

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-5

Technology Solutions and Problem Extensions

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions using technology
- (4) Problem Extensions

5. Let f be the function defined by $f(x) = e^x \cos x$.

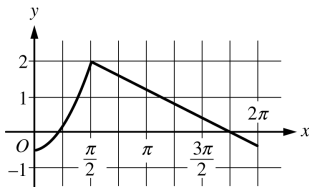
(a) Find the average rate of change of f on the interval $0 \leq x \leq \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \leq x \leq 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g' , the derivative of g , is shown

below. Find the value of $\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

- (a) The average rate of change of f on the interval $0 \leq x \leq \pi$ is

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^\pi - 1}{\pi}.$$

1 : answer

- (b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

2 : $\begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$

The slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$ is $e^{3\pi/2}$.

- (c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

x	$f(x)$
0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}e^{\pi/4}$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

3 : $\begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \frac{\pi}{4}, x = \frac{5\pi}{4} \\ \quad \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$

The absolute minimum value of f on $0 \leq x \leq 2\pi$ is $-\frac{1}{\sqrt{2}}e^{5\pi/4}$.

(d) $\lim_{x \rightarrow \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$$\lim_{x \rightarrow \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$$

By L'Hospital's Rule,

$$\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

$$3 : \begin{cases} 1 : g \text{ is continuous at } x = \frac{\pi}{2} \\ \text{and limits equal } 0 \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$$

Note: max 1/3 [1-0-0] if no limit notation attached to a ratio of derivatives

Part (a)

The average rate of change of f on the interval $[0, \pi]$:

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{-e^{\pi} - 1}{\pi}$$

The image shows a TI-84 Plus calculator screen. At the top, the display shows '1.1', 'ab5', and 'RAD'. Below this, the function $f(x) := e^x \cdot \cos(x)$ is entered. The screen is split to show the average rate of change calculation. On the left side, the expression $\frac{f(\pi) - f(0)}{\pi}$ is displayed. On the right side, the result $\frac{-(e^{\pi} + 1)}{\pi}$ is displayed. The word 'Done' is visible in the top right corner of the function editor area.

Part (b)

Slope of the tangent line at $x = \frac{3\pi}{2}$.

$$f'(x) = e^x \cos x - e^x \sin x$$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The image shows a TI-84 Plus calculator screen. At the top, the mode is set to RAD. The expression $\frac{d}{dx}(f(x))$ is entered, and the calculator displays the derivative formula $e^x \cdot \cos(x) - e^x \cdot \sin(x)$. Below this, the value of the derivative at $x = \frac{3\pi}{2}$ is calculated, showing $\frac{d}{dx}(f(x))|_{x=\frac{3\pi}{2}}$ and the result $e^{\frac{3\pi}{2}}$.

Part (c)

Find the absolute minimum value of f on the interval $[0, 2\pi]$.

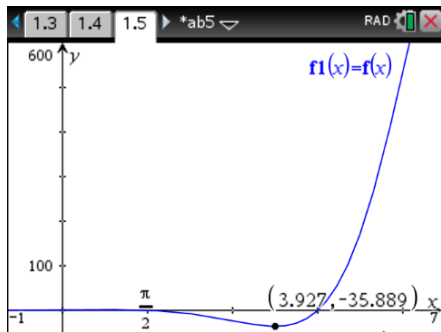
Calculator interface showing the equation $\text{solve}\left(\frac{d}{dx}(f(x))=0, x\right) | 0 \leq x \leq 2 \cdot \pi$ and the solutions $x = \frac{\pi}{4}$ or $x = \frac{5 \cdot \pi}{4}$.

	A a	B value	C num	D
=		=f(a[<i>i</i>])	=f(a[<i>i</i>])*1.	
1	0		1.	
2	$\pi/4$	$e^{(\pi/4)*\sqrt{2}}/2$	1.55088	
3	$5\pi/4$	$-e^{(5\pi/4)*\sqrt{2}}/2$	-35.8885	
4	2π	$e^{(2\pi)}$	535.492	
5				

Part (c)

Here is a graph of $y = f(x)$.

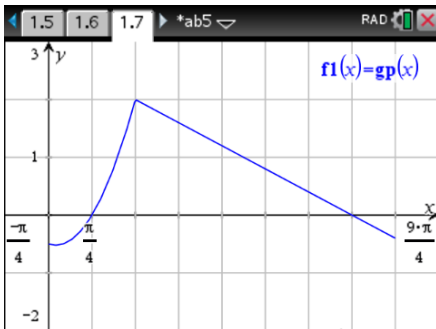
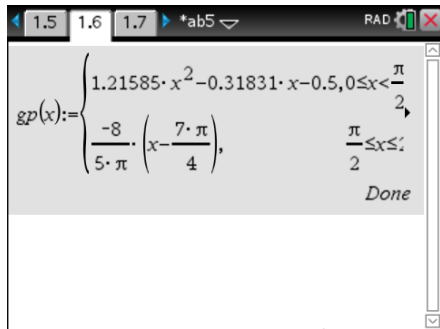
The the absolute minimum value is the y -coordinate of the labeled point.



Part (d)

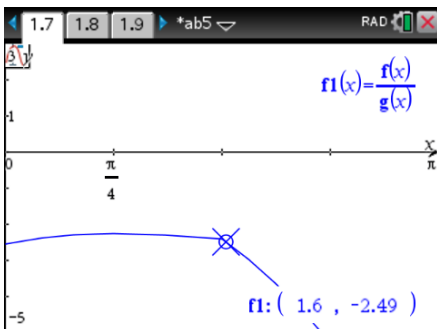
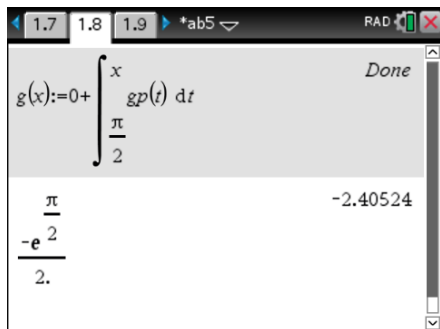
Find $\lim_{x \rightarrow \pi/2} \frac{f(x)}{g(x)}$

Consider this analytical definition of $g'(x)$.



Part (d)

Consider this definition of g , and a numerical approximation to the limit.



Example 1 Original g

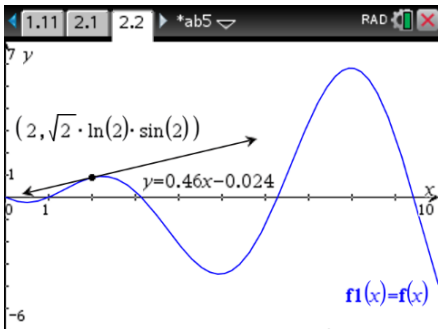
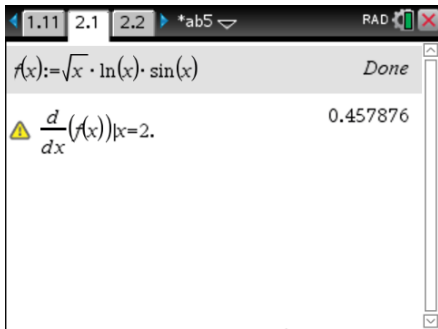
Sketch a graph of the function g .

Example 2 Function Analysis with Technology

Let the function f be defined by $f(x) = \sqrt{x} \ln x \sin x$ on the interval $1 \leq x \leq 10$.

- What is the slope of the tangent line at $x = 2$.
- Find the absolute minimum value and the absolute maximum value of f .
- Find the intervals on which the graph of f is concave up. Concave down.
- Find the inflection points on the graph of f .

Solution



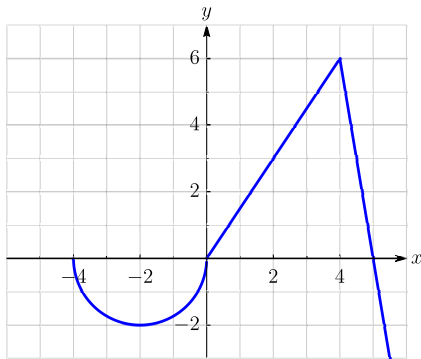
Example 3 L'Hospital One

Find the value of $\lim_{x \rightarrow \pi^+} \frac{\cos x \ln(x - \pi)}{\ln(e^x - e^\pi)}$ or state that it does not exist.

Example 4 L'Hospital Two

Let the function f be defined by $f(x) = x^3 + x^2 - 3x - 6$. Let g be a differentiable function such that $g(0) = 1$. The graph of g' , the derivative of g is

shown below. Find the value of $\lim_{x \rightarrow 2} \frac{f(x)}{g(x) - x^2}$.



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