TI in Focus: AP® Calculus

2018 AP® Calculus Exam: AB-5
Scoring Guidelines

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Student performance

(4) Interpretation

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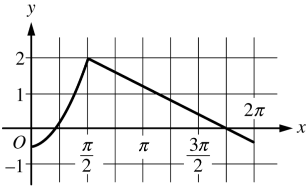
5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is $\dfrac{f(\pi) - f(0)}{\pi - 0} = \dfrac{-e^{\pi} - 1}{\pi}$.

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2}\sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$$

The slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$ is $e^{3\pi/2}$.

$$2 : \begin{cases} 1 : f'(x) \\ 1 : \text{slope} \end{cases}$$

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \dfrac{\pi}{4},\ x = \dfrac{5\pi}{4}$

x	$f(x)$
0	1
$\dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}e^{\pi/4}$
$\dfrac{5\pi}{4}$	$-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$.

$$3 : \begin{cases} 1 : \text{sets } f'(x) = 0 \\ 1 : \text{identifies } x = \dfrac{\pi}{4},\ x = \dfrac{5\pi}{4} \\ \quad\ \text{as candidates} \\ 1 : \text{answer with justification} \end{cases}$$

(d) $\lim_{x \to \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim_{x \to \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

$$\lim_{x \to \pi/2} \frac{f(x)}{g(x)} = \lim_{x \to \pi/2} \frac{f'(x)}{g'(x)} = \frac{-e^{\pi/2}}{2}.$$

3 : { 1 : g is continuous at $x = \frac{\pi}{2}$ and limits equal 0
1 : applies L'Hospital's Rule
1 : answer }

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

Student Performance

Part (a)

- Most students were able to write the difference quotient and earn this point.
- Some students found the average value of f.
- Some students found an average rate of change of f'.
- There were some incorrect decimal approximations of π.

Part(b)

- Most students knew to find the derivative of f
- Some students had difficulty with the product rule and the derivatives of transcendental functions.
- Some students did not evaluate trigonometric functions correctly.

Student Performance

Part (c)

- Most students understood the concept of critical values.
- Many students had difficulty identifying the critical values; Many found only one value.
- Many students had difficulty presenting a global argument to justify their response; Many considered only the critical values.
- Some students used poor communication, a local argument, or miscalculated values of f.

Part(d)

- Most students did not state g is continuous, to evaluate $\lim_{x \to \pi/2} g(x)$.
- Many students did not use correct limit notation.
- Some students equated $\lim_{x \to \pi/2} \dfrac{f(x)}{g(x)}$ to its indeterminate form.

Part (a) 1: answer

Any supporting work in the response must be correct.

Examples

(1) $\dfrac{f(\pi)-f(0)}{\pi}=\dfrac{-e^{\pi}-1}{\pi}$ 1

(2) $\dfrac{f(\pi)-f(0)}{\pi}$ 0

(3) $\dfrac{f(\pi)-f(0)}{\pi}=\dfrac{e^{\pi}\cos\pi-e^{0}\cos 0}{\pi}$ 1

(4) $\dfrac{e^{\pi}\cos\pi-e^{0}\cos 0}{\pi}$ 1

(5) $\dfrac{-e^{\pi}-1}{\pi}$ 1

(6) $\dfrac{1}{\pi}\displaystyle\int_0^{\pi} f'(x)\,dx=\dfrac{1}{\pi}[f(\pi)-f(0)]=\dfrac{-e^{\pi}-1}{\pi}$ 1

Part (b) 1: $f'(x)$

Correct expression for $f'(x)$.

Examples

(1) $f'(x) = e^x \cos x - e^x \sin x$ 1 - ?

(2) $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right)$ 1 - 1

(3) $f'(x) = e^x \cos x + e^x \sin x$ 0 - ?

Other sign errors.

(4) $f'(x) = e^x - \sin x + e^x \cos x$ alone: 0 - 0

presentation error clarified: 1 - ?

Part (b) 1: slope

Eligibility: f' in the form $\pm e^x \cos x \pm e^x \sin x$

$f'(x)$	Slope	Score
$e^x \cos x - e^x \sin x$	$e^{3\pi/2}$	1 - 1
$-e^x \cos x - e^x \sin x$	$e^{3\pi/2}$	0 - 1
$e^x \cos x + e^x \sin x$	$-e^{3\pi/2}$	0 - 1
$-e^x \cos x + e^x \sin x$	$-e^{3\pi/2}$	0 - 1

Note: tangent lines

(1) Second point awarded if correct slope is presented in a correct tangent line.

(2) Correct declared slope presented in an incorrect tangent line: earns second point.

(3) If an incorrect tangent line is identified as the answer: does not earn second point.

Part (c) 1: sets $f'(x) = 0$

Must consider $f'(x) = 0$.

Examples

(1)	$f'(x) = 0$ (nothing else)	1 - 0 - 0
(2)	$\cos x - \sin x = 0$	1 - ? - ?
(3)	$\sin x = \cos x$	1 - ? - ?
(4)	$f'(x)$ changes from negative to positive	1 - ? - ?
(5)	$f'(x)$ changes sign	1 - ? - ?
(6)	$f'(x) =$ incorrect stuff $= 0$	1 - ? - ?
(7)	Undeclared incorrect derivative $= 0$	0 - ? - ?

Part (c) 1: identifies $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$ as candidates

Eligibility: f' in the form $\pm e^x \cos x \pm e^x \sin x$

$f'(x)$	Candidates	
$e^x \cos x - e^x \sin x$	$x = \frac{\pi}{4},\ \frac{5\pi}{4};$	Earns second point
$-e^x \cos x + e^x \sin x$		
$e^x \cos x + e^x \sin x$	$x = \frac{3\pi}{4},\ \frac{7\pi}{4};$	Earns second point
$-e^x \cos x - e^x \sin x$		

Notes

(1) Any additional candidates (not endpoints) presented: does not earn second point.

(2) Candidate might only be presented in evaluation of f: earns second point.

Part (c) 1: answer with justification

Eligibility: Correct f'; Earned second point.

Interpretations

(1) Minimum value $-\frac{1}{\sqrt{2}}e^{5\pi/4}$ and a global argument: table of values.

(2) $-\frac{1}{\sqrt{2}}e^{5\pi/4}$ must be identified as the minimum value.

(3) Any incorrect function values: does not earn the third point.

(4) $f(0), f\left(\frac{\pi}{4}\right)$, and $f(2\pi)$ need not be explicitly calculated.
Only need to indicate that they are positive values.

(5) Minimum value can be identified in a point.
Still need a correct justification.

(6) Minimum occurs at $x = \frac{5\pi}{4}$: no sufficient for third point.

(7) $x = \frac{\pi}{4}$ could be excluded with a correct verbal argument.

(8) Local extrema arguments are not sufficient to earn the third point.

Part (d) 1: g is continuous at $x = \frac{\pi}{2}$ and limits equal 0

Interpretations

(1) g differentiable $\Longrightarrow$ g continuous: $\lim\limits_{x\to\pi/2} g(x) = g\left(\frac{\pi}{2}\right)$

(2) $$\lim_{x\to\pi/2} \frac{f(x) \rightarrow 0}{g(x) \rightarrow 0}$$

With g continuous: earns the first point.

(3) $\lim\limits_{x\to\pi/2} \dfrac{f(x)}{g(x)} = \dfrac{0}{0}$ Does not earn the first point.

Part (d) 1: applies l'Hospital's Rule

Interpretations

(1) An imported incorrect derivative of any form can earn the second point.

(2) Limit must be attached to a ratio of derivatives.

- $\displaystyle\lim_{x\to\pi/2} \frac{f'(x)}{g'(x)}$
- $\displaystyle\lim_{x\to\pi/2} \frac{e^x \cos x - e^x \sin x}{2}$
- $\displaystyle\lim_{x\to\pi/2} \frac{f'(x)}{2}$

(3) Missing $x \to \pi/2$: can still earn the point with supporting work.

(4) Maximum 1/3 [1 - 0 - 0] if no limit notation attached to a ratio of derivatives.

Part (d) 1: answer

Eligibility: Earned the second point.

Imported incorrect $f'(x)$ may still earn the third point.

$f'(x)$	Limit
$e^x \cos x - e^x \sin x$	$\dfrac{-e^{\pi/2}}{2}$
$-e^x \cos x - e^x \sin x$	$\dfrac{-e^{\pi/2}}{2}$
$-e^x \sin x$	$\dfrac{-e^{\pi/2}}{2}$
$e^x \cos x + e^x \sin x$	$\dfrac{e^{\pi/2}}{2}$
$-e^x \cos x + e^x \sin x$	$\dfrac{e^{\pi/2}}{2}$
Any imported incorrect derivative	Consistent answer

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