

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-6
Scoring Guidelines

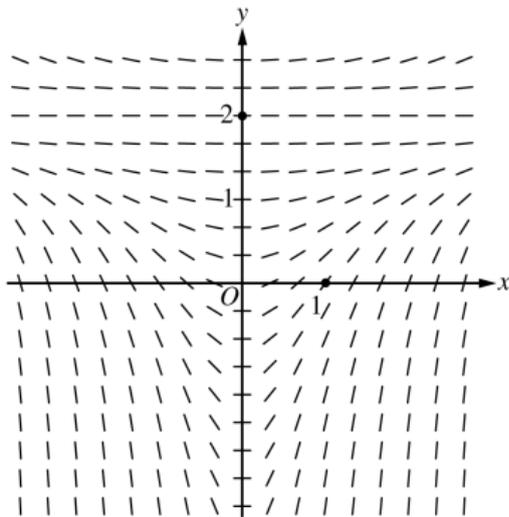
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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$.

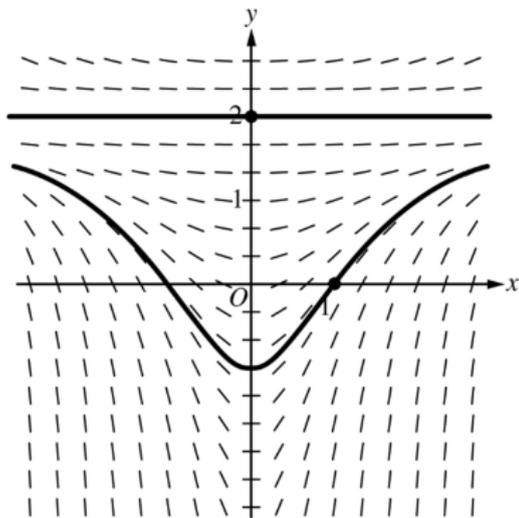
- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$.

- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

(a)



$$2: \begin{cases} 1: \text{solution curve through } (0, 2) \\ 1: \text{solution curve through } (1, 0) \end{cases}$$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

$$(b) \left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$$

An equation for the line tangent to the graph of $y = f(x)$ at

$$x = 1 \text{ is } y = \frac{4}{3}(x - 1).$$

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

$$2: \begin{cases} 1: \text{equation of tangent line} \\ 1: \text{approximation} \end{cases}$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x(y-2)^2$$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

Note: this solution is valid for $-\infty < x < \infty$.

- | | | |
|-----|---|---|
| 5 : | { | 1 : separation of variables |
| | | 2 : antiderivatives |
| | | 1 : constant of integration
and uses initial condition |
| | | 1 : solves for y |

Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Student Performance

Part (a)

- Some students did not sketch the solution curve through the point $(0, 2)$.
- Some confusion between the scale on the axes and the distance between line segments.

Part(b)

- Some students worked to find the solution curve, the work in part (c), to approximate $f(0.7)$.
- Most were successful at evaluating the derivative at $(1, 0)$ and in writing an equation of the tangent line.
- Some students made errors in simplification.

Student Performance

Part (c)

- Many students made mistakes while trying to solve for y before using the initial condition.
- Most students were able to find the antiderivative of the linear term.
- Many students introduced a natural logarithm for the antiderivative of the expression involving y .
- Some students did not separate the variables or did not separate the variables correctly.

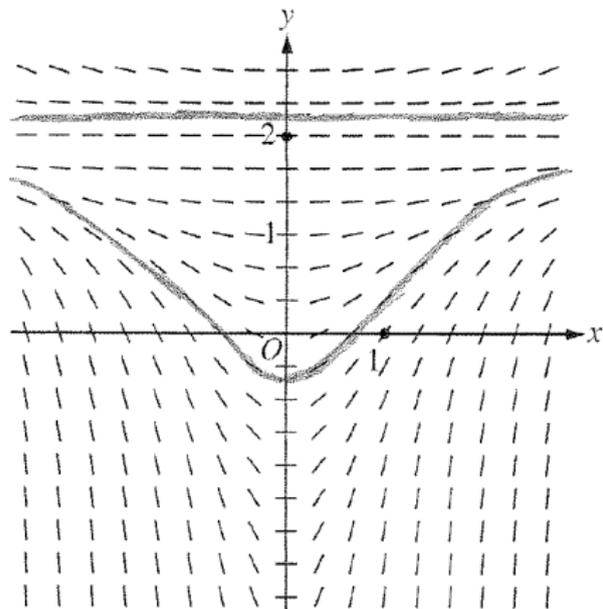
Part (a) 1: solution curve through (0, 2)

- (1) Horizontal line through (0, 2)

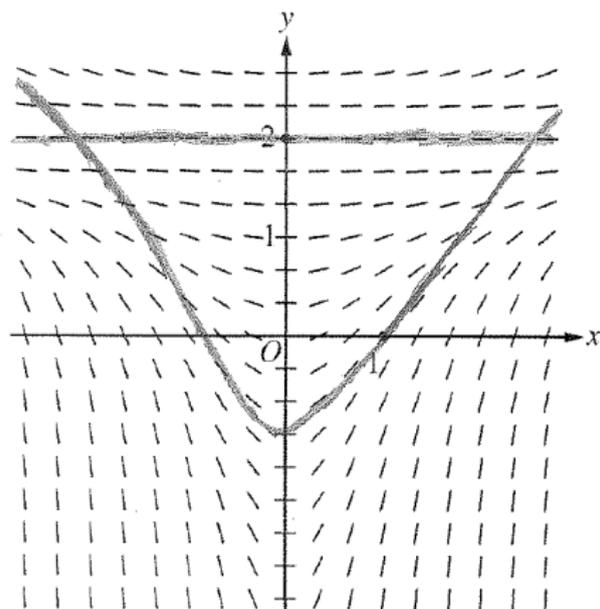
Part (a) 1: solution curve through (1, 0)

- (1) Solution curve through (1, 0) that does not directly contradict the slope field.
- (a) Continuous, extending from the left boundary to the right boundary.
 - (b) Relatively smooth.
 - (c) Does not cross $y = 2$.
- (2) Extends to the boundary interpretation:
Read across one gap with the student's solution curve.
- (3) Arrowheads: only read for the horizontal solution curve that does not extend to the boundary.

Part (a) Examples

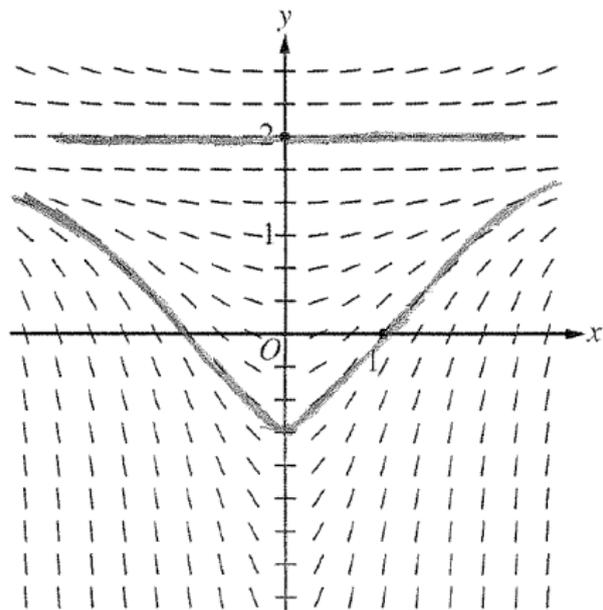


0 - 0

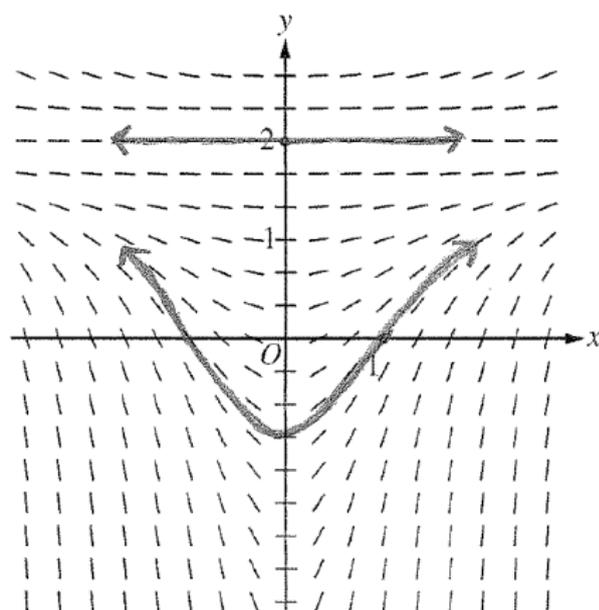


1 - 0

Part (a) Examples



1 - 1



1 - 0

Part (b) 1: equation of tangent line

- (1) Our correct tangent line: $y = \frac{4}{3}(x - 1) = \frac{4}{3}x - \frac{4}{3}$ or equivalent.
- (2) First appearance of a correct tangent line equation earns the point; cannot be lost.
- (3) Any subsequent error: student does not earn the approximation point.

Part (b) 1: approximation

- (1) Tangent line approximation: $y = \frac{4}{3}(0.7 - 1) = -0.4$
- (2) May earn 0 - 1 without writing an equation of the tangent line.
- (3) Bald -0.4 does not earn this point.
- (4) Approximation has many forms.
 $\frac{4}{3}(-0.3)$; $-\frac{12}{30}$; $-\frac{6}{15}$; $-\frac{2}{5}$
- (5) Additional work in (b) unrelated to the tangent line and the approximation is not read. However, depending on the work, it may be read later for part (c).

Part (c) 1: Separation of variables

(1) Would like to see:

$$\frac{dy}{(y-2)^2} = \frac{1}{3} dx \quad \text{or} \quad \frac{3}{(y-2)^2} dy = x dx \quad 1 - ? - ? - ?$$

(2) Other good separations

$$\frac{1}{(y-2)^2} \frac{dy}{dx} = \frac{x}{3}; \quad \int \frac{1}{(y-2)^2} = \int \frac{x}{3}; \quad \int \frac{1}{(y-2)^2} \frac{dy}{dx} dx = \int \frac{x}{3} dx$$

(3) Starts with: $-\frac{1}{y-2} = \frac{1}{6}x^2$ or $-\frac{3}{y-2} = \frac{1}{2}x^2$ 1 - 2 - ? - ?

(4) Bad separation. 0 - ? - ? - 0

(5) Special cases:

Missing x ; missing dy , dx , $\frac{dy}{dx}$;

mishandling/copy error of constant or of the difference.

(6) No separation: 0/5

Part (c) 2: antiderivatives

- (1) Eligibility:
earned first point, or special case, or
bad separation: all y on one side, all x on the other, handles dy/dx correctly,
one side correct.
- (2) Each point earned for the correct antiderivative.
Antiderivative points cannot be lost.
- (3) Linkage errors, and all subsequent errors: answer point.

Part (c) 1: constant of integration and uses initial condition

- (1) Eligibility: at least one antiderivative point earned, x and y are both present.
- (2) Earned for substitutions: $1 \rightarrow x$ and $0 \rightarrow y$.
And an arbitrary constant consistent with the presented antiderivatives in an equation that can be solved for the constant.
- (3) If the use of the initial condition $(1, 0)$
 - (a) Creates a non-real expression
 - (b) Creates a non-real C
 - (c) Creates an unsolvable equation
 - (d) Is used with a C not consistent with the presented antiderivativesThen, the point is not earned.

Part (c) 1: solves for y

(1) Point is awarded for our answer.

$$\frac{6}{-x^2 - 2} + 2; \quad \frac{2x^2 - 2}{x^2 + 2}; \quad -\frac{1}{\frac{1}{6}x^2 + \frac{1}{3}} + 2$$

$$-\frac{1}{\frac{x^2+2}{6}} + 2; \quad \left(-\frac{1}{6}x^2 - \frac{1}{3}\right)^{-1} + 2$$

(2) Any domains for the solution $y = f(x)$ are not considered.

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