TI in Focus: AP® Calculus

2018 AP® Calculus Exam: AB-3/BC-3
Technology Solutions and Problem Extensions

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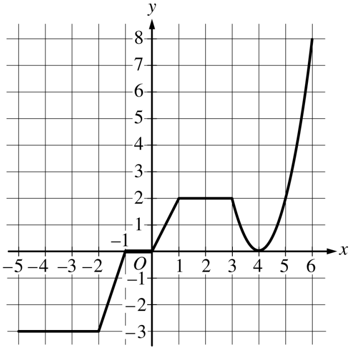
Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions using technology

(4) Problem Extensions



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$

$$= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$

$$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$$

$$= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$$

$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

Part (a)

$$\int_{-5}^{1} g(x)\,dx = f(1) - f(-5)$$ FTC; $f' = g$

$$f(-5) = f(1) - \int_{-5}^{1} g(x)\,dx$$ Solve for $f(-5)$.

$$= 3 - \left(-9 - \frac{3}{2} + 1\right)$$ Initial condition; geometry.

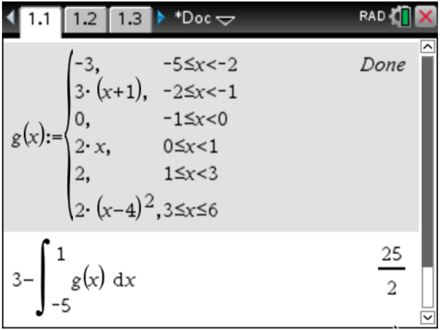
$$= 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$$ Simplify.

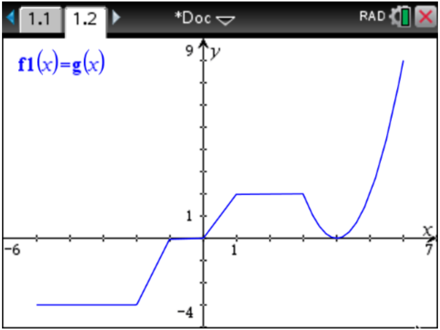
On the Scoring Standard:

$$f(-5) = f(1) + \int_{1}^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$$

Technology Solution

- Define the function g piecewise.
- Check the graph of g.
- Find the value of $f(-5)$.





Example 1 Sketch the Graph of f

For the functions g and f defined in this Free Response Question, we know that $f(1) = 3$ (given) and $f(-5) = \frac{25}{2}$ (part (a)).

(a) Find $f(t)$ for $t = -4, -3, -2, -1, 0, 2, 3, 4, 5, 6$.

(b) Use these values of f to sketch a rough graph of f.

(c) Explain the relationship between the graph of f and the graph of g.

(d) Find a analytical definition for the function f.

Part (b)

$$\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx \qquad \text{Split the integral.}$$

$$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx \qquad \text{Use definition of } g.$$

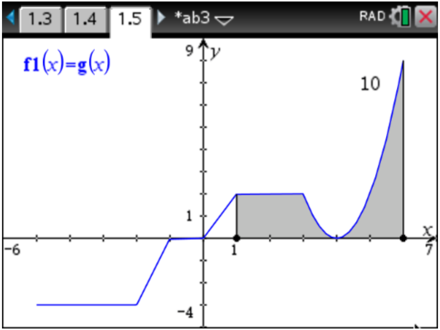
$$= 4 + \left[\frac{2}{3}(x-4)^3\right]_3^6 \qquad \text{Geometry; antiderivative.}$$

$$= 4 + \left[\frac{16}{3} - \left(-\frac{2}{3}\right)\right] \qquad \text{FTC.}$$

$$= 10$$

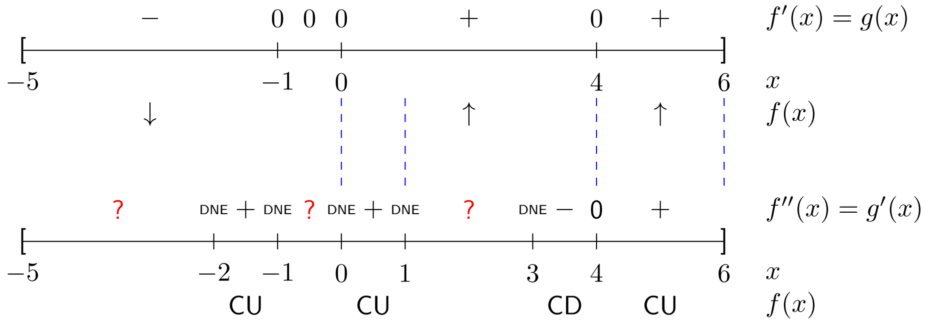
Technology Solution

- Define the function g piecewise.
- Compute the definite integral.



Part (c)

Sign Chart Solution:

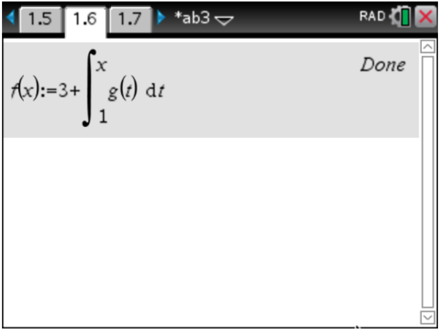


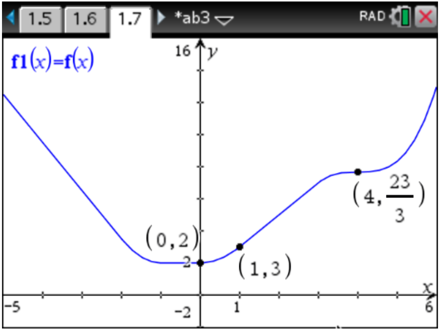
The graph of f is increasing and concave up: $(0, 1)$ and $(4, 6)$.

Note: Discuss the concavity of the graph of f on the interval $(1, 3)$.

Part (c)

Here is a graph of f.

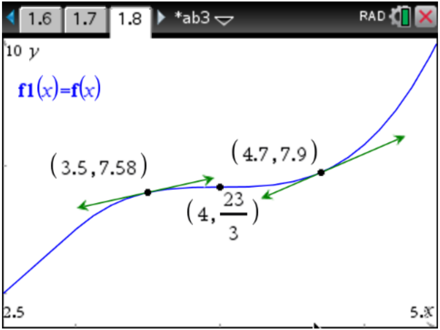




Part (d)

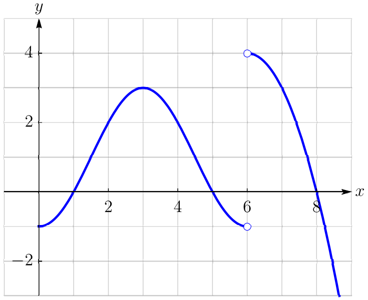
Using the Sign Chart:

The graph of f has a point of inflection at $x = 4$ because the graph changes from concave down to concave up at $x = 4$.



Example 2 Critical Thinking

The graph of the derivative of a continuous function f is shown.



(a) On what intervals is f increasing? Decreasing?

(b) At what values of x does f have a local maximum? Local minimum?

(c) On what intervals is f concave up? Concave down?

(d) Find the x-coordinate(s) of the inflection points.

(e) Assume $f(0) = 0$ and sketch a graph of f.

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