TI in Focus: AP® Calculus

2017 AP® Calculus Exam: BC-5
Scoring Guidelines

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Student performance

(4) Interpretation

(5) Common errors

(6) Specific scoring examples

5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

(b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\displaystyle\int_5^\infty f(x)\,dx$ or show that the integral diverges.

(d) Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

(a) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2}$

$f'(3) = \dfrac{(-3)(5)}{(18-21+5)^2} = -\dfrac{15}{4}$

2 : $f'(3)$

(b) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2} = 0 \Rightarrow x = \dfrac{7}{4}$

The only critical point in the interval $1 < x < 2.5$ has x-coordinate $\dfrac{7}{4}$.

f' changes sign from positive to negative at $x = \dfrac{7}{4}$.

Therefore, f has a relative maximum at $x = \dfrac{7}{4}$.

2 : $\begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \quad\ \text{with justification} \end{cases}$

(c) $$\int_5^\infty f(x)\,dx = \lim_{b\to\infty}\int_5^b \frac{3}{2x^2-7x+5}\,dx = \lim_{b\to\infty}\int_5^b\left(\frac{2}{2x-5}-\frac{1}{x-1}\right)dx$$

$$= \lim_{b\to\infty}\Big[\ln(2x-5)-\ln(x-1)\Big]_5^b = \lim_{b\to\infty}\left[\ln\left(\frac{2x-5}{x-1}\right)\right]_5^b$$

$$= \lim_{b\to\infty}\left[\ln\left(\frac{2b-5}{b-1}\right)-\ln\left(\frac{5}{4}\right)\right] = \ln 2 - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$

(d) f is continuous, positive, and decreasing on $[5, \infty)$.

The series converges by the integral test since $\int_5^\infty \frac{3}{2x^2 - 7x + 5}\, dx$ converges.

— OR —

$\frac{3}{2n^2 - 7n + 5} > 0$ and $\frac{1}{n^2} > 0$ for $n \geq 5$.

Since $\lim_{n \to \infty} \frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$ and the series $\sum_{n=5}^{\infty} \frac{1}{n^2}$ converges,

the series $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges by the limit comparison test.

2 : answer with conditions

Student Performance

(1) Part (a): successful with Quotient Rule and Chain Rule; connection between derivative and slope of a tangent line; presentation, linkage, algebra errors; dropped negative sign; attempts to differentiate the partial fraction decomposition.

(2) Part (b): most were able to identify potential critical points; difficult to justify; vague language in justifications; coping with $x = 1$ and $x = 5/2$.

(3) Part (c): conceptual understanding of improper integrals; communication and notational fluency lacking; lack of limit notation; properties of logarithms; arithmetic involving infinity; able to find the antiderivatives.

(4) Part (d): conceptual understanding of limit comparison test or integral test; stating of conditions to use certain tests; inconsistent notation or language; inappropriate test for convergence - informal methods.

Part (a) 2: $f'(3)$

To earn 1/2:

(1) Correct $f'(x)$ with incorrect value of $f'(3)$.

(2) $f'(x) = \dfrac{ax+b}{(2x^2-7x+5)^2}, \quad a \neq 0$ (conceptual understanding)

(3) $f'(x) = \dfrac{-3(4x-7)}{2x^2-7x+5}$ correctly evaluated at $x=3$.

(4) Special cases:

- Present equation of a tangent line as final answer.
- Linkage errors.

Part (b) 1: x-coordinate

(1) Earned for identifying the x-coordinate: $x = \frac{7}{4}$.

(2) Can be earned if consistent with eligible form of $f'(x)$ imported from part (a).

- $f'(x) = \dfrac{ax+b}{(2x^2-7x+5)^2}, \quad a \neq 0$
- $f'(x) = \dfrac{-3(4x-7)}{2x^2-7x+5}$

Part (b) 1: relative maximum with justification

(1) Must classify as a relative maximum and justify the answer.

(2) Justification examples:

- f' changes from positive to negative at $x = \dfrac{7}{4}$.
- $f''\left(\dfrac{7}{4}\right) < 0$
- f' is positive to the left of $x = \dfrac{7}{4}$ and negative to the right.

Part (b) 1: relative maximum with justification

These statements do not earn the justification point.

(1) Relative maximum at $x = \frac{7}{4}$ because f changes from increasing to decreasing there.

(2) The slope changes from positive to negative at $x = \frac{7}{4}$.

(3) At $x = \frac{7}{4}$ there is a relative maximum because the derivative changes from positive to negative.

Other incorrect or vague statements involve:

(1) $f'(x)$ increasing or decreasing.

(2) The slope or derivative of $f'(x)$.

(3) Inappropriate pronouns.

(4) Slope (of what?).

(5) Derivative (of what?).

Part (b) Notes

(1) Readers had a precise method for dealing with $x = 1$ and/or $x = 2.5$.

(2) Possible for a student with an incorrect $f'(x)$, but eligible form, to earn both points.

(3) Some common incorrect forms lead to a relative minimum, a correct conclusion.

Part (c) 1: antiderivative

(1) Earned for correct antiderivative only. Both parts correct.

(2) Do not need absolute value symbols.

(3) If $2\ln(2x-5)$ as an antiderivative: 0 - ? - 0

Part (c) 1: limit expression

(1) Earned when $\lim\limits_{b \to \infty}$ is correctly attached to the definite integral or an antiderivative.

- $\lim\limits_{b \to \infty} \int_5^b f(x)\, dx$
- $\lim\limits_{b \to \infty} \Big[\text{antiderivative}\Big]_5^b$

(2) Requires proper notation.

(3) Could use *wandering* or *late* limits.

Part (c) 1: answer

(1) Earned for our answer.

(2) Eligibility: limit notation correctly attached to an antiderivative.

Notes:

(1) The correct solution can be written in several different forms (properties of logarithms).

(2) Copy errors: read if change is subtraction to addition (integrand).

Part (d) 2: answer with conditions

(1) Students may us:
- Integral Test.
- Limit Comparison Test.
- Direct Comparison Test.

(2) Ratio Test: inconclusive.

(3) Restarts: not parallel solutions.

(4) Philosophy:
- In general, one point is earned by correctly executing one of these tests and drawing a correct (or consistent) conclusion.
- Some notational issues and the conditions of the test(s) are considered as part of the other point.

Part (d) 2: answer with conditions

Integral Test (Execution):

(1) Must either refer to part (c) or restate the integral from part (c).

(2) The conclusion must be consistent with their conclusion from part (c).

(3) May conclude *diverges* and still be eligible for both points.

Integral Test (Conditions):

(1) The function (integrand) must be continuous, positive, and decreasing.

(2) Explicit statement of the interval $[5, \infty)$ not required.
But, if specified it must be valid.

(3) Use of function versus series is positive, decreasing, and continuous.

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