

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-2
Scoring Guidelines

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples

2. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 3$.
- (b) Find the position of the particle at time $t = 3$.
- (c) Evaluate $\int_0^{3.5} v(t) dt$, and evaluate $\int_0^{3.5} |v(t)| dt$. Interpret the meaning of each integral in the context of the problem.
- (d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

$$2 : \begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$$

Student Performance

Part (a)

- Calculator active question, but many attempted a symbolic derivative for acceleration.
- Common errors: degree mode, velocity function entered incorrectly.
- Some did not provide the necessary communication step.

Part(b)

- Good, succinct communication.
- Some students did not use the initial condition.
- Linkage errors in reaching the final answer.

Student Performance

Part (c)

- Most students were able to use technology to evaluate the definite integrals.
- Most were able to identify the first as displacement and the second as total distance traveled.
- Some did not reference the time interval, or just referred to $t = 3.5$.
- Possible errors in entering the velocity function.

Part(d)

- Most common error involved incorrect derivative of $x_2(t)$: $2t$.
- Some students found an antiderivative.
- Some students failed to show the mathematical equation, or did not clearly indicate the setup, for the intersection time.

Part (a) 1: answer

- (1) The answer alone is not sufficient.
- (2) The student must indicate that $v'(3)$ is being computed.
- (3) Correct symbolic differentiation of $v(t)$ OK.

Inoculation

- (1) Degree mode error.
- (2) Decimal presentation error.

Part (b)

Find the position of the particle at time $t = 3$.

$$\begin{aligned}x(3) &= x(0) + \int_0^3 v(t) dt \\ &= -5 + \int_0^3 v(t) dt \\ &= -1.760213\end{aligned}$$

Scoring

$$3 : \left\{ \begin{array}{l} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{array} \right.$$

Part (b) Examples

$$(1) \int_0^3 v(t) dt - 5 = -1.760 \qquad 1 - 1 - 1$$

$$(2) \int_0^3 v(t) - 5 = -1.76 \qquad 0 - 1 - 1$$

$$(3) \int_0^3 (v(t) - 5) dt = -1.760 \qquad 0 - 0 - 0$$

$$(4) \int_0^3 v(t) - 5 = \text{incorrect or nothing} \qquad 0 - 0 - 0$$

$$(5) -5 + \int v(t) dt = -1.76 \qquad 0 - 0 - 1$$

Part (b) Examples

$$(1) \int_0^3 v(t) dt = 3.2397, \quad x(3) = -1.76 \qquad 1 - 1 - 1$$

$$(2) \int_0^3 v(t) dt, \quad x(3) = -1.76 \qquad 1 - 0 - 1$$

$$(3) -5 + \int_0^x v(t) dt \qquad 0 - 1 - 0$$

$$(4) -5 + \int_0^{3.5} v(t) dt = -1.76 \text{ or } -2.156 \qquad 0 - 1 - 1$$

$$(5) 5 + \int_0^3 v(t) dt = -1.76 \text{ or } 8.239 \text{ or } 8.24 \qquad 1 - 0 - 1$$

Part (c)

$$3 : \begin{cases} 1: \text{ answers} \\ 2: \text{ interpretations of } \int_0^{3.5} v(t) dt \text{ and } \int_0^{3.5} |v(t)| dt \end{cases}$$

$$\int_0^{3.5} v(t) dt = 2.844 \quad (\text{or } 2.843)$$

This integral represents the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

$$\int_0^{3.5} |v(t)| dt = 3.737$$

This integral represents the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

Part (c) 2: interpretations

Two ways to earn just 1 of the 2 points

(1) One complete and correct interpretation.

(2) These three items.

(a) Identify $\int_0^{3.5} v(t) dt$ as displacement.

(b) Identify $\int_0^{3.5} |v(t)| dt$ as distance traveled.

(c) No reference to time or an incorrect reference to time.

Part (c) 2: interpretations

$\int_0^{3.5} v(t) dt$ interpretation:

Must be describing the difference between the position of the particle at $t = 3.5$ and the position of the particle at $t = 0$.

$\int_0^{3.5} |v(t)| dt$ interpretation:

Must be describing the total distance traveled by the particle from $t = 0$ to $t = 3.5$.

Examples

- | | |
|---|--------|
| (1) net distance traveled \equiv displacement | OK |
| (2) change in position \equiv displacement | OK |
| (3) total distance \equiv distance traveled | OK |
| (4) distance | Not OK |

Part (d)

Solution

$$v(t) = x_2'(t)$$

$$v(t) = 2t - 1 \implies t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

Scoring

$$2 : \begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$$

Examples

- | | |
|---|-------|
| (1) $v(t) = 2t - 1$ | 1 - 0 |
| (2) $x_2'(t) = 2t - 1, \quad t = 1.571$ | 0 - 1 |
| (3) $2t - 1, \quad t = 1.571$ | 0 - 1 |
| (4) $x_2'(t) = 2t - 1$ | 0 - 0 |

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