TI in Focus: AP® Calculus

2017 AP® Calculus Exam: AB-1/BC-1
Technology Solutions and Problem Extensions

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions using technology

(4) Problem extensions

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

   (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

   (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

   (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

   (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

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1 : units in parts (a), (c), and (d)

(a) Volume $= \int_0^{10} A(h)\,dh$

$$\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$$
$$= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$$
$$= 176.3 \text{ cubic feet}$$

$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.

1 : overestimate with reason

(c) $\int_0^{10} f(h)\,dh = 101.325338$

The volume is 101.325 cubic feet.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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(d) Using the model, $V(h) = \int_0^h f(x)\,dx$.

$$\left.\frac{dV}{dt}\right|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$$
$$= f(5)\cdot 0.26 = 1.694419$$

When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.

$$3 : \begin{cases} 2 : \dfrac{dV}{dt} \\ 1 : \text{answer} \end{cases}$$

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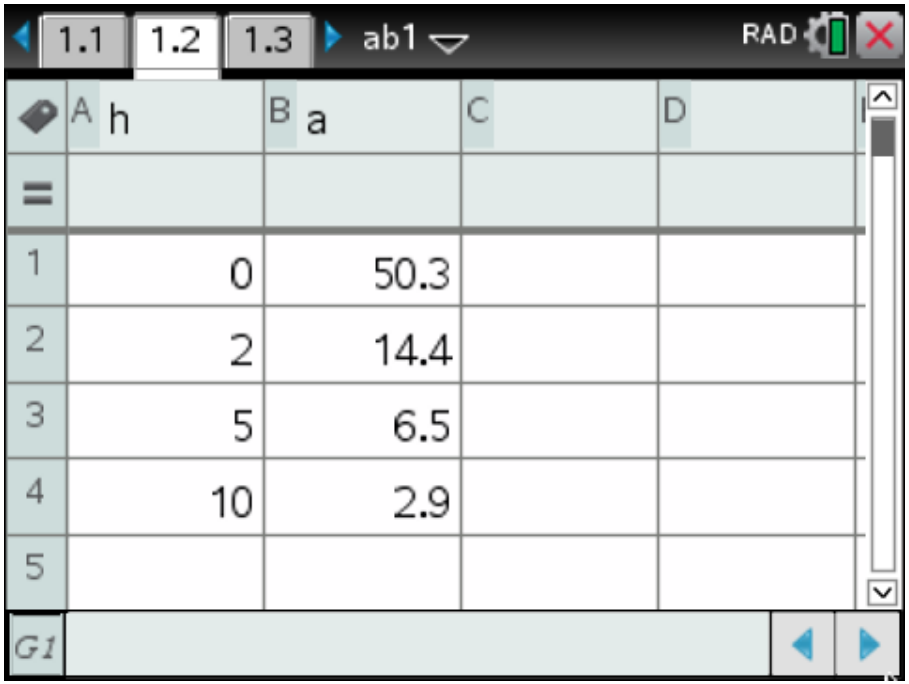
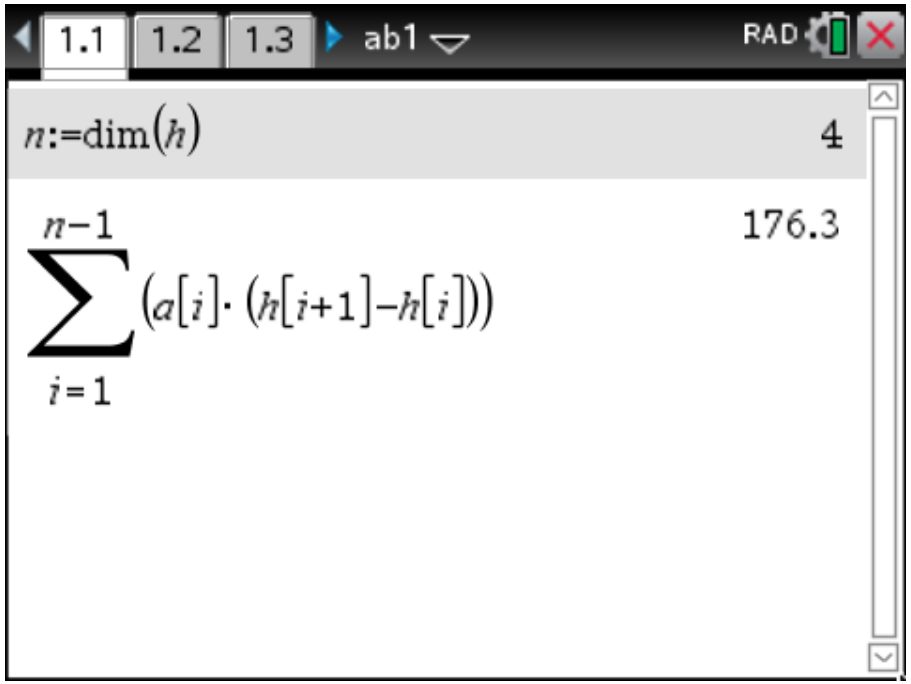
Part (a)

Use a left Riemann sum to approximate the volume of the tank.





Example 1 A Bigger Tank

Suppose the tank has a height of 15 feet. The area of the horizontal cross sections of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table.

h (feet)	0	2	5	10	12	15
$A(h)$ (sq ft)	50.3	14.4	6.5	2.9	2.2	1.4

(a) Use a right Riemann sum with five subintervals indicated by the data in the table to approximate the volume of the tank.

(b) Use a trapezoidal sum with five subintervals indicated by the data in the table to approximate the volume of the tank.

Solution

(a) $R_5 = (14.4)(2) + (6.5)(3) + (2.9)(5) + (2.2)(2) + (1.4)(3)$

$= 71.4$

(b) $T_5 = 2 \cdot \dfrac{50.3 + 14.4}{2} + 3 \cdot \dfrac{14.4 + 6.5}{2} + 5 \cdot \dfrac{6.5 + 2.9}{2} + 2 \cdot \dfrac{2.9 + 2.2}{2}$

$+ 3 \cdot \dfrac{2.2 + 1.4}{2}$

$= 130.05$

Technology Solution

2.2 | 3.1 | 4.1 | *ab1 | RAD

	A h	B a	C	D
=				
1	0	50.3		
2	2	14.4		
3	5	6.5		
4	10	2.9		
5	12	2.2		
6	15	1.4		

G4

3.1 | 4.1 | 4.2 | *ab1 | RAD

$$\sum_{i=1}^{n-1}\left(a[i+1]\cdot\left(h[i+1]-h[i]\right)\right)$$

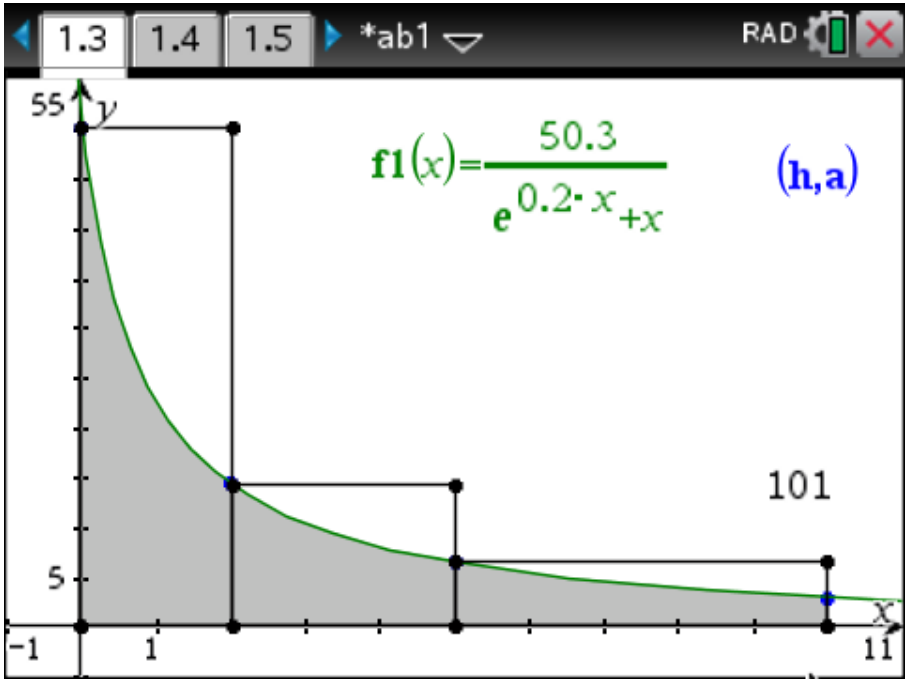
71.4

$$\sum_{i=1}^{n-1}\left(\frac{\left(h[i+1]-h[i]\right)\cdot\left(a[i]+a[i+1]\right)}{2}\right)$$

130.05

Part (b)

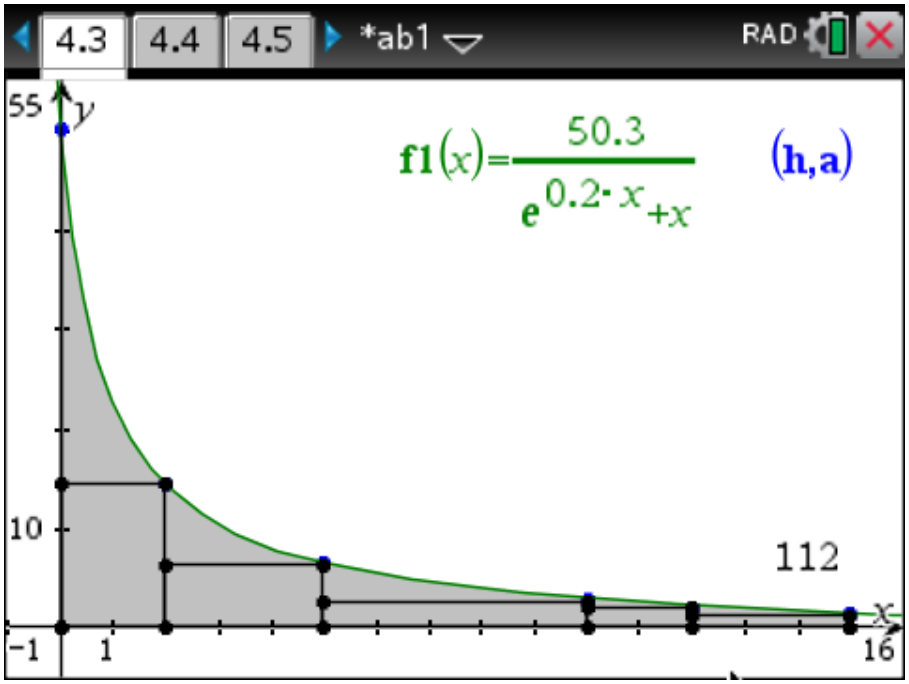
Does the approximation in part (a) overestimate or underestimate the volume of the tank?

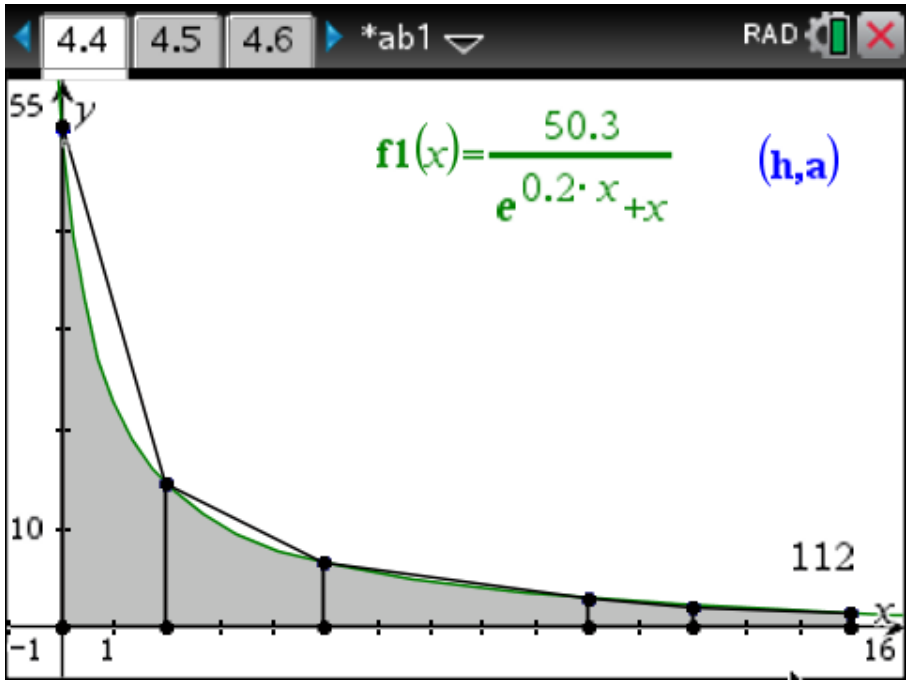


The function A is decreasing on the interval $[0, 11]$.

The left Riemann sum is an overestimate.

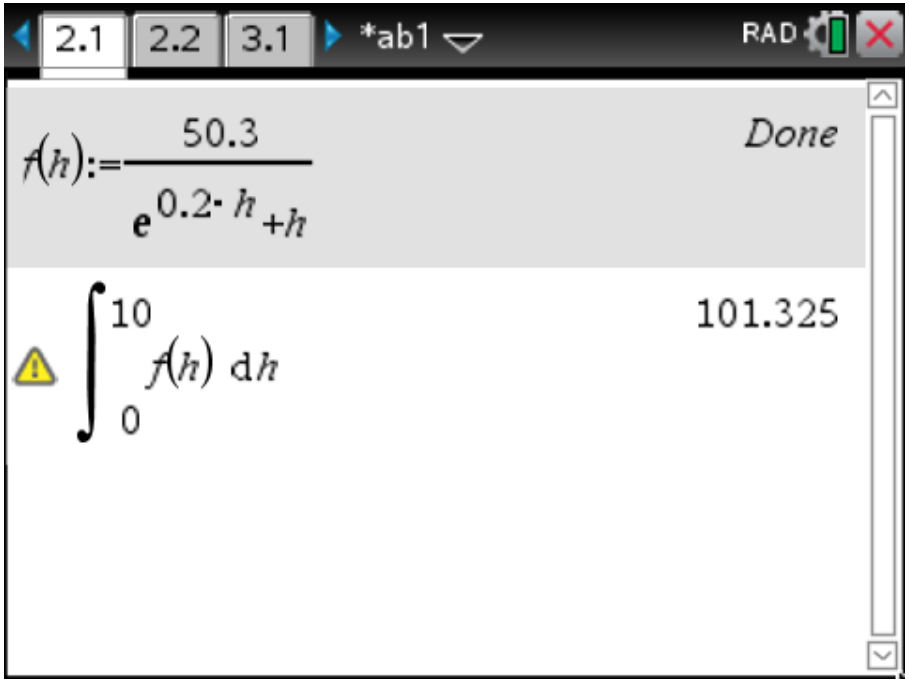
Visualizing the right Riemann sum and the trapezoidal sum.





Part (c)

Based on the model, find the volume of the tank.



Example 2 Extend the Model

Suppose the tank has a height of 10 feet. The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$.

Consider the definite integral $\displaystyle\int_0^{10} f(h)\,dh$ which represents the volume of the tank, or equivalently, the area under the graph of f, between 0 and 10.

(a) Complete the following table.

n	2	4	8	16	32
L_n					
R_n					
M_n					

Example 2 (Continued)

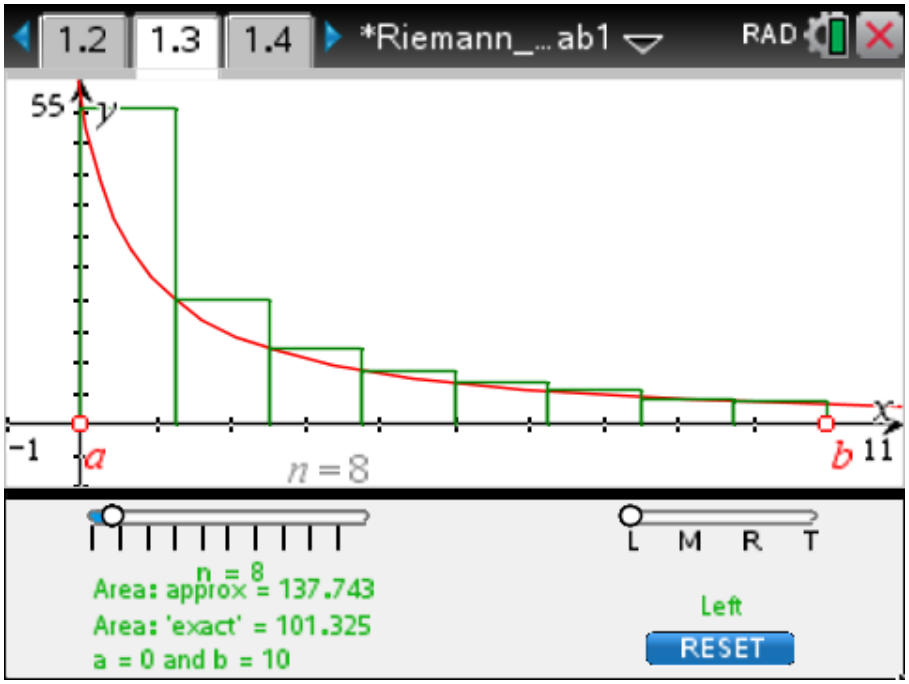
(b) How do your results change in relation to the exact volume as n increases?

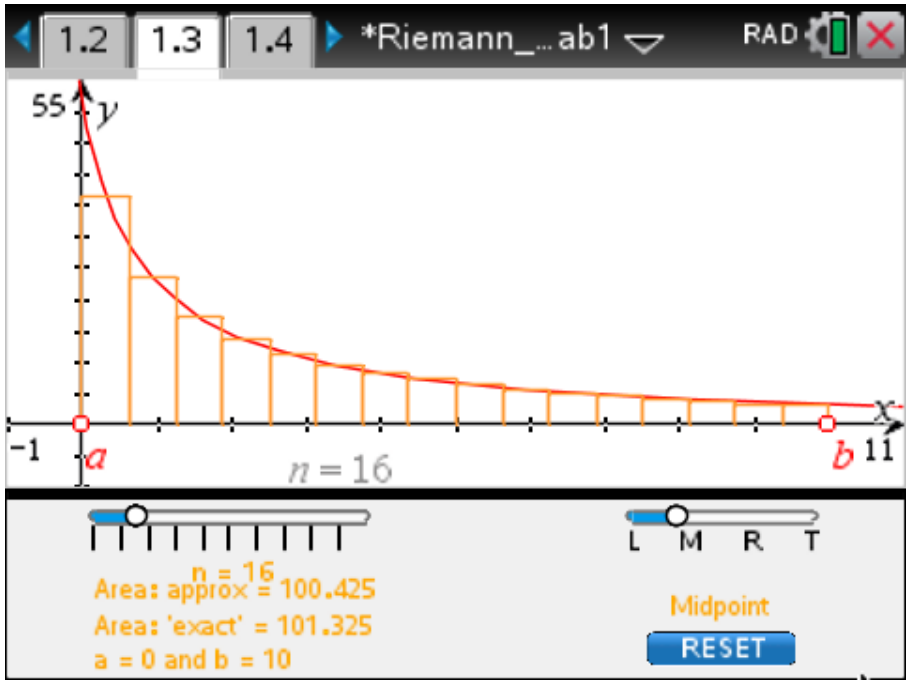
(c) As n increases, the left (right) Riemann sum returns an overestimate (underestimate) of the exact volume. Explain this result. What characteristic of the function f can be used to justify your answer?

(d) Explain why the midpoint sum in this case is a better estimate of the exact volume (rather than the left or right Riemann sum). That is, why is the midpoint sum closer to the exact answer than the left or right Riemann sum for fixed values of n?

(e) Can you suggest another, better sum to estimate the area based on different geometric figures (other than rectangles)? Explain why this method would produce a more accurate estimate of the area.

Riemann_Sums Activity and tns File





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