TI in Focus: AP® Calculus

2018 AP® Calculus Exam: BC-2
Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

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Outline

(1) Free Response Question
(2) Scoring Guidelines
(3) Student performance
(4) Interpretation
(5) Common errors
(6) Specific scoring examples

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

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(a) $p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$

There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters.

The number of plankton cells, in millions, in the entire column of water is given by $\int_{0}^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh$.

Because $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$$

The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.

$$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$$

(d) $\int_{0}^{1} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$

The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$$

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Student Performance

Part (a)

- Good use of technology, few decimal presentation errors.
- Some students started to differentiate analytically, but most eventually used technology.
- Some students struggled with the units and the interpretation.
- Three parts to the meaning:
  25, rate, density of plankton.

Part(b)

- Most students presented an integral involving $p(h)$.
- Some students considered the volume of a solid of revolution (πr^2).
- Some students did not correctly incorporate nor address the cross-sectional area of 3.

Student Performance

Part (c)

- Few students expressed the total number of plankton as an integral expression.
- Many students used $u(h)$ rather than $f(h)$.
- Many students did not use the cross-sectional area (3).
- Many students did not conclude $\int_{30}^{K} f(h)\,dh \leq \int_{30}^{\infty} u(h)\,dh$.

Part(d)

- Most students were able to start this problem: arc length in a parametric equations setting.
- Most students were able to evaluate the definite integral using technology.
- Some students made mistakes in presentation and notation: parentheses and copy errors.

Part (a) 1: answer

- Looking for -1.179. (Could be presented as a restart.)
- Our answer only.
- The units are tied to the second point.
- Bald answer: OK only in this part of the problem.

Examples

(1) $p'(h) = 0.4e^{-0.0025h^2} - 0.001h^3e^{-0.0025h^2}$ 0 - ?

(2) $0.2(50)e^{-0.0025(25^2)} + 0.2(25^2)e^{-0.0025(25^2)} \cdot (-0.005(25))$ 1 - ?

(3) $p'(h) = (0.4h - 0.001h^3)(0.99750312)^{h^2}$ 0 - ?

(4) $p'(25) = (0.4(25) - 0.001(25^3))(0.99750312)^{25^2}$ 1 - ?

Part (a) 1: meaning with units

(1) Units can be attached to the answer rather than in the description.

(2) Meaning of $p'(25)$ has three parts, so four parts to this point.

- At 25 meters, at a depth of 25.
- the word **rate**.
- The function is a **density** function.
- Units: million cells per cubic meter per meter.

(3) Read for the second point with an incorrect answer.
Must be consistent. 0 - 1 possible.

Part (a) Correct Samples

(1) At a depth of 25 meters, the density of plankton is decreasing at a rate of 1.179 millions of cells per cubic meter per meter of depth.

(2) $p'(25) = -1.179$ millions of cells per cubic meter per meter $p'(25)$ is the instantaneous rate of change in the density of plankton cells at $h = 25$ meters.

(3) At a depth of 25 meters the density is decreasing at a rate of 1.179 millions of cells/m^4.

(4) When the height of the plankton cells are 25m, the density is decreasing at a rate of 1.179 million cells per cubic meter per meter.

Part (a) Issues

If any one of the four items is missing or incorrect, the the response does not earn the point.

(1) The density of plankton cells is changing at a rate of -1.179 millions of cells/m^3/m.

(2) At a depth of 25m the density of plankton cells is changing at -1.179 million cells per cubic meter per meter.

(3) The concentration of plankton is decreasing at 1.179 millions of cells per cubic meter as depth increases by 1 meter at a depth of 25 meters.

(4) The density of plankton cells is decreasing at a rate of -1.179 million cells/cubic m^2 when $h = 25$m.

(5) $p'(25)$ represents the number of plankton cells in millions per cubic meter per meter of depth at 25 meters of depth.

(6) $p'(25) = -1.179$ millions of cells per cubic meter per second This is the rate of change of density of plankton cells at depth of 25 meters.

Part (b) integrand

Incorrect nonzero constants: does not earn the answer point.

(1) $\int_0^{30} 3p(h)\,dh$	1 - ?
(2) $\int_0^{30} p(h)\,dh$	1 - ?
(3) $\int_0^{10} 0.2h^2e^{-0.0025h^2}\,dh$	1 - 0
(4) $\int_0^{30} 0.6h^2e^{-0.0025h^2}\,dh$	1 - ?
(5) $\int_0^{30} \pi p(h)$	1 - 0
(6) $\int_0^{30} 3p'(h)\,dh$	0 - 0
(7) $\int_0^{30} \pi[p(h)]^2\,dh$	0 - 0

Part (b) 1: answer

(1) 1675; 1675 million; 1,675,000,000

(2) Point is earned for the correct answer only if the integrand is in the from $k\,p(h)$.

(3) Bald answers do not earn this point.

(4) These answers earn the point.

1675.4; 1675.41; 1675.414; 1675.415; 1675.414#; 1675.415#

(5) These answers do not earn the point:

1676; 1675.42; 1675.427

Part (c) 1: integral expression

(1) Common responses:

$\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh$ 1 - ? - ?

$1675 + \int_{30}^{K} 3f(h)\,dh$ 1 - ? - ?

(2) Eligibility criteria for importing an integral or numerical value.

(3) Common issues:

- Uses $u(h)$ rather than $f(h)$.
- Missing one or both 3s.
- ∞ instead of K as upper limit.
- Both integrals start at 30 or 0.

Part (c) 1: compares improper integral

(1) Must correctly compare an integral involving $f(h)$ to an improper integral involving $u(h)$.

(2) Comparing $\int_{30}^{K} u(h)\,dh \leq \int_{30}^{\infty} u(h)\,dh$ is insufficient.

(3) Comparing $f(h)$ and $u(h)$ outside of an integral context is insufficient.

(4) Comparison can be done analytically or verbally.

(5) Answers that do not earn the point: generally mention $f(h) \leq u(h)$ without a definite integral.

Part (c) 1: explanation

(1) Eligibility: response must deal with both 3s.

(2) Must show two items.

   (a) Produce a numeric value from the integrals (1990 or consistent) that is less than 2000.

   (b) State that the numeric value is the bound on the number of plankton/integral expression
   (rather than the actual number of plankton).

(3) If the response does not address both 3s: cannot earn this point.

Part (d) 1: integrand

Examples

(1) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ 1 - ?

(2) $\int_0^1 \sqrt{(662\sin(5t))^2 + (880\cos(6t))^2}\, dt$ 1 - ?

(3) $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ 1 - ?

(4) $\int_0^1 \sqrt{(x')^2 + (y')^2}\, dt$ 1 - ?

Parentheses issues: response does not earn the first point but eligible for the answer point with our correct answer.

Copy errors: response does not earn the first point but eligible for answer point.

Part (d) 1: total distance

(1) 757.456 or 757.455

(2) Bald answer does not earn the point.

(3) Degree mode?

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