Type II Error and the Power of a Test
Mike Koehler

A new car has been released for which the manufacturer reports that the car gets 23 mpg for city driving. A consumer group feels that the true mileage is lower and performs the following hypothesis test, using $\alpha = .05$.

Assumptions: The gas mileage in the sample of 30 cars comes from a simple random sample drawn from the population of all mileage and the population standard deviation is known.

H_0: $\mu = 23$
H_a: $\mu < 23$ $\qquad n = 30 \qquad \sigma = 1.2$

Car's actual performance

	Car's mpg = 23	Car's mpg < 23
Reject H_0	Type I error	Correct decision
Fail to reject H_0	Correct decision	Type II error

What is a Type I error in the context of this problem?

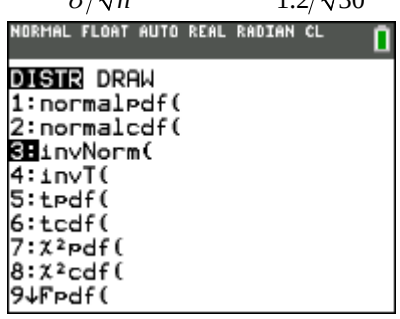
What is a Type II error in the context of this problem?

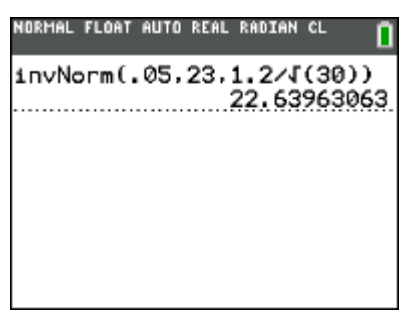
Use the random variable $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ as the basis for deciding whether or not to reject the null hypothesis H_0. The rejection region is the set of all values of the test statistic z that lead us to reject the null hypothesis. In this case, the rejection region consists of all z-values to the left of -1.645.

It is easier to find the probability of Type II errors if the decision criterion is in terms of $\bar{x}$ instead of z.
Find the ***decision value*** in terms of $\bar{x}$ by solving the following equation.

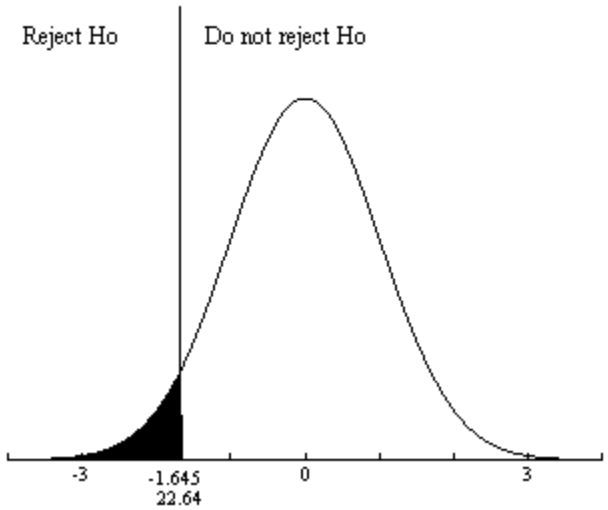
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \Rightarrow -1.645 = \frac{\bar{x} - 23}{1.2/\sqrt{30}}$$

Use inverse normal to find the value of $\bar{x}$.





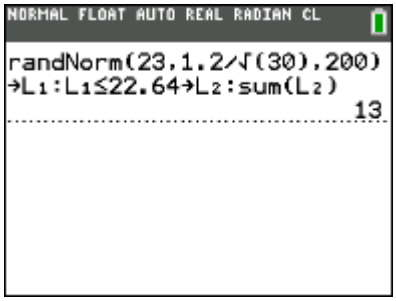
Reject H_0 if $\bar{x} \le 22.64$, otherwise do not reject H_0. See figure below.



Problem 1: Use simulation to determine the probability of a Type I error.

A Type I error occurs if the mean is 23 mpg and H_0 is rejected.
Follow the screen at the right to simulate the occurrence of Type I error.
The distribution of sample means will form a normal distribution with mean 23 and standard deviation $1.2/\sqrt{30}$.
Type I error occurs when the sample mean is less than or equal to 22.64 if the mean is 23.

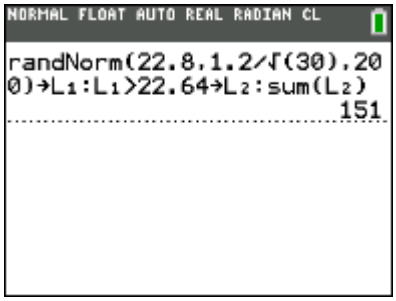
[When the simulation was run for a total of 1000 sample means, H_0 was rejected a total of 52 times.]



Problem 2: Use simulation to determine the probability of a Type II error if the true mean is 22.8.

A Type II error occurs if the mean is 22.8 mpg and H_0 is not rejected.
Assume the new distribution has a standard deviation of 1.2 also.
Follow the screen at the right to simulate the occurrence of a Type II error.
The distribution of sample means will form a normal distribution with mean 22.8 and standard deviation $1.2/\sqrt{30}$.
Type II error occurs when the sample mean is greater than 22.64 if the mean is not 23.

[When the simulation was run for a total of 1000 sample means, the results failed to reject H_0 a total of 765 times.]



© Mike Koehler

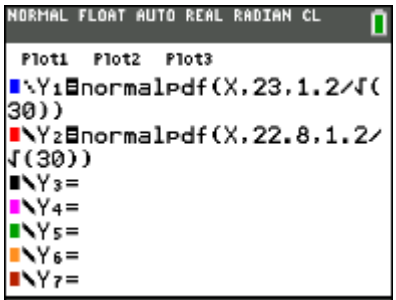
Problem 3: Find the probability β of a Type II error if the true mean is 22.8.

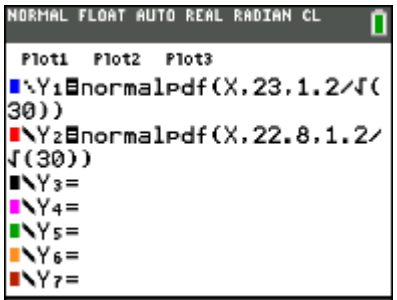
Find the probability of not rejecting H_0 if it is in fact the case that the mean in 22.8.
Probability ($\bar{x}$ > 22.64) given that the distribution is $N\left(22.8, 1.2/\sqrt{30}\right)$.

Draw the normal probability density curve for functions with means of 23 and 22.8
Press [Y=], [2nd][DISTR]
1: normalpdf(.

84+C [WINDOW][21.6,24.24] by [0,2]

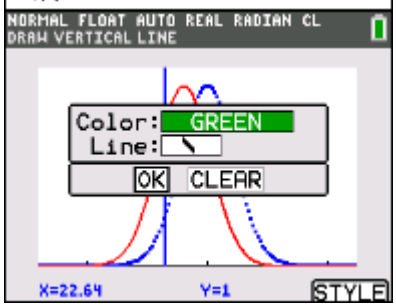
84+ [WINDOW][21.86,23.74] by [-.5,2]

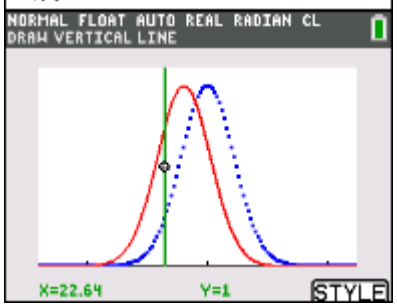




Draw a vertical line at 22.64 using [2nd][DRAW] 4**:Vertical**

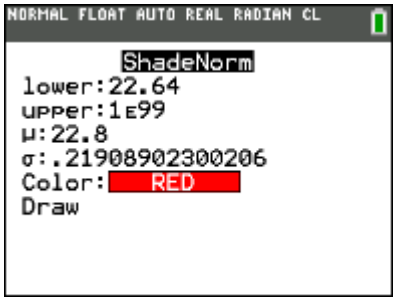
Use **STYLE** (GRAPH) to select a color for the line.

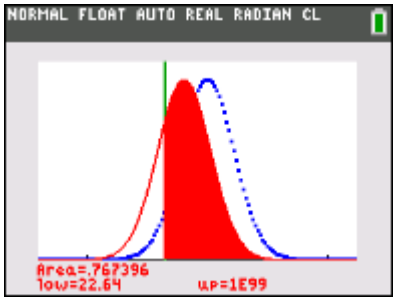




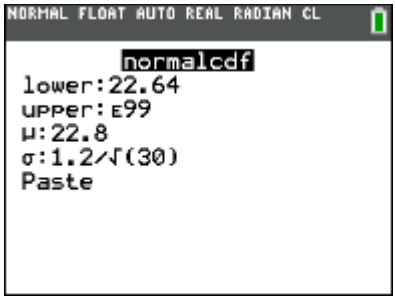
Use **ShadeNorm(** to shade the region where H_0 will not be rejected, that is the area to the right of 22.64, under the assumption that the mean is 22.8.

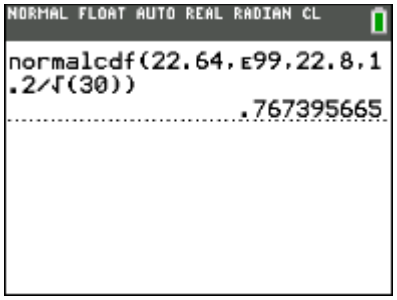
The area of this region is .767.





This value can also be found from the home screen using normalcdf.





The probability of a Type II error is .767. There is a 77% chance that the consumer group will fail to reject the null hypothesis that the average gas mileage is 23 when the true mileage is 22.8.

The theoretical result is close to the result of the simulation.

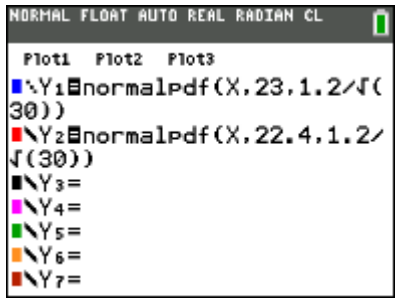
Problem 4: Find the probability β of a Type II error if the true mean is 22.4

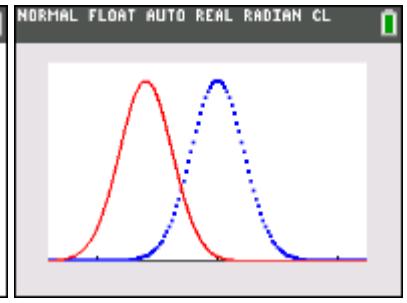
Find the probability of failing to reject H_0 if it is in fact the case that the mean in 22.4.
Probability ($\bar{x}$ > 22.64) given that the distribution is $N\left(22.4, 1.2/\sqrt{30}\right)$.

Draw the normal probability density functions for means of 23 and 22.4

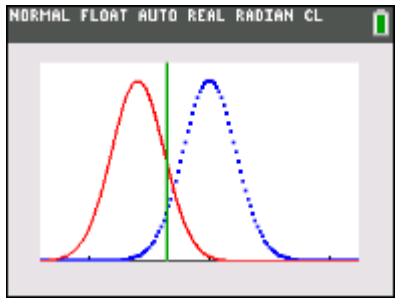
84+C [WINDOW][21.6,24.24] by [0,2]

84+ [WINDOW] [21.86,23.74] by [-.5,2].



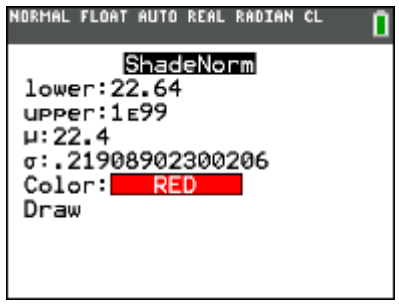


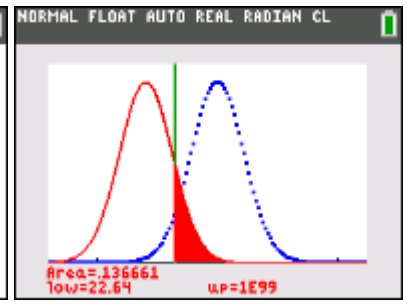
Draw a vertical line at 22.64 using [2nd][DRAW] **4:Vertical** as shown in problem 3.



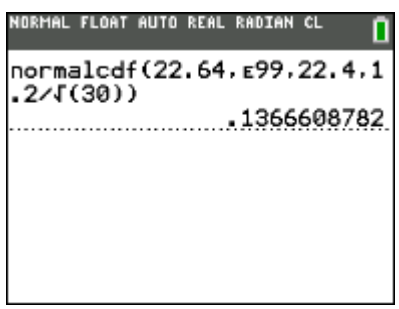
Use **ShadeNorm(** to shade the region where H_0 will not be rejected, that is the area to the right of 22.64, under the assumption that the mean is 22.4.

Probability ($\bar{x}$ > 22.64 | N (22.4,1.2/$\sqrt{30}$)





The area of this region is .1367



The probability of a Type II error is .1367. There is a 13.7% chance that the consumer group will fail to reject the null hypothesis that the average gas mileage is 23 when the true mileage is 22.4.