TI in Focus: AP® Calculus

2017 AP® Calculus Exam: AB-1/BC-1
Scoring Guidelines

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Student performance

(4) Interpretation

(5) Common errors

(6) Specific scoring examples

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

   (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

   (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

   (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

   (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

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1 : units in parts (a), (c), and (d)

(a) Volume $= \int_0^{10} A(h)\,dh$

$$\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$$
$$= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$$
$$= 176.3 \text{ cubic feet}$$

$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.

1 : overestimate with reason

(c) $\int_0^{10} f(h)\,dh = 101.325338$

The volume is 101.325 cubic feet.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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(d) Using the model, $V(h) = \int_0^h f(x)\,dx$.

$$\left.\frac{dV}{dt}\right|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= f(5)\cdot 0.26 = 1.694419$$

When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.

$$3 : \begin{cases} 2 : \dfrac{dV}{dt} \\ 1 : \text{answer} \end{cases}$$

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Student Performance

(1) Contextual and calculator active: usually challenging.

(2) Students did reasonably well with units (except part (d)).

(3) Most students were able to construct a left Riemann sum.

(4) Most were able to recognize volume as the integral of cross-sectional area.

(5) Part (d): difficult to start with the correct expression for $V(h)$.

1: units in parts (a), (c), and (d)

(1) Units must be correct in all parts: (a), (c), and (d).

(2) Must have at least one unit connected with a number to award this point.

Part (a) 1: left Riemann sum

(1) Must present a convincing left Riemann sum.

(2) Philosophy: a sum of products.

(3) 5/6 rule: 5 of the 6 numbers correct:

$$2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5$$

$$C(2 \cdot A(0) + 3 \cdot A(2) + 5 \cdot A(5))$$

Part (a) 1: approximation

(1) $2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5$ (both points)

(2) 176.3 with supporting work.

(3) $100.6 + 43.2 + 32.5$ (no convincing left Riemann sum, communication)

(4) 176.3 alone is a bald answer: no credit

Part (a): Examples

(1) $2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5$ 1 - 1

(2) $2 \cdot 50.3 + 3 \cdot 14.4 + 4 \cdot 6.5$ 1 - 0

(3) $2A(0) + 3A(2) + 5A(5) = 176.3$ 1 - 1

(4) $2A(0) + 3A(2) + 5A(5) = 176$ (inoculation) 1 - 0

(5) $2A(0) + 3A(2) + 5A(5)$ 1 - 0

(6) $100.6 + 43.2 + 32.5$ 0 - 1

Part (b) 1: overestimate with reason

Philosophy: reason must involve the behavior of A.

Acceptable (earns the point):

(1) Overestimate because A is decreasing.

(2) Overestimate because the function is decreasing.

(3) Overestimate because the graph is decreasing.

(4) Overestimate because the area is decreasing.

Part (b) 1: overestimate with reason

Unacceptable (does not earn the point):

(1) Overestimate because A' is negative.

(2) Overestimate because the slope of A is negative.

(3) Overestimate because the rectangles are above the graph.

(4) Overestimate because the function is decreasing and concave up.

(Statements involving concavity will not earn the point.)

(5) Overestimate because the left endpoints are higher than the function.

Part (c) 1: integral; 1: answer

(1) "Integral" means "definite integral" and the correct one. 1 - ?

(2) $k \cdot \int_0^{10} f(h)\,dh$ for $k \neq 0$, $k \neq 1$ 1 - 0

(3) $k + \int_0^{10} f(h)\,dh$ for $k \neq 0$ 1 - 0

(4) $\pi \cdot \int_0^{10} f(h)\,dh = 101.325$ 1 - 0

(5) 101.325 (no integral, bald) 0 - 0

(6) $\int f(h)\,dh = 101.325$ (communication) 0 - 1

Part (d) 2: $\dfrac{dV}{dt}$

Philosophy

- The computation of $\dfrac{dV}{dt}$ involves the Chain Rule and the FTC.
- Each of these concepts is worth exactly 1 point.

Use of A instead of f

(1) $\dfrac{dV}{dt} = A(h) \cdot \dfrac{dh}{dt}$ 2 - ?

(2) $\dfrac{dV}{dt} = A(h) \cdot \dfrac{dh}{dt} \approx f(h) \cdot \dfrac{dh}{dt} = \dfrac{50.3}{e+5} \cdot 0.26$ 2 - 1

(3) $\dfrac{dV}{dt} = A(h) \cdot \dfrac{dh}{dt} = 6.5 \cdot 0.26 = 1.69$ 2 - 0

Chain Rule without FTC

(1) $\dfrac{dV}{dt} = \dfrac{dV}{dh} \cdot \dfrac{dh}{dt}$ (no other work)

(2) $\dfrac{dV}{dt} = f(10) \cdot \dfrac{dh}{dt}$ or $\dfrac{dV}{dt} = A(10) \cdot \dfrac{dh}{dt}$

(3) $\dfrac{dV}{dh} = W$ (incorrect) $\implies \dfrac{dV}{dt} = W \cdot \dfrac{dh}{dt}$

FTC without Chain Rule

(1) $\dfrac{dV}{dh} = f(h)$

(2) $dV = A(h)\,dh$

(3) $\dfrac{dV}{dh} = f(5)$

(4) $\dfrac{dV}{dt} = \dfrac{f(h)}{dh/dt}$

Part (d) 1: answer

Examples

(1) $\dfrac{dV}{dh} \cdot \dfrac{dh}{dt} = 1.694$ 2 - 1

(2) $\dfrac{50.3}{e+5} \cdot 0.26$ 2 - 1

(3) $f(5) \cdot 0.26 = 1.694$ 2 - 1

(4) $f(5) \cdot 0.26$ 2 - 0

(5) $\dfrac{dV}{dh} = \dfrac{50.3}{e+5} \implies 1.694$ 1 - 1

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