

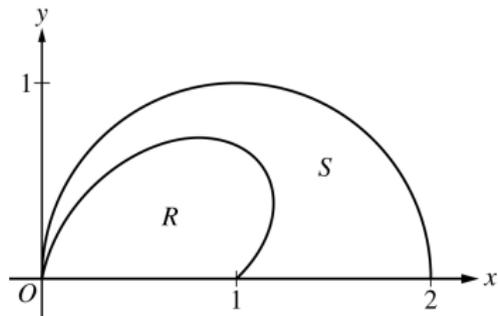
# TI in Focus: AP<sup>®</sup> Calculus

2017 AP<sup>®</sup> Calculus Exam: BC-2  
Scoring Guidelines

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## Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples



2. The figure above shows the polar curves  $r = f(\theta) = 1 + \sin \theta \cos(2\theta)$  and  $r = g(\theta) = 2 \cos \theta$  for  $0 \leq \theta \leq \frac{\pi}{2}$ . Let  $R$  be the region in the first quadrant bounded by the curve  $r = f(\theta)$  and the  $x$ -axis. Let  $S$  be the region in the first quadrant bounded by the curve  $r = f(\theta)$ , the curve  $r = g(\theta)$ , and the  $x$ -axis.
- (a) Find the area of  $R$ .
- (b) The ray  $\theta = k$ , where  $0 < k < \frac{\pi}{2}$ , divides  $S$  into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of  $k$ .

- (c) For each  $\theta$ ,  $0 \leq \theta \leq \frac{\pi}{2}$ , let  $w(\theta)$  be the distance between the points with polar coordinates  $(f(\theta), \theta)$  and  $(g(\theta), \theta)$ . Write an expression for  $w(\theta)$ . Find  $w_A$ , the average value of  $w(\theta)$  over the interval  $0 \leq \theta \leq \frac{\pi}{2}$ .
- (d) Using the information from part (c), find the value of  $\theta$  for which  $w(\theta) = w_A$ . Is the function  $w(\theta)$  increasing or decreasing at that value of  $\theta$ ? Give a reason for your answer.

$$(a) \frac{1}{2} \int_0^{\pi/2} (f(\theta))^2 d\theta = 0.648414$$

The area of  $R$  is 0.648.

$$(b) \int_0^k ((g(\theta))^2 - (f(\theta))^2) d\theta = \frac{1}{2} \int_0^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) d\theta$$

— OR —

$$\int_0^k ((g(\theta))^2 - (f(\theta))^2) d\theta = \int_k^{\pi/2} ((g(\theta))^2 - (f(\theta))^2) d\theta$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{integral expression} \\ \quad \text{for one region} \\ 1 : \text{equation} \end{cases}$$

$$(c) w(\theta) = g(\theta) - f(\theta)$$

$$w_A = \frac{\int_0^{\pi/2} w(\theta) d\theta}{\frac{\pi}{2} - 0} = 0.485446$$

The average value of  $w(\theta)$  on the interval  $\left[0, \frac{\pi}{2}\right]$  is 0.485.

$$(d) w(\theta) = w_A \text{ for } 0 \leq \theta \leq \frac{\pi}{2} \Rightarrow \theta = 0.517688$$

$$w(\theta) = w_A \text{ at } \theta = 0.518 \text{ (or } 0.517).$$

$$w'(0.518) < 0 \Rightarrow w(\theta) \text{ is decreasing at } \theta = 0.518.$$

$$3 : \begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$$

$$2 : \begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$$

## Student Performance

- (1) Part (a): used formula for area, but difficulty in evaluating the definite integral (technology); polar coordinate system versus Cartesian coordinate system.
- (2) Part (b): difficult to interpret a region divided into two equal regions by a ray; additive property of integrals; confusion with area of functions in Cartesian coordinates; area between to polar curves.
- (3) Part (c): knew the formula for average value, but unable to write  $w(\theta)$ ; understanding of radial distance (the meaning of  $r$  in polar coordinates); conceptual meaning of average value.
- (4) Part (d): trouble with technology, solving  $w(\theta) = w_A$ ; calculus-based reasons for the conclusion.

**Part (a) 1: integral**

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- (1) Awarded for the integral expression with the correct limits (constant not included in this point).

•  $\int_0^{\pi/2} [f(\theta)]^2 d\theta$  1 - ?

•  $\int_0^{\pi/2} [(1 + \sin \theta \cos(2\theta))]^2 d\theta$  1 - ?

- (2) Nonzero constant multiple of the correct integral.

•  $\int_0^{\pi/2} [cf(\theta)]^2 d\theta$  1 - ?

•  $\int_0^{\pi/2} [(c + c \sin \theta \cos(2\theta))]^2 d\theta$  1 - ?

**Part (a) 1: integral**

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- (1) An indefinite integral does not earn this point. Eligible for 2nd point.

$$\int [f(\theta)]^2 d\theta \qquad 0 - ?$$

- (2) A definite integral of  $r^2$  does not earn this point. Eligible for 2nd point.

$$\int_0^{\pi/2} r^2 d\theta \qquad 0 - ?$$

- (3) An earned integral point is banked.

- (4) Incorrect or missing constant: cannot earn the answer point.

**Part (a) 1: integral**

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Missing parenthesis/parentheses:

- (1) In the presence of our correct answer, student earns both points.

$$\frac{1}{2} \int_0^{\pi/2} 1 + \sin \theta \cos(2\theta)^2 d\theta = 0.648 \quad 1 - 1$$

- (2) Without our correct answer, student does not earn either point.

$$\frac{1}{2} \int_0^{\pi/2} 1 + \sin \theta \cos(2\theta)^2 d\theta (= 0.652) \quad 0 - 0$$

**Part (a) 1: answer**

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- (1) Bald answers earn no points.
- (2) Awarded for our answer only:

$$0.648 \quad \text{or} \quad \frac{15\pi - 16}{48}$$

**Part (b) 1: integral expression for one region**

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Awarded for the integral expression with the correct limits, where one limit is  $k$ .

$$(1) \int_0^k [(g(\theta))^2 - (f(\theta))^2] d\theta \quad 1 - ?$$

$$(2) \int_0^k [(2 \cos \theta)^2 - (1 + \sin \theta \cos(2\theta))^2] d\theta \quad 1 - ?$$

$$(3) \int_k^{\pi/2} [(g(\theta))^2 - (f(\theta))^2] d\theta \quad 1 - ?$$

$$(4) \int_k^{\pi/2} [(2 \cos \theta)^2 - (1 + \sin \theta \cos(2\theta))^2] d\theta \quad 1 - ?$$

**Part (b) 1: integral expression for one region**

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- (1) An integral without one limit of  $k$  does not earn the 1st point.

$$\frac{1}{2} \int_0^{\pi/2} [(g(\theta))^2 - (f(\theta))^2] d\theta \quad ? - ?$$

- (2) Misspellings OK:  $x$  for  $k$ .
- (3) The integrand must be a nonzero constant multiple of the correct integral to earn the 1st point.
- (4) An earned integral point is banked.
- (5) Missing parenthesis/parentheses (communication): eligible for the 1st point but does not earn the equation point.
- (6) Algebra or other mistakes: does not earn the second point.

**Part (b) 1: equation**

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Awarded for a correct version of our equation (not an expression).

$$(1) \int_0^k [(g(\theta))^2 - (f(\theta))^2] d\theta = \frac{1}{2} \int_0^{\pi/2} [(g(\theta))^2 - (f(\theta))^2] d\theta \quad 1 - 1$$

(Or using  $g(\theta) = 2 \cos \theta$  and  $f(\theta) = 1 + \sin \theta \cos(2\theta)$ )

$$(2) \int_0^k [(g(\theta))^2 - (f(\theta))^2] d\theta = \int_k^{\pi/2} [(g(\theta))^2 - (f(\theta))^2] d\theta \quad 1 - 1$$

(Or using  $g(\theta) = 2 \cos \theta$  and  $f(\theta) = 1 + \sin \theta \cos(2\theta)$ )

$$(3) \int_0^k [(g(\theta))^2 - (f(\theta))^2] d\theta = 0.922 \quad 1 - 1$$

**Part (c) 1:  $w(\theta)$** 

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(1) Awarded for a correct expression for  $w(\theta)$ .

Must be explicitly stated, not simply included in an integral.

•  $w(\theta) = g(\theta) - f(\theta)$  1 - ? - ?

•  $w(\theta) = 2 \cos \theta - (1 + \sin \theta \cos(2\theta))$  1 - ? - ?

•  $w(\theta) = 2 \cos \theta - 1 - \sin \theta \cos(2\theta)$  1 - ? - ?

(2) Missing parentheses or incorrect distribution: does not earn the 1st point.

•  $w(\theta) = 2 \cos \theta - 1 + \sin \theta \cos(2\theta)$  0 - ? - ?

## Part (c) 1: integral

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(1) Awarded for the integral expression with the correct limits.

(2) Do not need to declare  $w(\theta)$  to earn the 2nd point.

(3) Incorrect  $w(\theta)$  can still earn this point.

Eligible for 3rd point but must get our answer.

$$\bullet \int_0^{\pi/2} \pm w(\theta) d\theta \quad ? - 1 - ?$$

$$\bullet \int_0^{\pi/2} \pm [g(\theta) - f(\theta)] d\theta \quad ? - 1 - ?$$

(4) Incorrect or missing constant: will not earn the 3rd point.

**Part (c) 1: average value**

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- (1) Bald answers earn no points.
- (2) Awarded for our answer only:

$$0.485 \quad \text{or} \quad \frac{14 - 3\pi}{3\pi}$$

**Part (d) 1: solves  $w(\theta) = w_A$** 

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Awarded for correctly solving for  $\theta$ : 0.518 (or 0.517).

**Part (d) 1: answer with reason**

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Awarded for decreasing conclusion with reason.

- |  |       |
|--|-------|
| (1) Decreasing because $w'(0.518) < 0$ .                     | 1 - 1 |
| (2) $\theta = 0.518$ . Decreasing because $w'(\theta) < 0$ . | 1 - 1 |
| (3) Decreasing because $w'(0.518)$ is negative.              | 1 - 1 |

Note:

Do not need to calculate a value for  $w'$  to earn the second point.

However, accept  $w'(\theta) \in [-0.6, -0.5]$ .

Any value outside this interval does not earn the 2nd point.

## Part (d)

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Importing:

If the average value point was not earned in part (c), the student may be eligible for both points in part (d) by importing from (c).

Eligibility:

- (1) Correct expression for  $w(\theta)$  in part (c).
- (2)  $0 < w_A < 1$
- (3)  $\theta$  must lie in the interval  $\left[0, \frac{\pi}{2}\right]$  (or  $[0, 1.571]$ ).

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