

## TI in Focus: AP<sup>®</sup> Calculus

2017 AP<sup>®</sup> Calculus Exam: BC-5

Technology Solutions and Problem Extensions

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## Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions using technology
- (4) Problem extensions

5. Let  $f$  be the function defined by  $f(x) = \frac{3}{2x^2 - 7x + 5}$ .
- (a) Find the slope of the line tangent to the graph of  $f$  at  $x = 3$ .
- (b) Find the  $x$ -coordinate of each critical point of  $f$  in the interval  $1 < x < 2.5$ . Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (c) Using the identity that  $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$ , evaluate  $\int_5^{\infty} f(x) dx$  or show that the integral diverges.
- (d) Determine whether the series  $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$  converges or diverges. State the conditions of the test used for determining convergence or divergence.

$$(a) f'(x) = \frac{-3(4x-7)}{(2x^2-7x+5)^2}$$

$$f'(3) = \frac{(-3)(5)}{(18-21+5)^2} = -\frac{15}{4}$$

2 :  $f'(3)$ 

$$(b) f'(x) = \frac{-3(4x-7)}{(2x^2-7x+5)^2} = 0 \Rightarrow x = \frac{7}{4}$$

The only critical point in the interval  $1 < x < 2.5$  has  $x$ -coordinate  $\frac{7}{4}$ .

$f'$  changes sign from positive to negative at  $x = \frac{7}{4}$ .

Therefore,  $f$  has a relative maximum at  $x = \frac{7}{4}$ .

2 :  $\begin{cases} 1 : x\text{-coordinate} \\ 1 : \text{relative maximum} \\ \text{with justification} \end{cases}$

$$(c) \int_5^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_5^b \frac{3}{2x^2-7x+5} dx = \lim_{b \rightarrow \infty} \int_5^b \left( \frac{2}{2x-5} - \frac{1}{x-1} \right) dx$$

$$= \lim_{b \rightarrow \infty} \left[ \ln(2x-5) - \ln(x-1) \right]_5^b = \lim_{b \rightarrow \infty} \left[ \ln\left(\frac{2x-5}{x-1}\right) \right]_5^b$$

$$= \lim_{b \rightarrow \infty} \left[ \ln\left(\frac{2b-5}{b-1}\right) - \ln\left(\frac{5}{4}\right) \right] = \ln 2 - \ln\left(\frac{5}{4}\right) = \ln\left(\frac{8}{5}\right)$$

3 :  $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{limit expression} \\ 1 : \text{answer} \end{cases}$

(d)  $f$  is continuous, positive, and decreasing on  $[5, \infty)$ .

The series converges by the integral test since  $\int_5^{\infty} \frac{3}{2x^2 - 7x + 5} dx$  converges.

— OR —

$$\frac{3}{2n^2 - 7n + 5} > 0 \text{ and } \frac{1}{n^2} > 0 \text{ for } n \geq 5.$$

Since  $\lim_{n \rightarrow \infty} \frac{\frac{3}{2n^2 - 7n + 5}}{\frac{1}{n^2}} = \frac{3}{2}$  and the series  $\sum_{n=5}^{\infty} \frac{1}{n^2}$  converges,

the series  $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$  converges by the limit comparison test.

2 : answer with conditions

**Part (a)**

$$f(x) = \frac{3}{2x^2 - 7x + 5} = 3(2x^2 - 7x + 5)^{-1}$$

$$f'(x) = 3 \cdot (-1) \cdot (2x^2 - 7x + 5)^{-2} \cdot (4x - 7) = \frac{-3(4x - 7)}{(2x^2 - 7x + 5)^2}$$

$$f'(3) = \frac{(-3)(5)}{(18 - 21 + 5)^2} = -\frac{15}{4}$$

The screenshot shows a TI-84 Plus calculator interface. At the top, it displays "1.1", "\*Doc", and "RAD". The main display area shows the function definition:  $f(x) := \frac{3}{2 \cdot x^2 - 7 \cdot x + 5}$ . Below this, the derivative is shown:  $\frac{d}{dx}(f(x)) = \frac{-3 \cdot (4 \cdot x - 7)}{(2 \cdot x^2 - 7 \cdot x + 5)^2}$ . At the bottom, the derivative evaluated at  $x=3$  is shown:  $\frac{d}{dx}(f(x))|_{x=3} = \frac{-15}{4}$ . The word "Done" is visible in the top right corner of the calculator screen.

### Example 1 Tangent Line Approximation

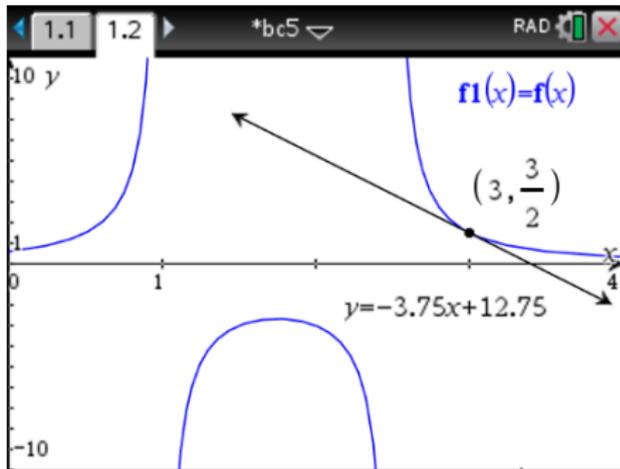
- (a) Write an equation for the tangent line to the graph of  $y = f(x)$  at the point where  $x = 3$ .
- (b) Use the equation of the tangent line to approximate  $f(3.2)$ .
- (c) Is this approximation an overestimate or underestimate? Justify your answer.

### Solution

$$(a) f(3) = \frac{3}{18 - 21 + 5} = \frac{3}{2}$$

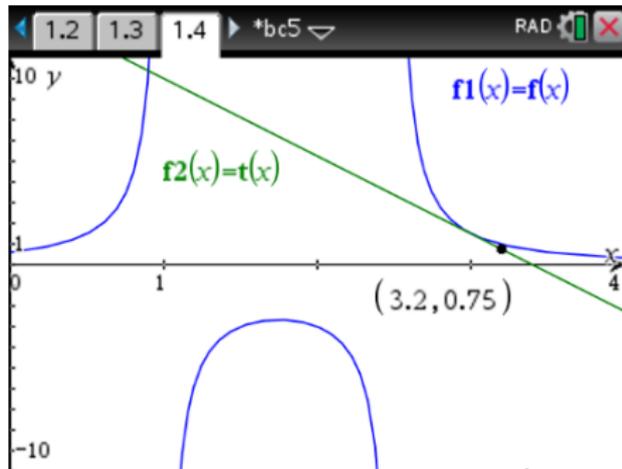
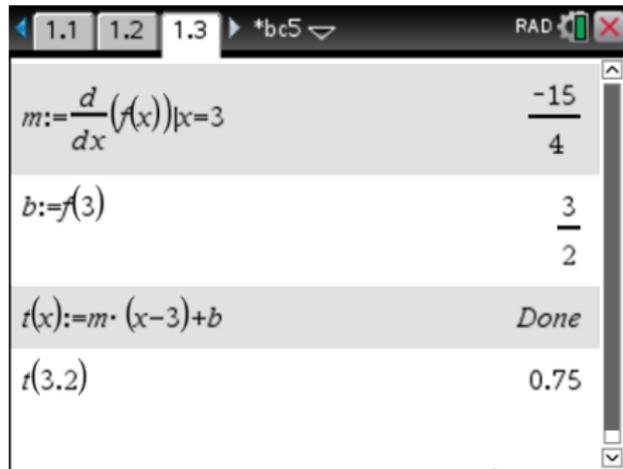
$$f'(3) = -\frac{15}{4}$$

$$\begin{aligned} y &= -\frac{15}{4}(x - 3) + \frac{3}{2} \\ &= -3.75x + 12.75 \end{aligned}$$



## Solution

$$(b) y(3.2) = -\frac{15}{4} \left( \frac{1}{5} \right) + \frac{3}{2} = -\frac{3}{4} + \frac{3}{2} = \frac{3}{4}$$



**Solution**

$$(c) f''(x) = \frac{18(4x^2 - 14x + 13)}{(2x^2 - 7x + 5)^3} = \frac{18(4x^2 - 14x + 13)}{(x-1)^3(2x-5)^3}$$

$$\text{For } x > \frac{5}{2} \implies f''(x) > 0$$

The approximation is an underestimate since the graph of  $f$  is concave up.

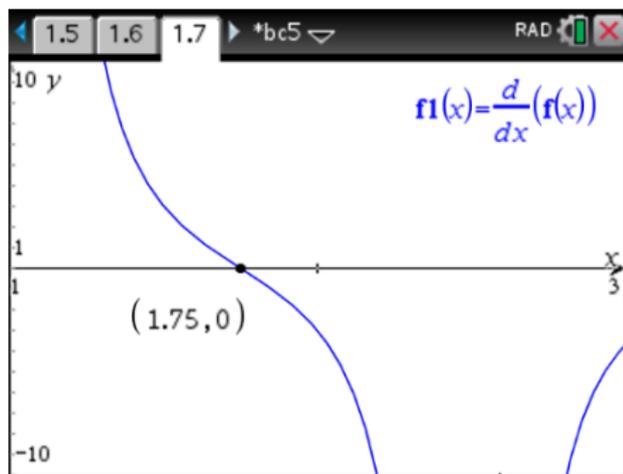
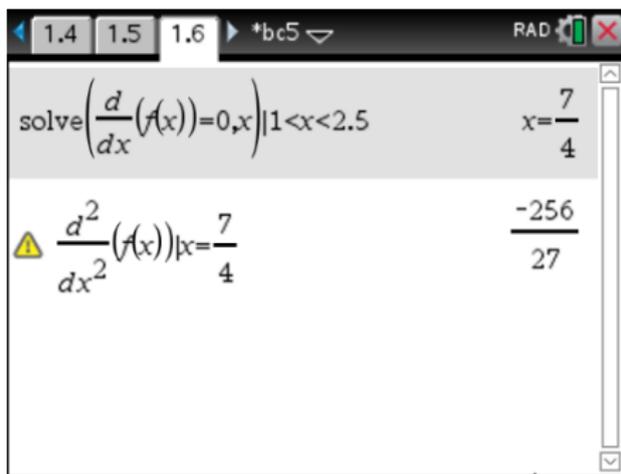
A screenshot of a TI-84 Plus calculator interface. The top status bar shows the mode set to 'RAD' and the angle mode set to '\*bc5'. The main display area shows the derivative of a function  $f(x)$  with respect to  $x^2$ ,  $\frac{d^2}{dx^2}(f(x))$ , as  $\frac{18 \cdot (4 \cdot x^2 - 14 \cdot x + 13)}{(2 \cdot x^2 - 7 \cdot x + 5)^3}$ . Below this, the 'factor' function is applied to the same expression, resulting in  $\frac{18 \cdot (4 \cdot x^2 - 14 \cdot x + 13)}{(x-1)^3 \cdot (2 \cdot x - 5)^3}$ . The calculator interface includes a numeric keypad at the bottom and a scroll bar on the right side of the display area.

**Part (b)**

The only critical point in the interval:  $x = \frac{7}{4}$

$f'$  changes sign from positive to negative at  $x = \frac{7}{4}$ .

Therefore,  $f$  has a relative maximum at  $x = \frac{7}{4}$ .



## Example 2 If Some (Analysis of the Graph) is Good, More is Better

- (a) Are there any other critical points of  $f$ ? If so, classify each point as the location of a relative minimum, a relative maximum, or neither. If not, why not?
- (b) Find all the horizontal and vertical asymptotes on the graph of  $f$ .
- (c) Find the intervals on which the graph of  $f$  is concave up, concave down. Find the inflection points.

## Part (c)

Find the Partial Fraction Decomposition (PFD) for  $f$ .

$$\frac{3}{(2x-5)(x-1)} = \frac{A}{2x-5} + \frac{B}{x-1}$$

$$3 = A(x-1) + B(2x-5)$$

$$x = 1 : 3 = B(-3) \implies B = -1$$

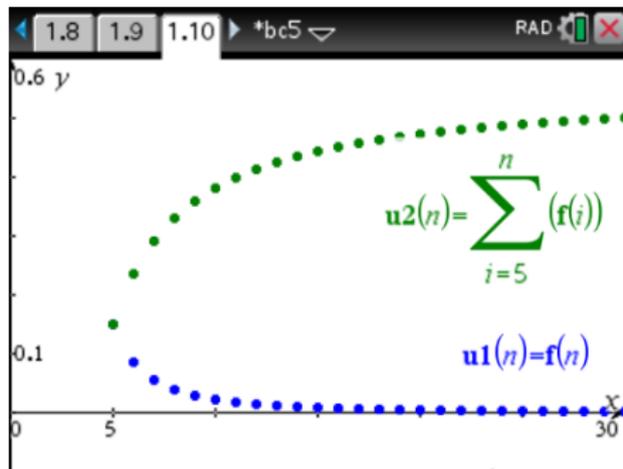
$$x = \frac{5}{2} : 3 = A\left(\frac{3}{2}\right) \implies A = 2$$

$$\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x-5} - \frac{1}{x-1}$$

The screenshot shows a TI-84 Plus calculator interface. At the top, the mode is set to RAD. The input field contains the expression  $\text{expand}(f(x))$ . The result displayed is  $\frac{2}{2 \cdot x - 5} - \frac{1}{x - 1}$ . Below this, the integral  $\int_5^{\infty} f(x) dx$  is shown, with the result  $-\ln\left(\frac{5}{8}\right)$ .

## Part (d)

Consider the sequence and the partial sums, graphically and numerically.



A	n	B	part_sum	C	D
1	10		0.381299		
2	100		0.537771		
3	500		0.549954		
4	1000		0.551459		
5	10000		0.552811		

### Example 3 More Approximations

Consider the series  $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$

- Find the 20th partial sum,  $s_{20}$ .
- Use the Remainder Estimate for the Integral Test to estimate the error in using  $s_{20}$  as an approximation to the sum of the series.
- Find a value of  $n$  so that  $s_n$  is within 0.0001 of the sum.

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