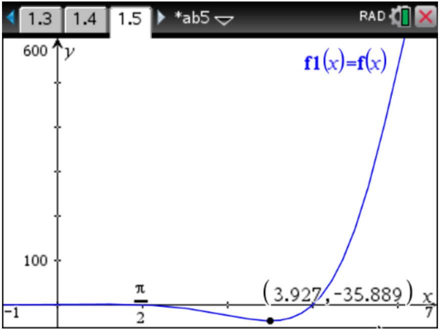




Part (c)

Here is a graph of $y = f(x)$.

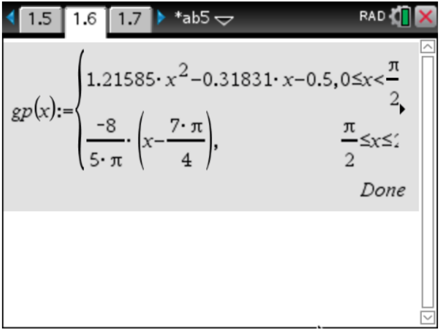
The the absolute minimum value is the y-coordinate of the labeled point.

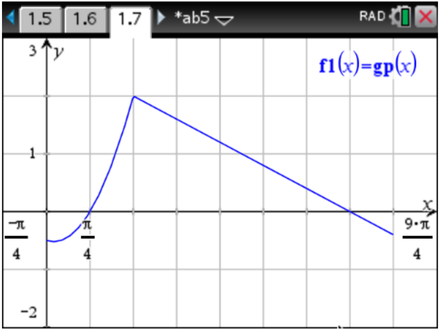


Part (d)

Find $\displaystyle\lim_{x\to\pi/2}\frac{f(x)}{g(x)}$

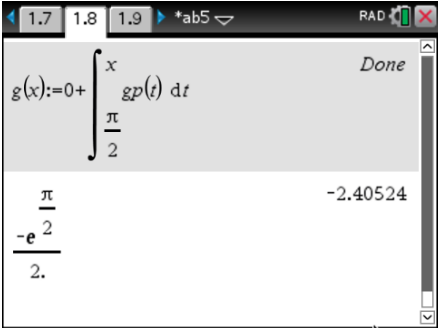
Consider this analytical definition of $g'(x)$.

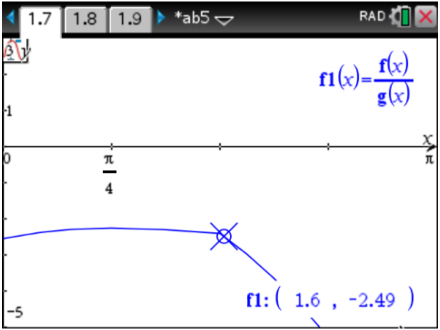




Part (d)

Consider this definition of g, and a numerical approximation to the limit.





Example 1 Original g

Sketch a graph of the function $\boldsymbol{g}$.

Example 2 Function Analysis with Technology

Let the function f be defined by $f(x) = \sqrt{x}\ln x \sin x$ on the interval $1 \le x \le 10$.

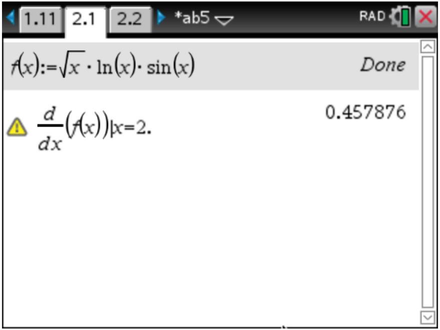
(a) What is the slope of the tangent line at $x = 2$.

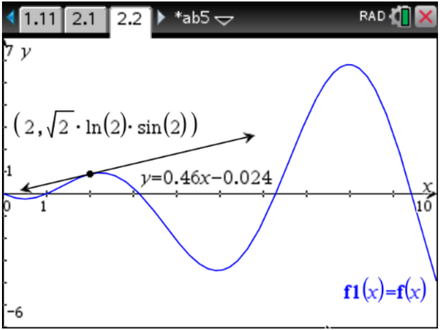
(b) Find the absolute minimum value and the absolute maximum value of f.

(c) Find the intervals on which the graph of f is concave up. Concave down.

(d) Find the inflection points on the graph of f.

Solution



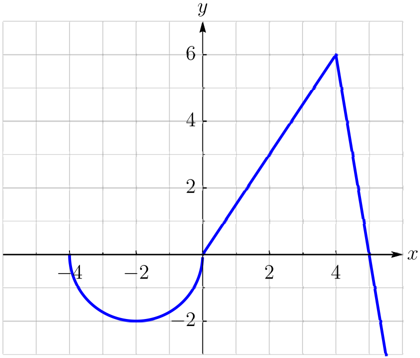


Example 3 L'Hospital One

Find the value of $\displaystyle\lim_{x\to\pi^+} \frac{\cos x \ln(x-\pi)}{\ln(e^x - e^\pi)}$ or state that it does not exist.

Example 4 L'Hospital Two

Let the function f be defined by $f(x) = x^3 + x^2 - 3x - 6$. Let g be a differentiable function such that $g(0) = 1$. The graph of g', the derivative of g is shown below. Find the value of $\lim_{x \to 2} \dfrac{f(x)}{g(x) - x^2}$.



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