

2017 AP Calculus Exam: AB-6

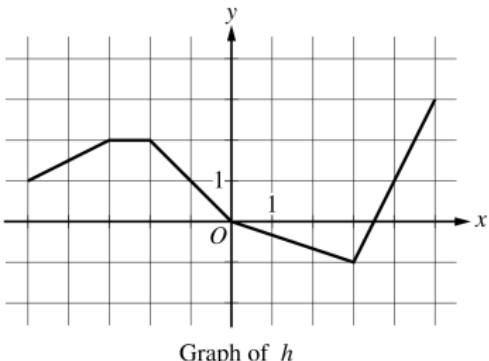
Technology Uses and Problem Extensions

Stephen Kokoska

Bloomsburg University
Former Chief Reader, AP Calculus

AB-6

x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3



6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- (a) Find the slope of the line tangent to the graph of f at $x = \pi$.
- (b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- (c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- (d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

AB-6

(a) $f'(x) = -2\sin(2x) + \cos x e^{\sin x}$

2 : $f'(\pi)$

$$f'(\pi) = -2\sin(2\pi) + \cos \pi e^{\sin \pi} = -1$$

(b) $k'(x) = h'(f(x)) \cdot f'(x)$

2 : $\begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$

$$k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$$

$$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$$

(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$

3 : $\begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$

$$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$$

$$= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$$

(d) g is differentiable. $\Rightarrow g$ is continuous on the interval $[-5, -3]$.

2 : $\begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification,} \\ \quad \text{using Mean Value Theorem} \end{cases}$

$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$

Therefore, by the Mean Value Theorem, there is at least one value c , $-5 < c < -3$, such that $g'(c) = -4$.

AB-6**Part (a)****Technology Solution**

- (1) Define the function f .
- (2) Find the derivative of f when $x = \pi$.

The image shows a graphing calculator screen with the following details:

- Top status bar: Shows "1.1" and "1.2" with arrows, the mode setting "*ab6", and RAD mode.
- Input field: Displays the function $f(x):=\cos(2\cdot x)+e^{\sin(x)}$ and a "Done" button.
- Calculation area:
 - Shows the derivative expression $\frac{d}{dx}(f(x))|_{x=\pi}$ with the result "-1".
 - Shows the second derivative expression $\frac{d}{dx}(f(x))$ with the result $\cos(x)\cdot e^{\sin(x)} - 2\cdot \sin(2\cdot x)$.

AB-6**Part (a) Problem Extensions**

Find $f''(x)$ and $f''(\pi)$.

$$f'(x) = -2 \sin(2x) + \cos x e^{\sin x}$$

$$\begin{aligned} f''(x) &= -2 \cdot 2 \cos(2x) + [-\sin x e^{\sin x} + \cos x \cdot \cos x e^{\sin x}] \\ &= -4 \cos(2x) - \sin x e^{\sin x} + \cos^2 x e^{\sin x} \end{aligned}$$

$$\begin{aligned} f''(\pi) &= -4 \cos(2\pi) - \sin \pi e^{\sin \pi} + \cos^2 \pi e^{\sin \pi} \\ &= -4(1) - 0 \cdot e^0 + (-1)^2 \cdot e^0 \\ &= 4 + 1 = -3 \end{aligned}$$

AB-6**Part (a) Problem Extensions****Technology Solution**

1.1 1.2 *ab6 ▾ RAD DEG

$$\frac{d^2}{dx^2}(f(x))$$
$$((\cos(x))^2 - \sin(x)) \cdot e^{\sin(x)} - 4 \cdot \cos(2 \cdot x)$$
$$\frac{d^2}{dx^2}(f(x))|_{x=\pi} = -3$$

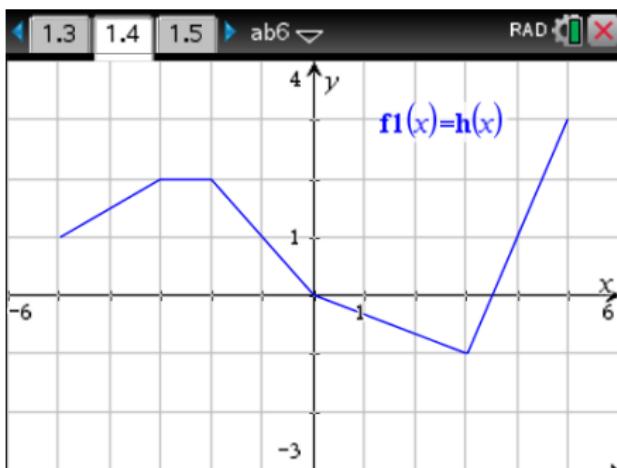
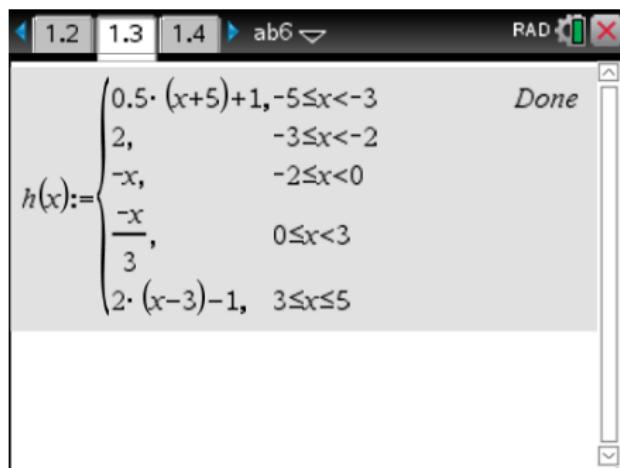
AB-6**Part (a) Problem Extensions**

- (1) Let j be the function defined by $j(x) = f(x^2)$. Find $j'(x)$ and $j'(\sqrt{\pi})$.
- (2) Let L be the function defined by $L(x) = \ln(f(x))$. Find $L'(x)$ and $L'(\pi)$.
- (3) Let s be the function defined by $s(x) = \sin(f(x))$. Find $s'(x)$ and $s'(\pi/2)$.

AB-6**Part (b)****Technology Solution**

It is possible to define the piecewise function h .

We can also graph h .



AB-6**Part (c) Problem Extensions**

Let n be the function defined by $n(x) = \frac{g(-2x)}{h(x)}$. Find $n'(x)$.

$$n'(x) = \frac{h(x)(-2)g'(-2x) - h'(x)g(-2x)}{[h(x)]^2}$$

$$n'(2) = \frac{\left(-\frac{2}{3}\right)(-2)(-1) - \left(-\frac{1}{3}\right)(5)}{\left(\frac{2}{3}\right)^2}$$

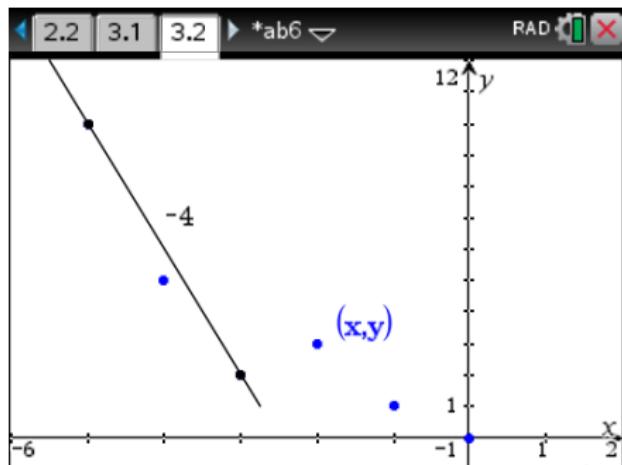
$$= \frac{-\frac{4}{3} + \frac{5}{3}}{\frac{4}{9}} = \frac{3}{4}$$

AB-6**Part (d)****Technology Solution**

Plot the points on the graph of g .

Construct the line joining the points $(-5, 10)$ and $(-3, 2)$.

Find the slope of the line.



AB-6**Problem Extensions**

Let H be the function defined by $H(x) = \int_0^x h(t) dt$.

- (1) Find the values $H(3)$ and $H(-3)$.
- (2) Find the intervals on which H is both increasing and concave up.
- (3) Find the critical numbers of H and classify each as a relative minimum, a relative maximum, or neither. Justify your answer.
- (4) Find the absolute minimum and the absolute maximum value of H on the closed interval $[-5, 5]$.