TI in Focus: AP® Calculus

2017 AP® Calculus Exam: BC-5
Technology Solutions and Problem Extensions

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions using technology

(4) Problem extensions

5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

(b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x)\,dx$ or show that the integral diverges.

(d) Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

(a) $$f'(x) = \frac{-3(4x-7)}{\left(2x^2 - 7x + 5\right)^2}$$

$$f'(3) = \frac{(-3)(5)}{(18 - 21 + 5)^2} = -\frac{15}{4}$$

$2 : f'(3)$

(b) $$f'(x) = \frac{-3(4x-7)}{\left(2x^2 - 7x + 5\right)^2} = 0 \Rightarrow x = \frac{7}{4}$$

The only critical point in the interval $1 < x < 2.5$ has x-coordinate $\frac{7}{4}$.

f' changes sign from positive to negative at $x = \frac{7}{4}$.

Therefore, f has a relative maximum at $x = \frac{7}{4}$.

$$2 : \begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \quad\ \text{with justification} \end{cases}$$

(c) $$\int_5^\infty f(x)\,dx = \lim_{b\to\infty} \int_5^b \frac{3}{2x^2 - 7x + 5}\,dx = \lim_{b\to\infty} \int_5^b \left(\frac{2}{2x-5} - \frac{1}{x-1}\right) dx$$

$$= \lim_{b\to\infty} \Big[\ln(2x-5) - \ln(x-1)\Big]_5^b = \lim_{b\to\infty} \left[\ln\left(\frac{2x-5}{x-1}\right)\right]_5^b$$

$$= \lim_{b\to\infty} \left[\ln\left(\frac{2b-5}{b-1}\right) - \ln\left(\frac{5}{4}\right)\right] = \ln 2 - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$$

$$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$$

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\displaystyle\int_5^\infty \frac{3}{2x^2 - 7x + 5}\,dx$ converges.

— OR —

$\dfrac{3}{2n^2 - 7n + 5} > 0$ and $\dfrac{1}{n^2} > 0$ for $n \geq 5$.

Since $\displaystyle\lim_{n\to\infty} \frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$ and the series $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n^2}$ converges,

the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges by the limit comparison test.

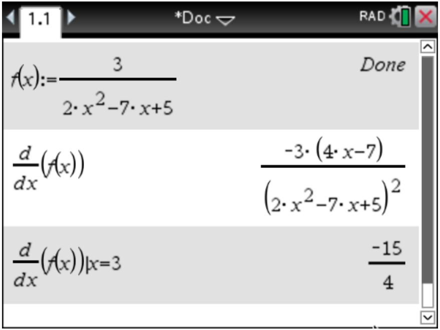
2 : answer with conditions

Part (a)

$$f(x) = \frac{3}{2x^2 - 7x + 5} = 3(2x^2 - 7x + 5)^{-1}$$

$$f'(x) = 3 \cdot (-1) \cdot (2x^2 - 7x + 5)^{-2} \cdot (4x - 7) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2}$$

$$f'(3) = \frac{(-3)(5)}{(18 - 21 + 5)^2} = -\frac{15}{4}$$



Example 1 Tangent Line Approximation

(a) Write an equation for the tangent line to the graph of $y = f(x)$ at the point where $x = 3$.

(b) Use the equation of the tangent line to approximate $f(3.2)$.

(c) Is this approximation an overestimate or underestimate? Justify your answer.

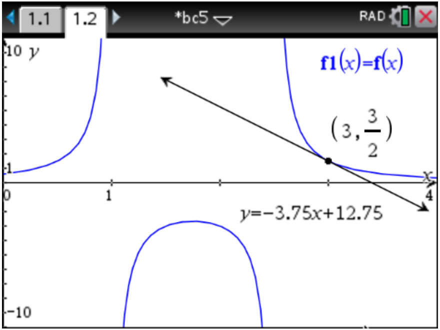
Solution

(a) $f(3) = \dfrac{3}{18 - 21 + 5} = \dfrac{3}{2}$

$$f'(3) = -\frac{15}{4}$$

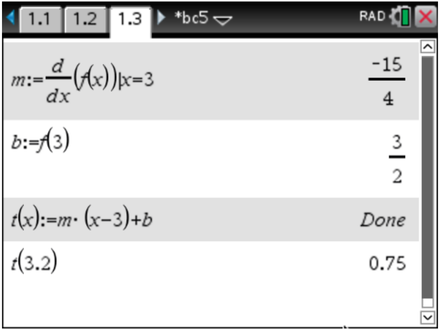
$$y = -\frac{15}{4}(x-3) + \frac{3}{2}$$

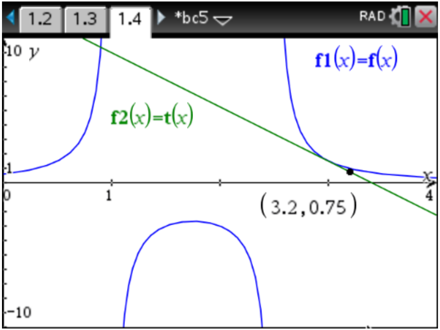
$$= -3.75x + 12.75$$



Solution

(b) $$y(3.2) = -\frac{15}{4}\left(\frac{1}{5}\right) + \frac{3}{2} = -\frac{3}{4} + \frac{3}{2} = \frac{3}{4}$$



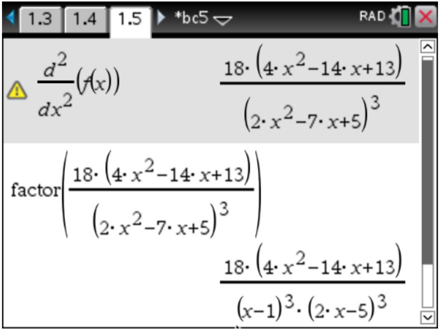


Solution

(c) $$f''(x) = \frac{18(4x^2 - 14x + 13)}{(2x^2 - 7x + 5)^3} = \frac{18(4x^2 - 14x + 13)}{(x-1)^3(2x-5)^3}$$

For $x > \frac{5}{2} \implies f''(x) > 0$

The approximation is an underestimate since the graph of f is concave up.

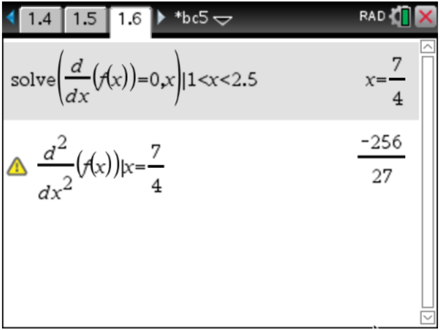


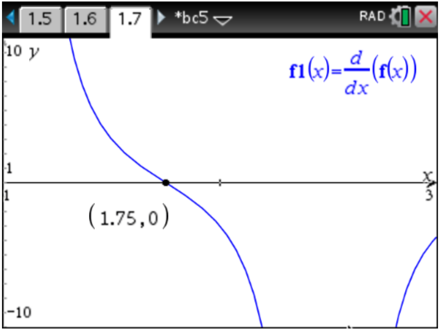
Part (b)

The only critical point in the interval: $x = \dfrac{7}{4}$

f' changes sign from positive to negative at $x = \dfrac{7}{4}$.

Therefore, f has a relative maximum at $x = \dfrac{7}{4}$.





Example 2 If Some (Analysis of the Graph) is Good, More is Better

(a) Are there any other critical points of f? If so, classify each point as the location of a relative minimum, a relative maximum, or neither. If not, why not?

(b) Find all the horizontal and vertical asymptotes on the graph of f.

(c) Find the intervals on which the graph of f is concave up, concave down. Find the inflection points.

Part (c)

Find the Partial Fraction Decomposition (PFD) for f.

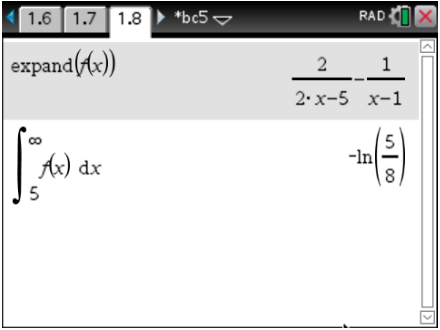
$$\frac{3}{(2x-5)(x-1)} = \frac{A}{2x-5} + \frac{B}{x-1}$$

$$3 = A(x-1) + B(2x-5)$$

$$x = 1: \;\; 3 = B(-3) \implies B = -1$$

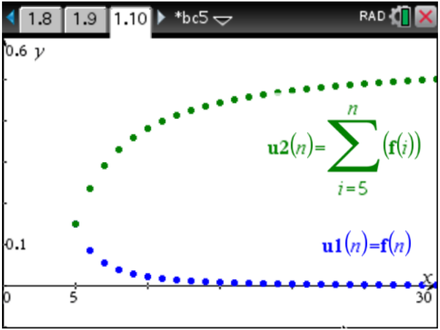
$$x = \frac{5}{2}: \;\; 3 = A\left(\frac{3}{2}\right) \implies A = 2$$

$$\frac{3}{2x^2-7x+5} = \frac{2}{2x-5} - \frac{1}{x-1}$$



Part (d)

Consider the sequence and the partial sums, graphically and numerically.



	A n	B part_sum	C	D
1	10	0.381299		
2	100	0.537771		
3	500	0.549954		
4	1000	0.551459		
5	10000	0.552811		

Example 3 More Approximations

Consider the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$

(a) Find the 20th partial sum, s_{20}.

(b) Use the Remainder Estimate for the Integral Test to estimate the error in using s_{20} as an approximation to the sum of the series.

(c) Find a value of n so that s_n is within 0.0001 of the sum.

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