TI in Focus: AP® Calculus

2018 AP® Calculus Exam: AB-3/BC-3
Scoring Guidelines

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Outline

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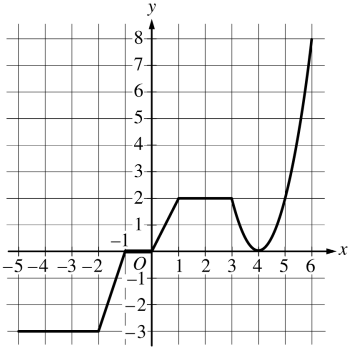

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$ $= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$ $= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$ $= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$	$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$
(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.	$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$
(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

Student Performance

Part (a)

- Some students could not incorporate the initial condition, $f(1) = 3$.
- Some students could not calculate $\displaystyle\int_{-1}^{5} g(x)\,dx$.
- If used geometry correctly: some had difficulty using the initial condition.
- Attempts at an analytical solution.

Part(b)

- Many students had difficulty dealing with a piecewise defined function. For example: use of $g(x) = 2(x-4)^2$ on the entire interval $[1, 6]$.
- $\displaystyle\int_{1}^{3} g(x)\,dx$ computed as 5. Use of $[0, 3]$?
- If students found an antiderivative, most were successful.
- Some students found a derivative.

Student Performance

Part (c)

- Most students were successful.
- If only one interval: $(4, 6)$ with supporting reason.
- Errors in reason often related to the concavity of the graph of f.
- Vague language: the slope, the function.

Part(d)

- Many students were able to identify $x = 4$.
- Many had difficulty providing a reason for the point of inflection.
- Common (incorrect) answer: $x = 3,\ 4$.

Part (a) 1: integral

(1) Integrals that earn this point (with dx or implicit dx);

$$\int_{-5}^{1} g(x)\,dx; \quad \int_{1}^{-5} g(x)\,dx; \quad \int_{-5}^{1} f'(x)\,dx; \quad \int_{1}^{-5} f'(x)\,dx$$

(2) If no definite integral presented: arithmetic that earns this point:

$$-9-\frac{3}{2}+1; \quad -1+9+\frac{3}{2}; \quad -10\frac{1}{2}+1; \quad -1+10\frac{1}{2}$$

(3) Examples that do not earn this point:

$$\int_{-5}^{1} 2(x-4)^2\,dx; \quad 3-\left(-9-\frac{1}{2}\right)$$

Part (a) 1: answer

(1) $\dfrac{25}{2}$ with supporting work, may or may not have earned first point.

(2) Examples:

$$3-\left(-9-\frac{3}{2}+1\right); \quad 3-\left(-10\frac{1}{2}+1\right); \quad 3-\left(-\frac{19}{2}\right); \quad 2+\frac{21}{2}$$

(3) Linkage error: does not earn the answer point.

(4) Other solutions:

- Use of different initial condition, for example, $f(0)=2$.
- Successive accumulation of area starting from $x=1$.
- Analytic solution.

Part (a) Examples

(1) $f(-5) = 12.5$ 0 - 0

(2) $3 - \left(-\frac{19}{2}\right)$ 0 - 1

(3) $3 + \left(\frac{21}{2} - 1\right)$ 1 - 1

(4) $-9 + \frac{3}{2} + 1$ 0 - ?

(5) $f(1) + \int_{1}^{-5} g(x)\,dx = 3 - \left(-\frac{19}{2}\right)$ 1 - 1

(6) $\int_{-5}^{1} g(x)\,dx$ 1 - ?

Part (b) 1: split at $x = 3$

(1) $\displaystyle\int_a^3 g(x)\,dx + \int_3^6 g(x)\,dx, \quad -5 \le a < 3$

(2) $\displaystyle\int_a^3 g(x)\,dx$ with $\displaystyle\int_3^6 g(x)\,dx,$ with $-5 \le a < 3$

(3) $\displaystyle 4 + \int_3^6 g(x)\,dx$

(4) May split more: $\displaystyle\int_1^3 g(x)\,dx + \int_3^4 g(x)\,dx + \int_4^6 g(x)\,dx$

Part (b) 1: antiderivative of $2(x-4)^2$

(1) Our antiderivative:

$$\frac{2}{3}(x-4)^3; \quad \frac{2}{3}u^3 \text{ where } u = x-4; \quad \frac{2}{3}x^3 - 8x^2 + 32x$$

(2) If there is no symbolic antiderivative, first presentation is evaluated:

$$\frac{2}{3}(6-4)^3 - \frac{2}{3}(3-4)^3; \quad \frac{2}{3}(8-(-1)); \quad \frac{16}{3} - \left(-\frac{2}{3}\right)$$

Part (b) 1: answer

(1) Eligibility: earns the antiderviatve point.

(2) Earned for our answer: 10.

(3) Linkage error: does not earn the answer point.

Exception: $4 + \left.\frac{2}{3}u^3\right]_3^6 = 4 + \left.\frac{2}{3}(x-4)^3\right]_3^6 = 10$

(4) Other issues: missing dx, copy errors.

Part (b) Examples

(1) $\int_1^6 2(x-4)^2\,dx = \frac{2}{3}(x-4)^3\Big]_1^6 = \frac{70}{3}$ — 0 - 1 - 0

(2) $4 + \int_3^6 2(x-4)^2\,dx = 2 + \frac{2}{3}(x-4)^3\Big]_3^6 = 10$ — 1 - 1 - 1

(3) $4 + \int_3^6 2(x-4)^2\,dx = \frac{2}{3}(x-4)^3\Big]_3^6$ — 1 - 1 - 0

(4) $5 + \int_3^6 2(x-4)^2\,dx = 5 + \frac{2}{3}(x-4)^3\Big]_3^6 = 11$ — 1 - 1 - 0

(5) $\int_3^6 g(x)\,dx = \frac{2}{3}x^3 - 8x^2 + 32x\Big]_3^6 = 6$ — 0 - 1 - 0

Part (c) 1: intervals

(1) May include or exclude the endpoints.

(2) Various notation acceptable: $(0,1)$, $(4,6)$; $0 \leq x \leq 1$ and $x \geq 4$

(3) Not acceptable:

$(0,1)$ and $(4,b)$ where $b > 6$ or $b = \infty$.

Part (c) 1: reason

(1) Eligibility: one of the following.

- Our intervals: $(0, 1)$ and $(4, 6)$
- Exactly one of our intervals and a subset of the other interval.
- Exactly one of our intervals.
- $(0, 1)$ and $(4, b)$ where $b > 6$ or $b = \infty$.

(2) Earned for correct reason using f', g, and/or g'.

- $f' > 0$ and f' increasing.
- Graph of g is above the x-axis and g is increasing.
- $g > 0$ and the slope of g is positive.
- f' is positive and g' is positive.

(3) Explicit statement of $g' = f''$ required to use f'' positive.

(4) Vague references do not earn the second point: function, slope, derivative, the graph.

Part (d) 1: answer

(1) Earns the point:

$$x = 4; \quad \left(4, 7\frac{2}{3}\right); \quad x = 4 \text{ and any subset of } [-5,-2] \cup [-1,0] \cup [1,3]$$

(2) Does not earn the point: $(4, 0)$

Part (d) 1: reason

(1) Eligibility: answer point or $(4, 0)$.

(2) Acceptable reasons:

- g changes increasing to decreasing or vice versa.
- f' changes decreasing to increasing.
- g' changes sign.
- g' changes negative to positive.
- The slope of g changes positive to negative or vice versa.
- g has a local minimum at $x = 4$.

Part (d) 1: reason

(1) Does not earn the second point:

- f changes concavity at $x = 4$.
- g changes direction at $x = 4$.
- Vague, non-specific references to: function, slope, derivative.

(2) Scoring procedure for $x = 4$ and any subset of $[-5, -2] \cup [-1, 0] \cup [1, 3]$

(3) Other issues: spelling, POI

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