PAM HARRIS

equipping math teachers with content & pedagogy for student success

@pwharris
www.MathIsFigureOutAble.com
facebook.com/MathIsFigureOutAble
pharris@byu.net

MATH IS FIGUREOUTABLE!

REAL MATH

@pwharris

PAM HARRIS

~~FAKE~~ MATH

MATHEMATIZE

MENTOR MATHEMATICIANS

EXPERIENCE MATHEMATIZING

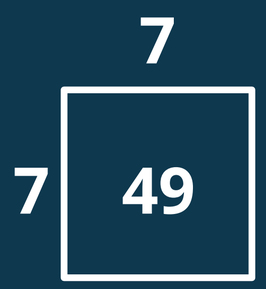
8×7

8×7

7 x 7

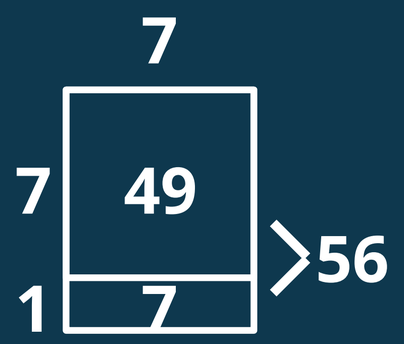
8×7

7 x 7



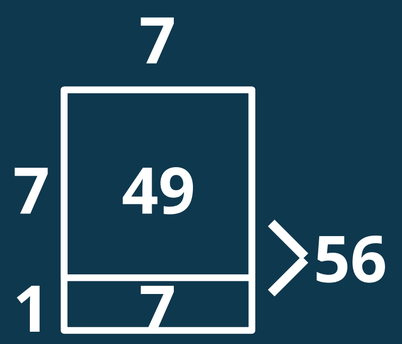
8×7

7 x 7



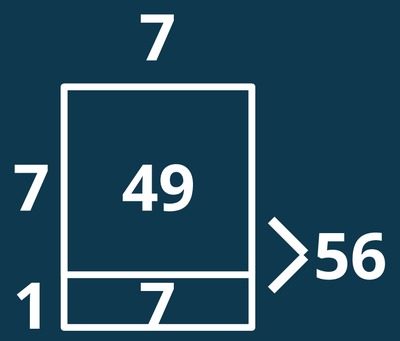
$8 \times 7 = 56$

7 x 7



$8 \times 7 = 56$

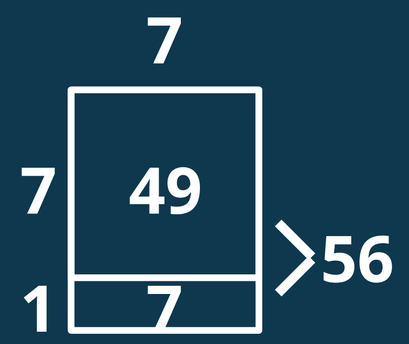
7 x 7



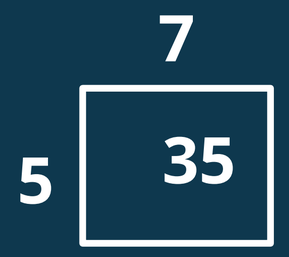
5 x 7

$8 \times 7 = 56$

7 x 7

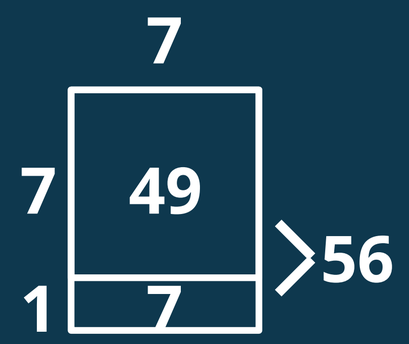


5 x 7

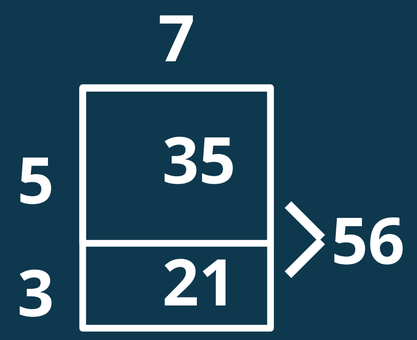


$8 \times 7 = 56$

7 x 7

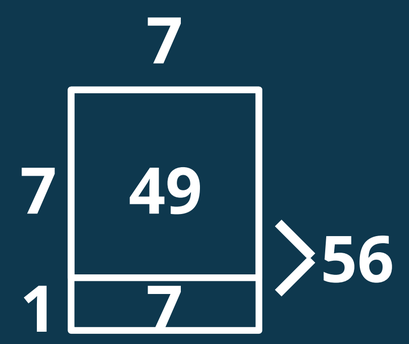


5 x 7



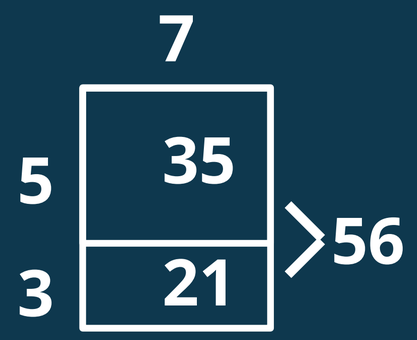
$8 \times 7 = 56$

7 x 7



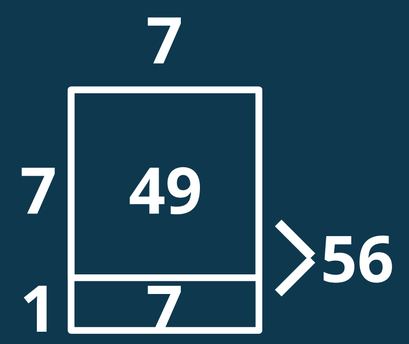
8 x 8

5 x 7

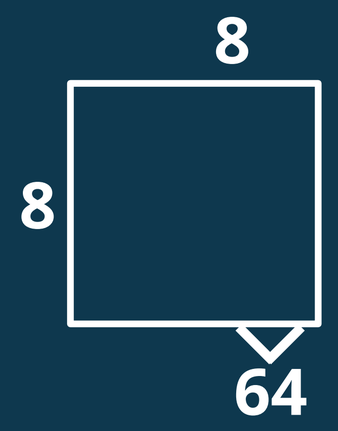


$8 \times 7 = 56$

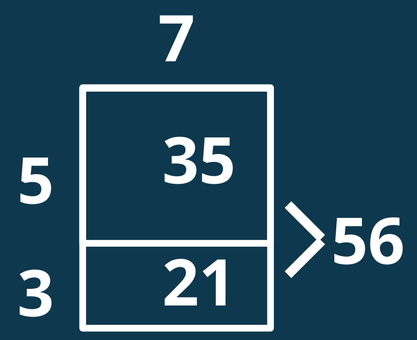
7 x 7



8 x 8

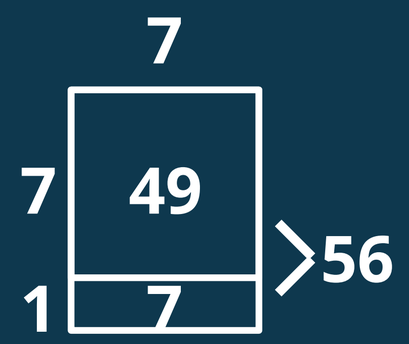


5 x 7

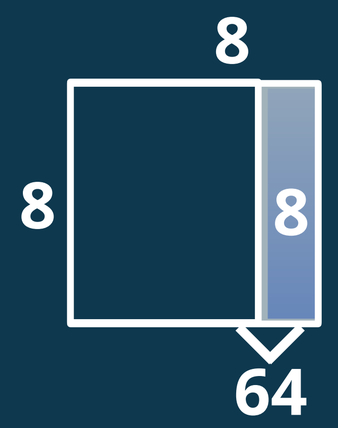


$8 \times 7 = 56$

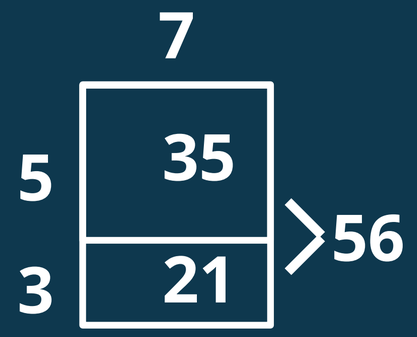
7 x 7



8 x 8

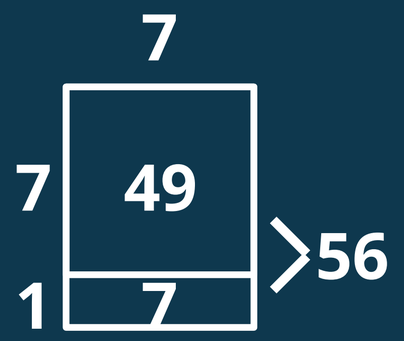


5 x 7

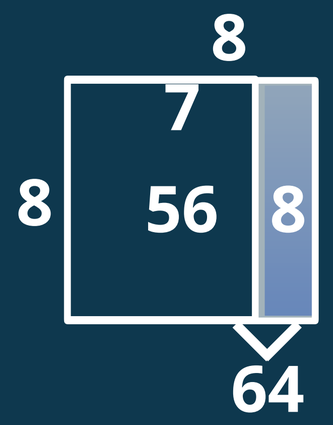


$8 \times 7 = 56$

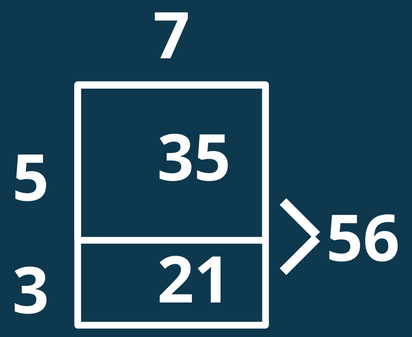
7 x 7



8 x 8

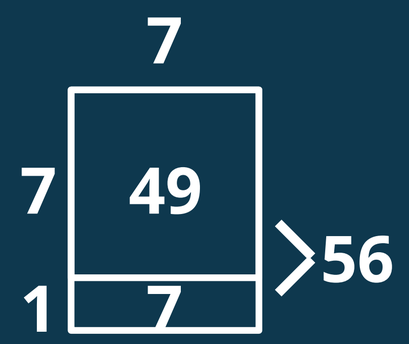


5 x 7

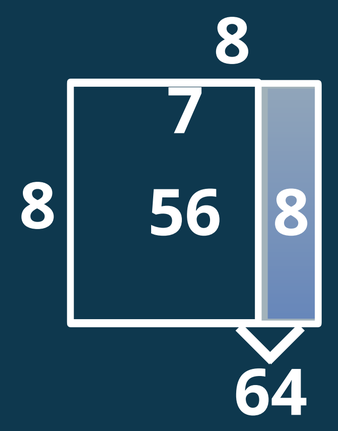


$8 \times 7 = 56$

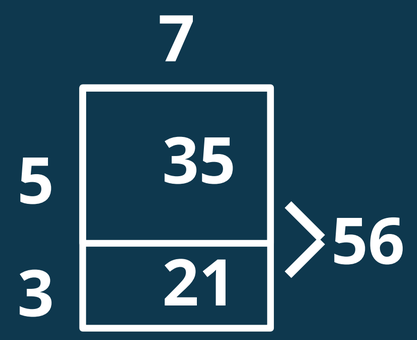
7 x 7



8 x 8



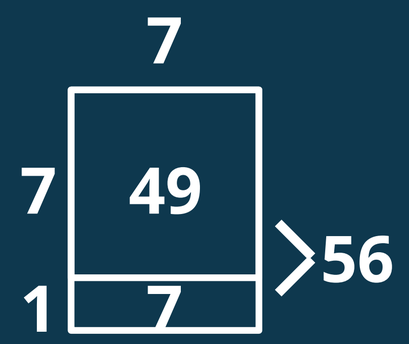
5 x 7



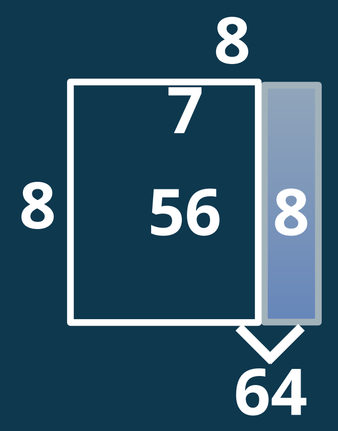
8 x 5

$8 \times 7 = 56$

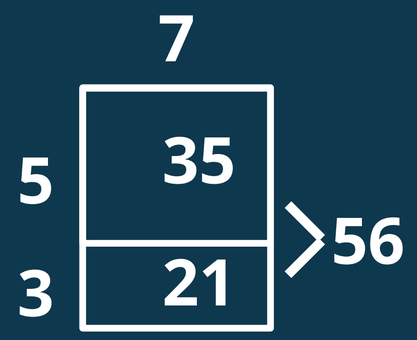
7 x 7



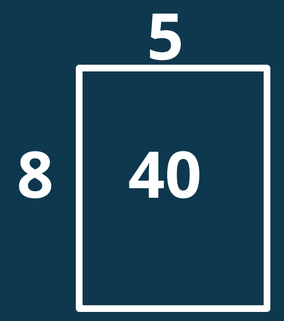
8 x 8



5 x 7

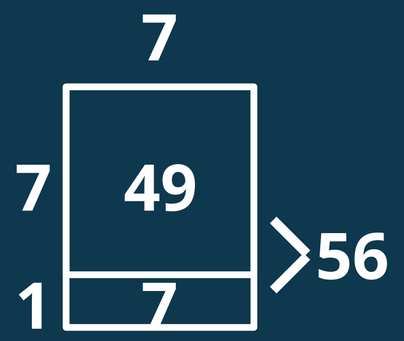


8 x 5

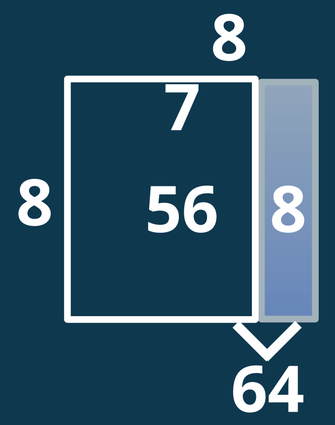


$8 \times 7 = 56$

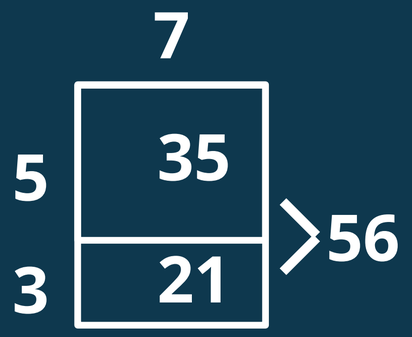
7 x 7



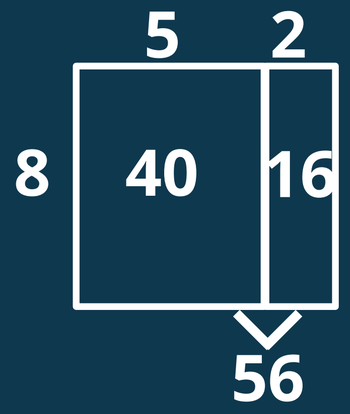
8 x 8



5 x 7



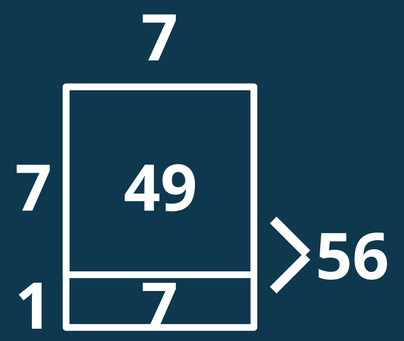
8 x 5



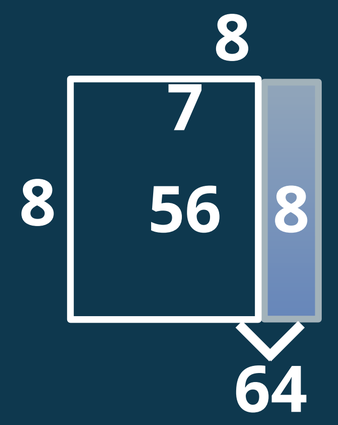
$8 \times 7 = 56$

80×7

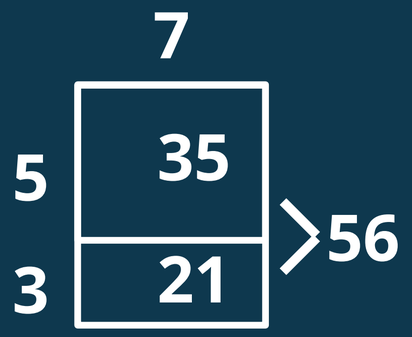
7 x 7



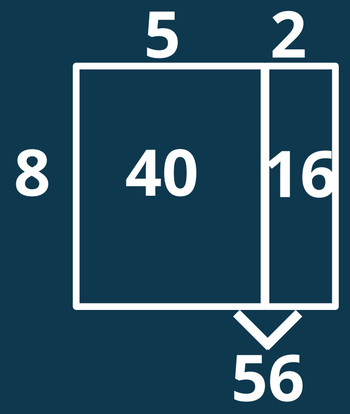
8 x 8



5 x 7



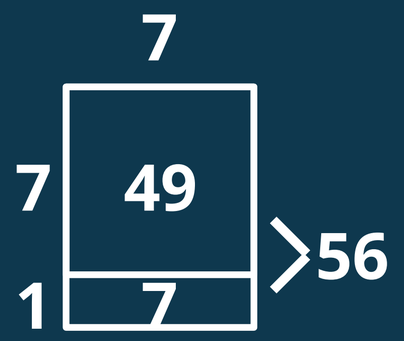
8 x 5



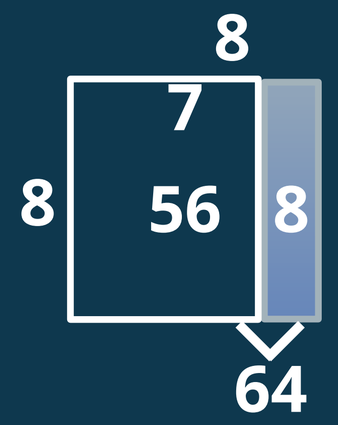
$8 \times 7 = 56$

$80 \times 7 = 560$

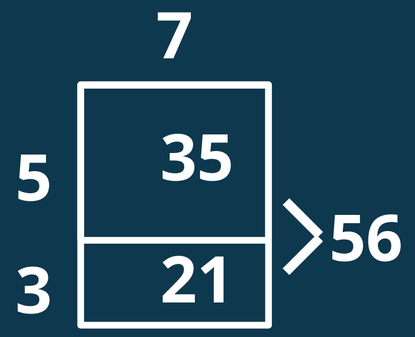
7 x 7



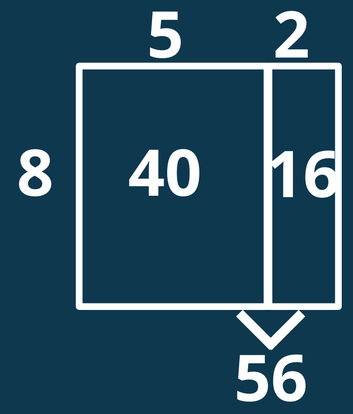
8 x 8



5 x 7



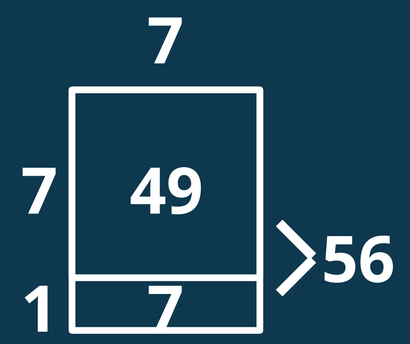
8 x 5



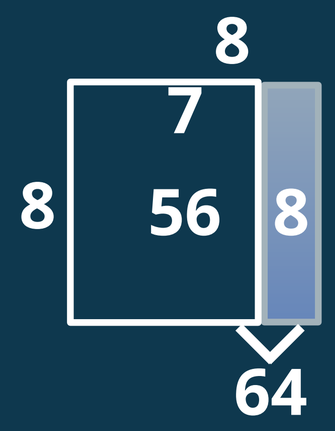
$8 \times 7 = 56$

$80 \times 7 = 560$

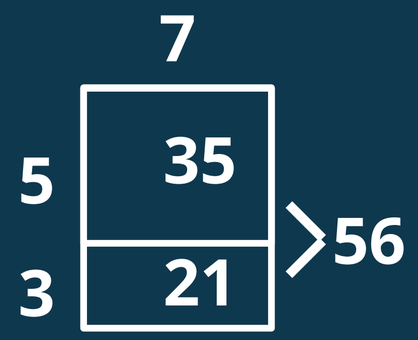
7 x 7



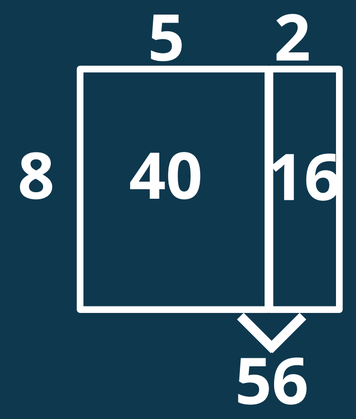
8 x 8



5 x 7



8 x 5

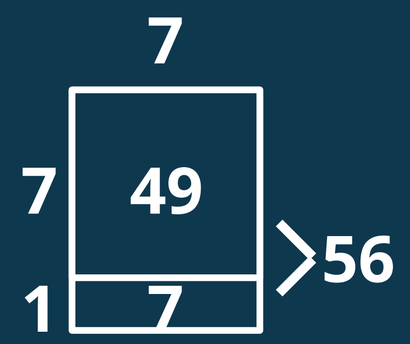


$8 \times 7 = 56$

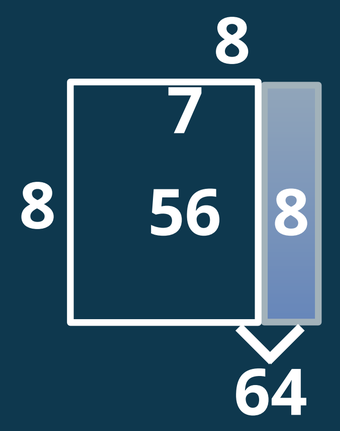
$80 \times 7 = 560$

79×7

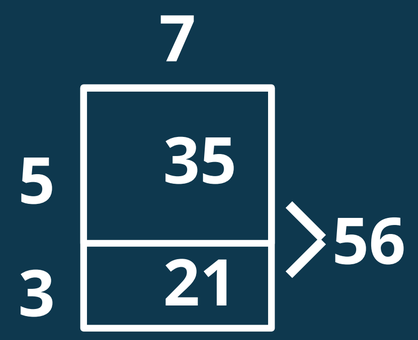
7 x 7



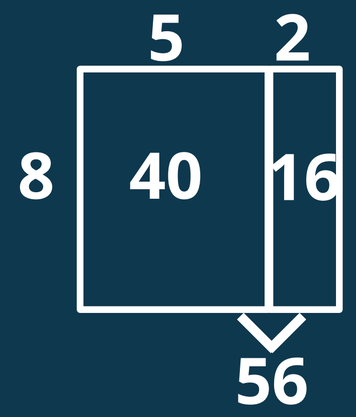
8 x 8



5 x 7



8 x 5

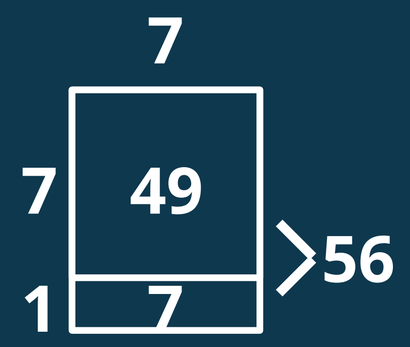


$8 \times 7 = 56$

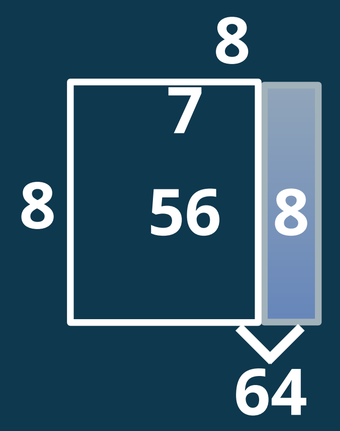
$80 \times 7 = 560$

$79 \times 7 = 553$

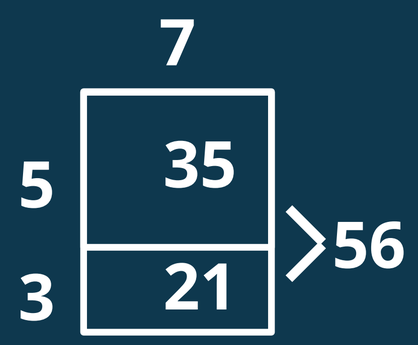
7 x 7



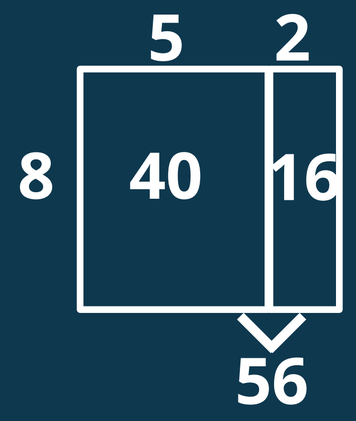
8 x 8



5 x 7



8 x 5



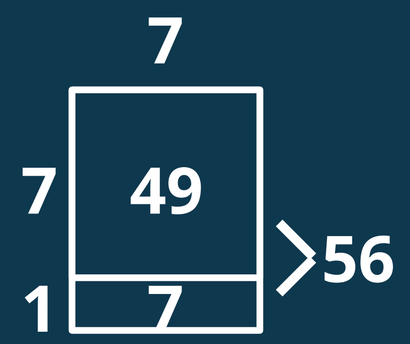
$8 \times 7 = 56$

$80 \times 7 = 560$

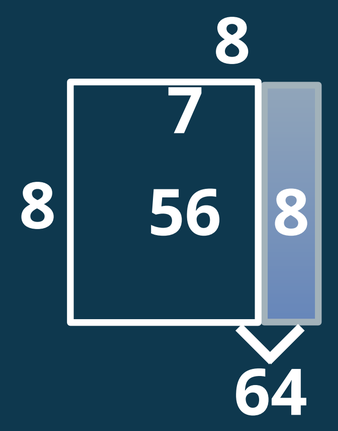
$79 \times 7 = 553$

6×7

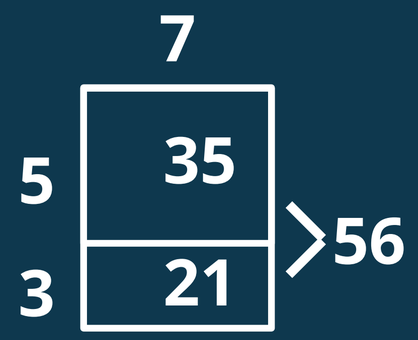
7 x 7



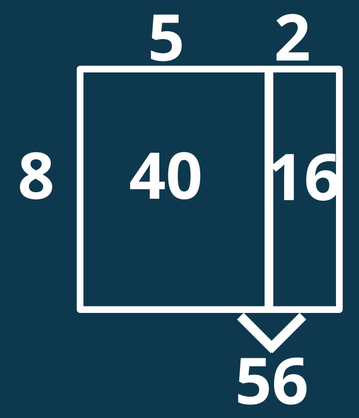
8 x 8



5 x 7



8 x 5



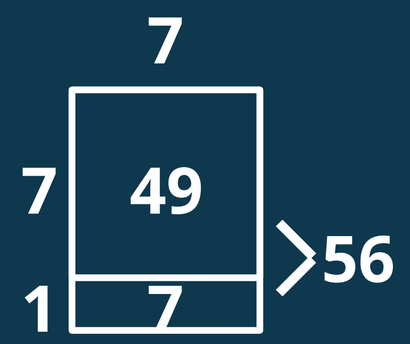
$8 \times 7 = 56$

$80 \times 7 = 560$

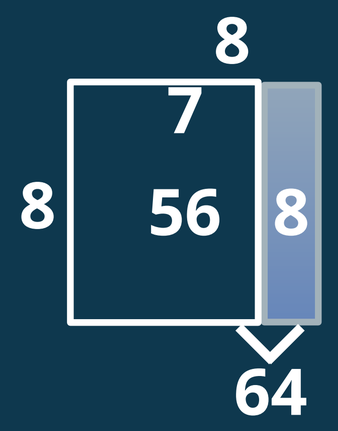
$79 \times 7 = 553$

$6 \times 7 = 42$

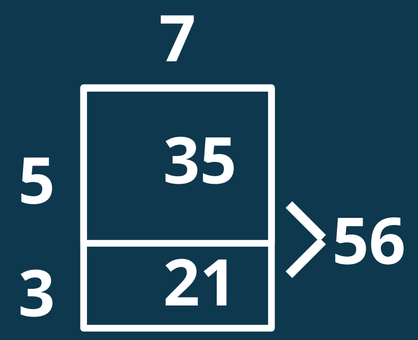
7 x 7



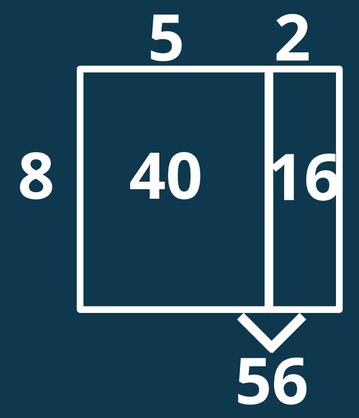
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

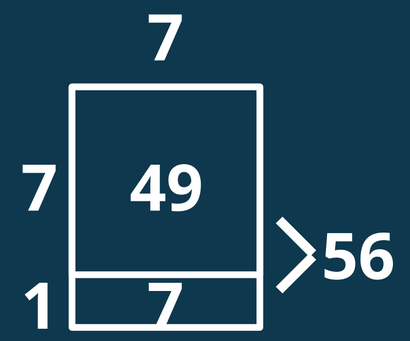
$80 \times 7 = 560$

$79 \times 7 = 553$

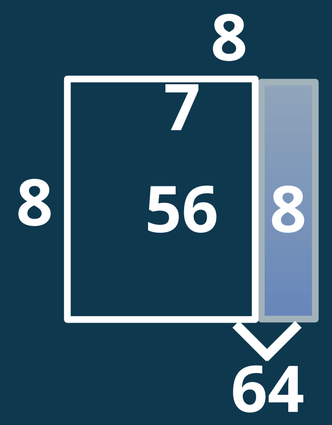
$6 \times 7 = 42$

6×70

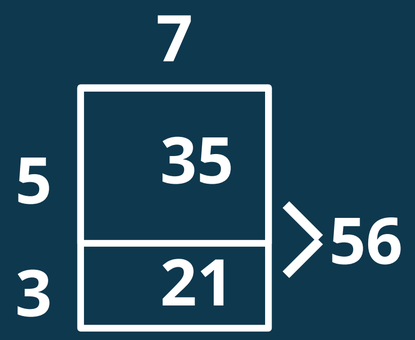
7 x 7



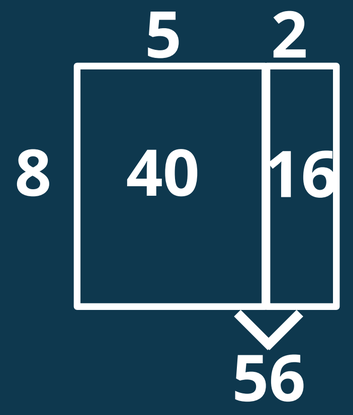
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

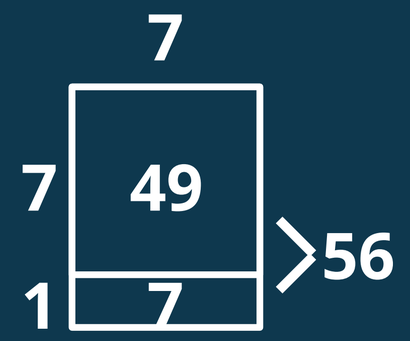
$80 \times 7 = 560$

$79 \times 7 = 553$

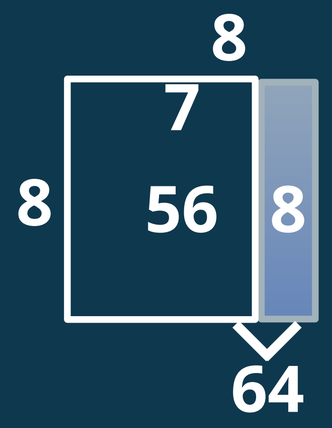
$6 \times 7 = 42$

$6 \times 70 = 420$

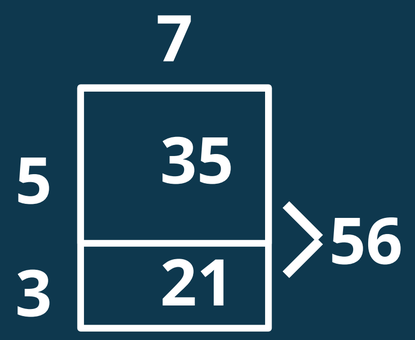
7 x 7



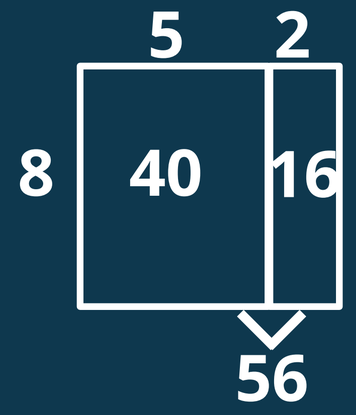
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

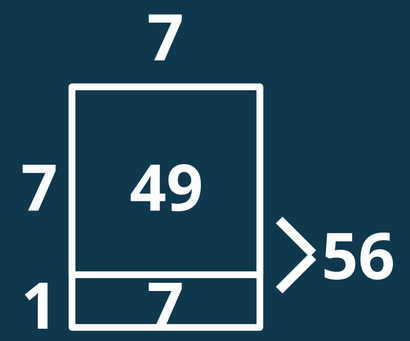
$79 \times 7 = 553$

$6 \times 7 = 42$

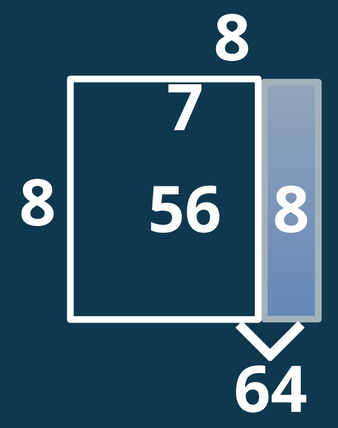
$6 \times 70 = 420$

6×69

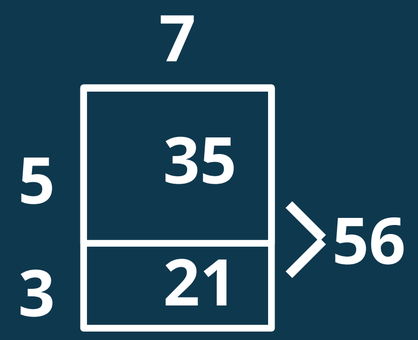
7 x 7



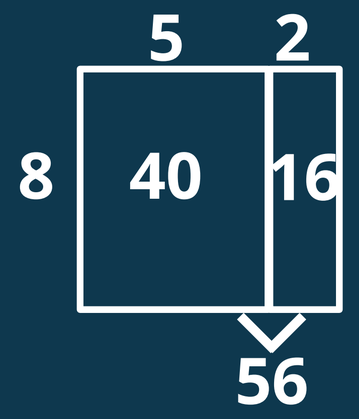
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

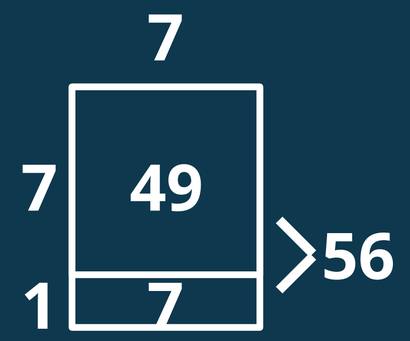
$79 \times 7 = 553$

$6 \times 7 = 42$

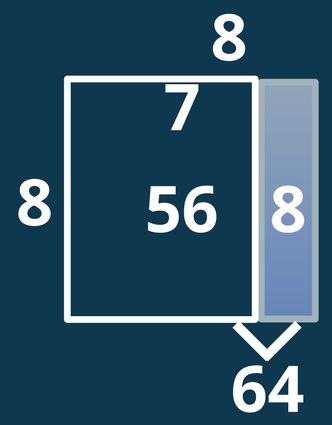
$6 \times 70 = 420$

$6 \times 69 = 414$

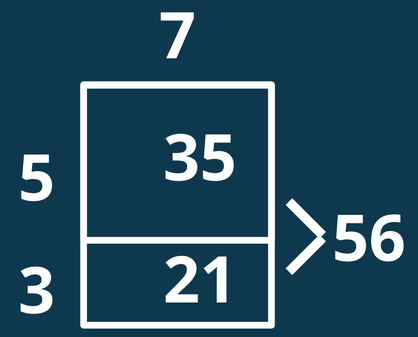
7 x 7



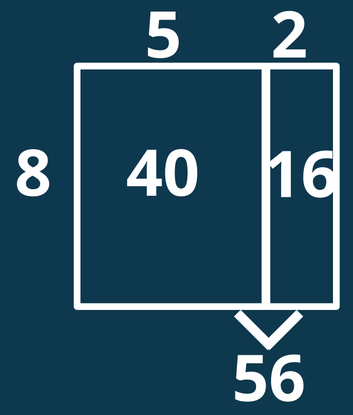
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

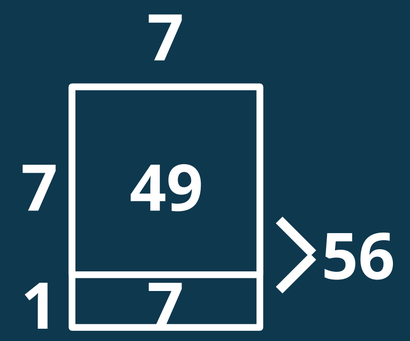
$79 \times 7 = 553$

$6 \times 7 = 42$

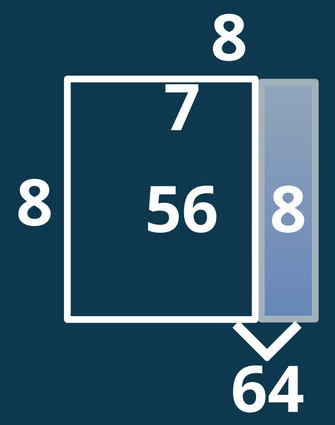
$6 \times 70 = 420$

$6 \times 69 = 414$

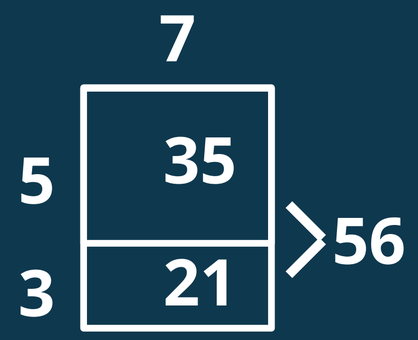
7 x 7



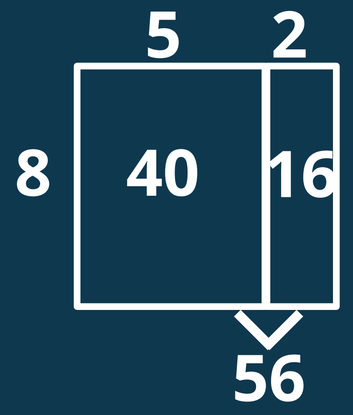
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

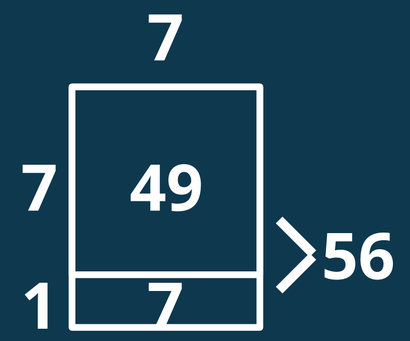
$79 \times 7 = 553$

$6 \times 7 = 42$

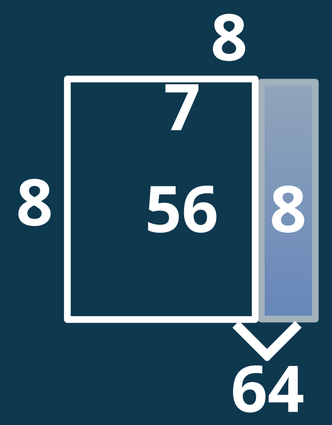
$6 \times 70 = 420$

$6 \times 69 = 414$

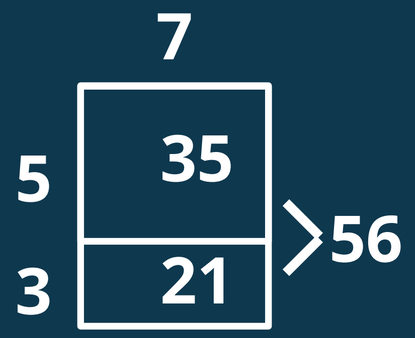
7 x 7



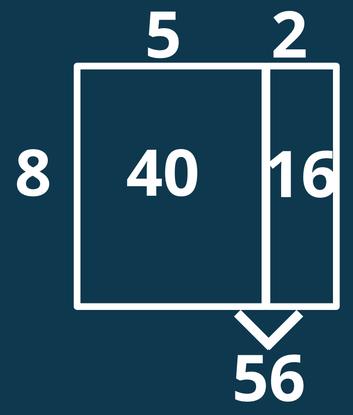
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

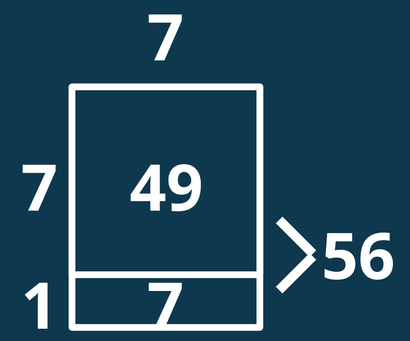
$79 \times 7 = 553$

$6 \times 7 = 42$

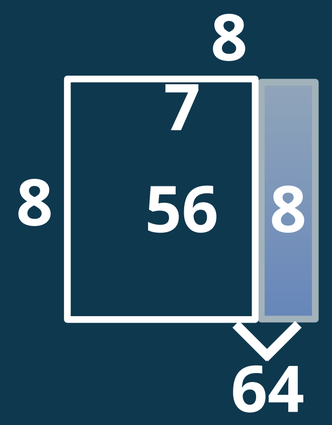
$6 \times 70 = 420$

$6 \times 69 = 414$

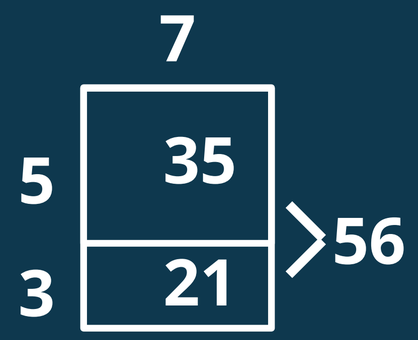
7 x 7



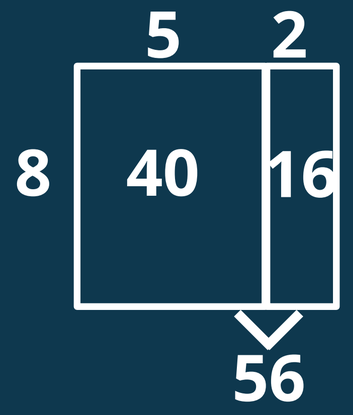
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

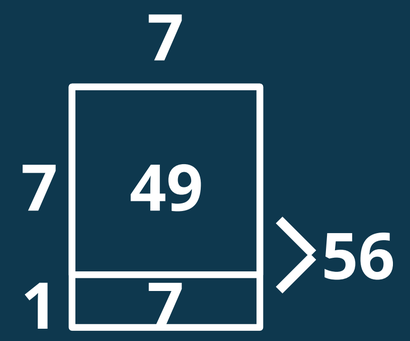
$79 \times 7 = 553$

$6 \times 7 = 42$

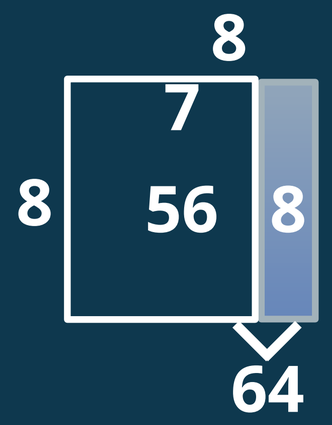
$6 \times 70 = 420$

$6 \times 69 = 414$

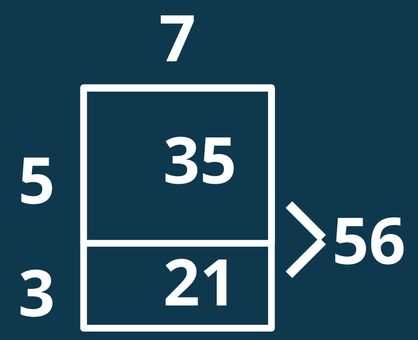
7 x 7



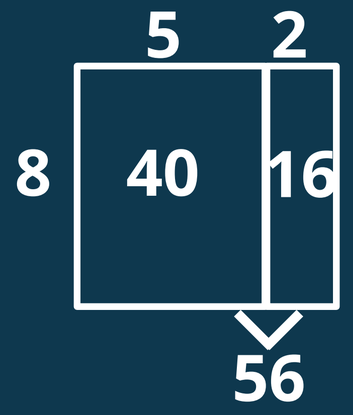
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

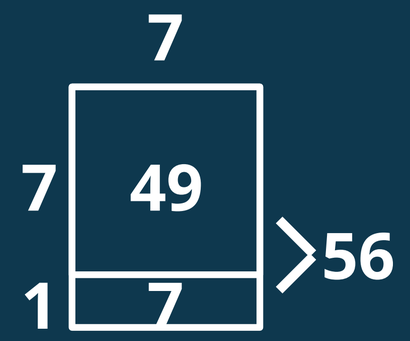
$79 \times 7 = 553$

$6 \times 7 = 42$

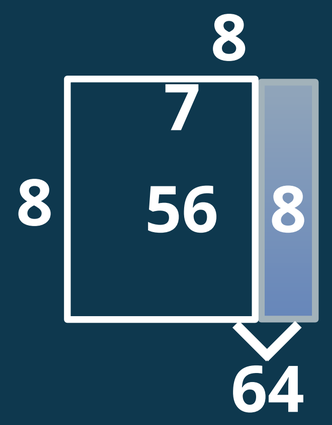
$6 \times 70 = 420$

$6 \times 69 = 414$

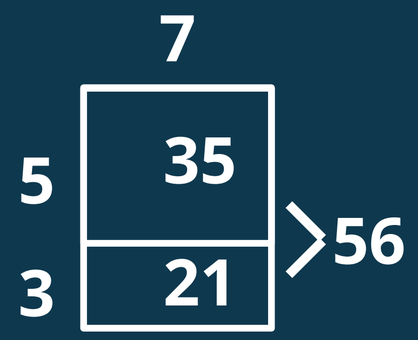
7 x 7



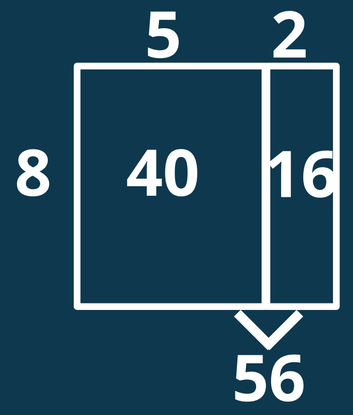
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

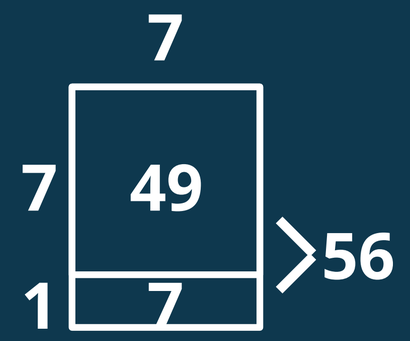
$79 \times 7 = 553$

$6 \times 7 = 42$

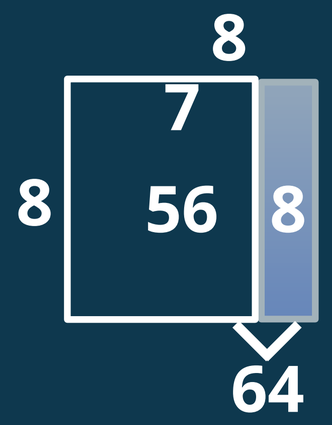
$6 \times 70 = 420$

$6 \times 69 = 414$

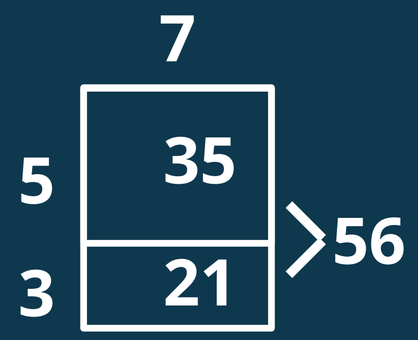
7 x 7



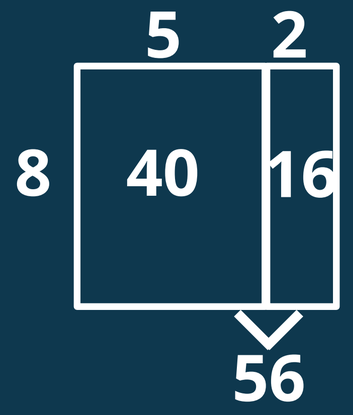
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

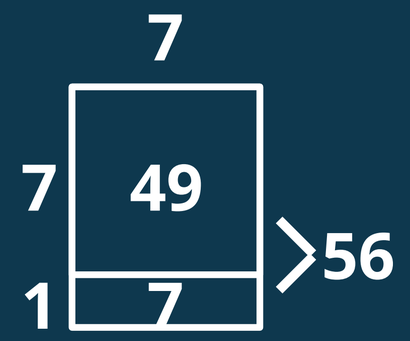
$79 \times 7 = 553$

$6 \times 7 = 42$

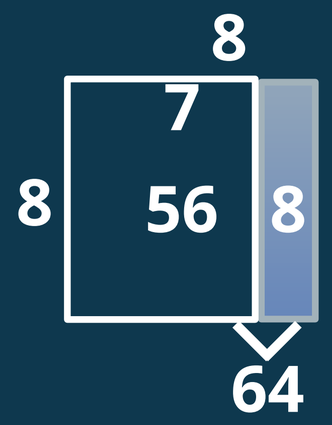
$6 \times 70 = 420$

$6 \times 69 = 414$

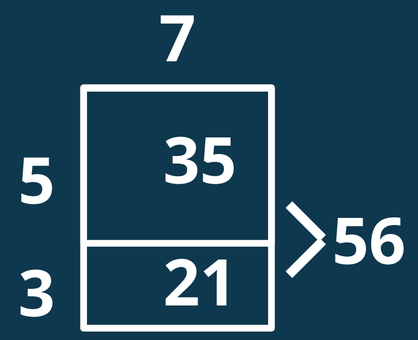
7 x 7



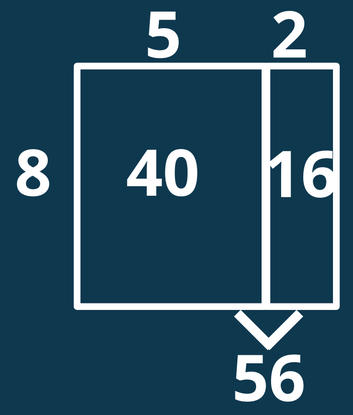
8 x 8



5 x 7



8 x 5



8×49

$8 \times 7 = 56$

$80 \times 7 = 560$

$79 \times 7 = 553$

$6 \times 7 = 42$

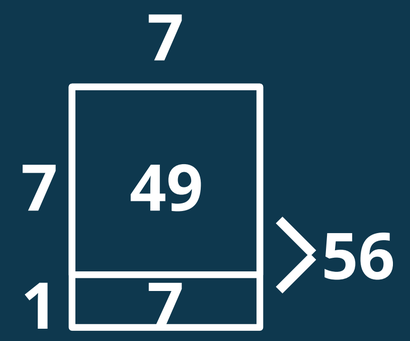
$6 \times 70 = 420$

$6 \times 69 = 414$

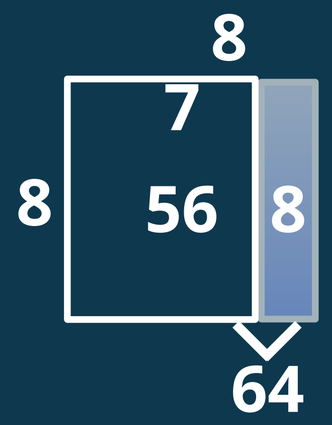
$8 \times 5 = 40$

8×49

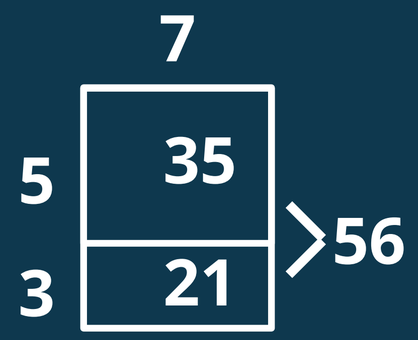
7 x 7



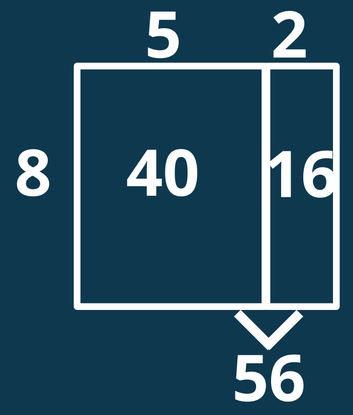
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

$79 \times 7 = 553$

$6 \times 7 = 42$

$6 \times 70 = 420$

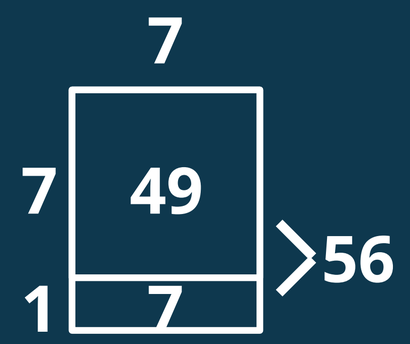
$6 \times 69 = 414$

$8 \times 5 = 40$

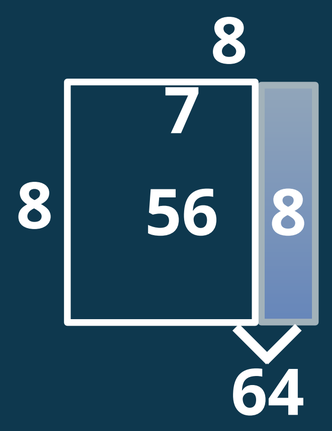
$8 \times 50 = 400$

8×49

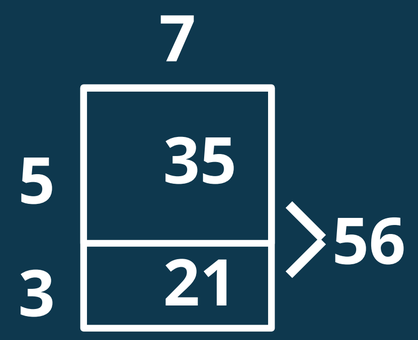
7 x 7



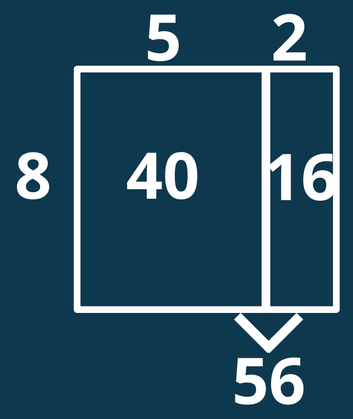
8 x 8



5 x 7



8 x 5



$8 \times 7 = 56$

$80 \times 7 = 560$

$79 \times 7 = 553$

$6 \times 7 = 42$

$6 \times 70 = 420$

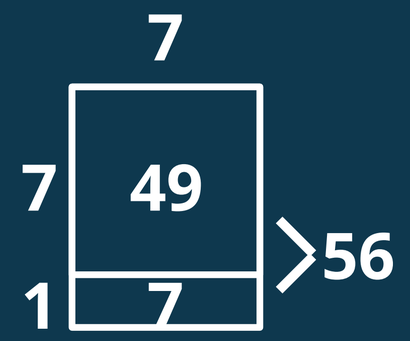
$6 \times 69 = 414$

$8 \times 5 = 40$

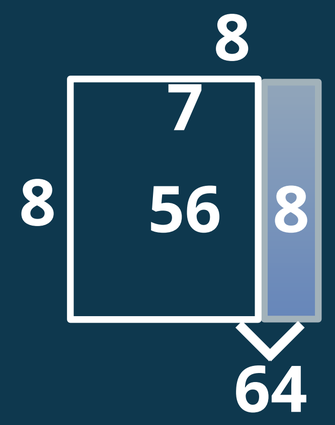
$8 \times 50 = 400$

$8 \times 49 = 392$

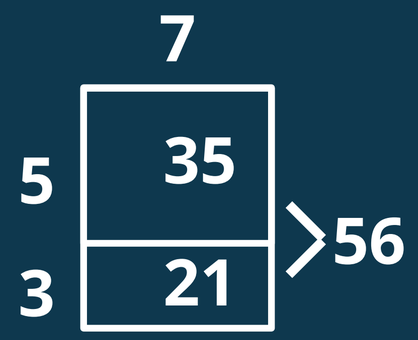
7 x 7



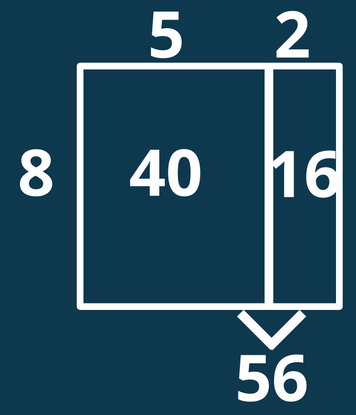
8 x 8



5 x 7

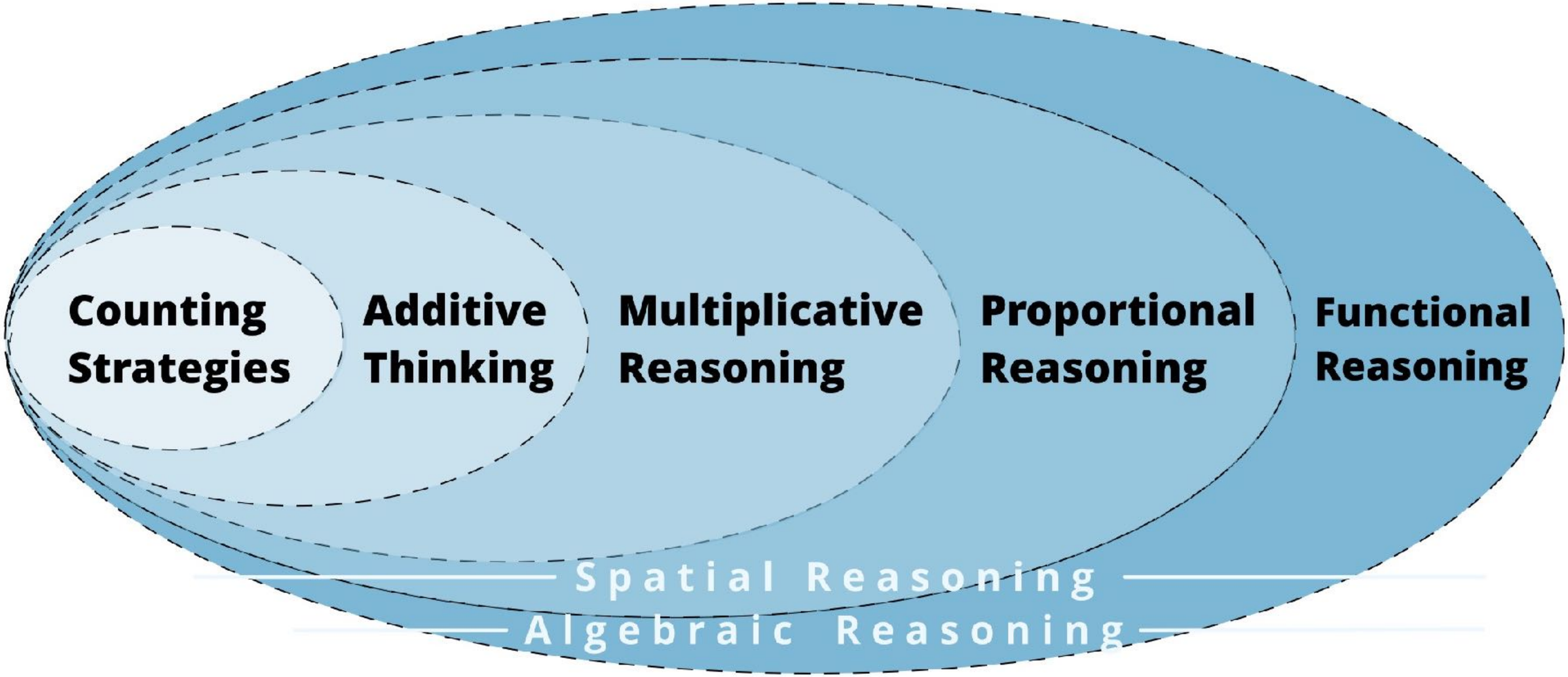


8 x 5



MATHEMATIZE

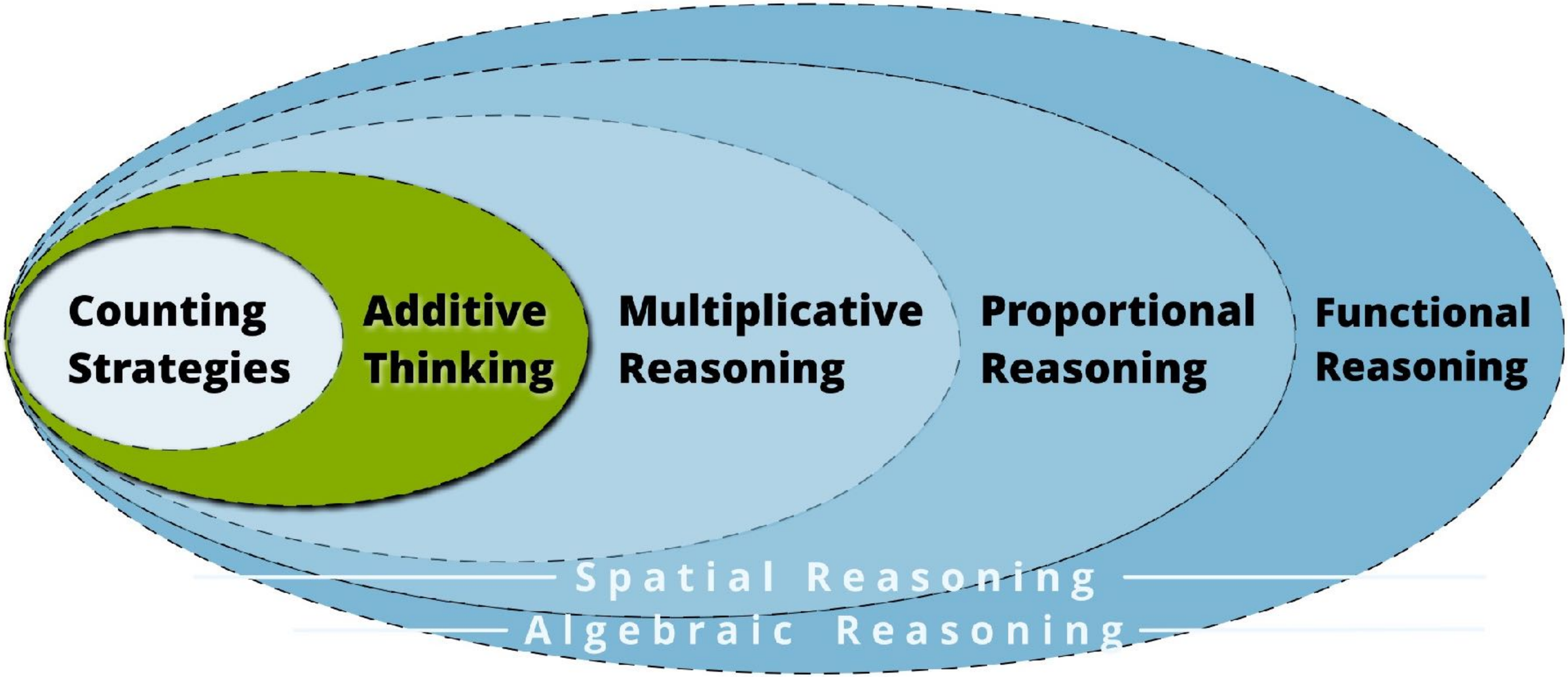
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

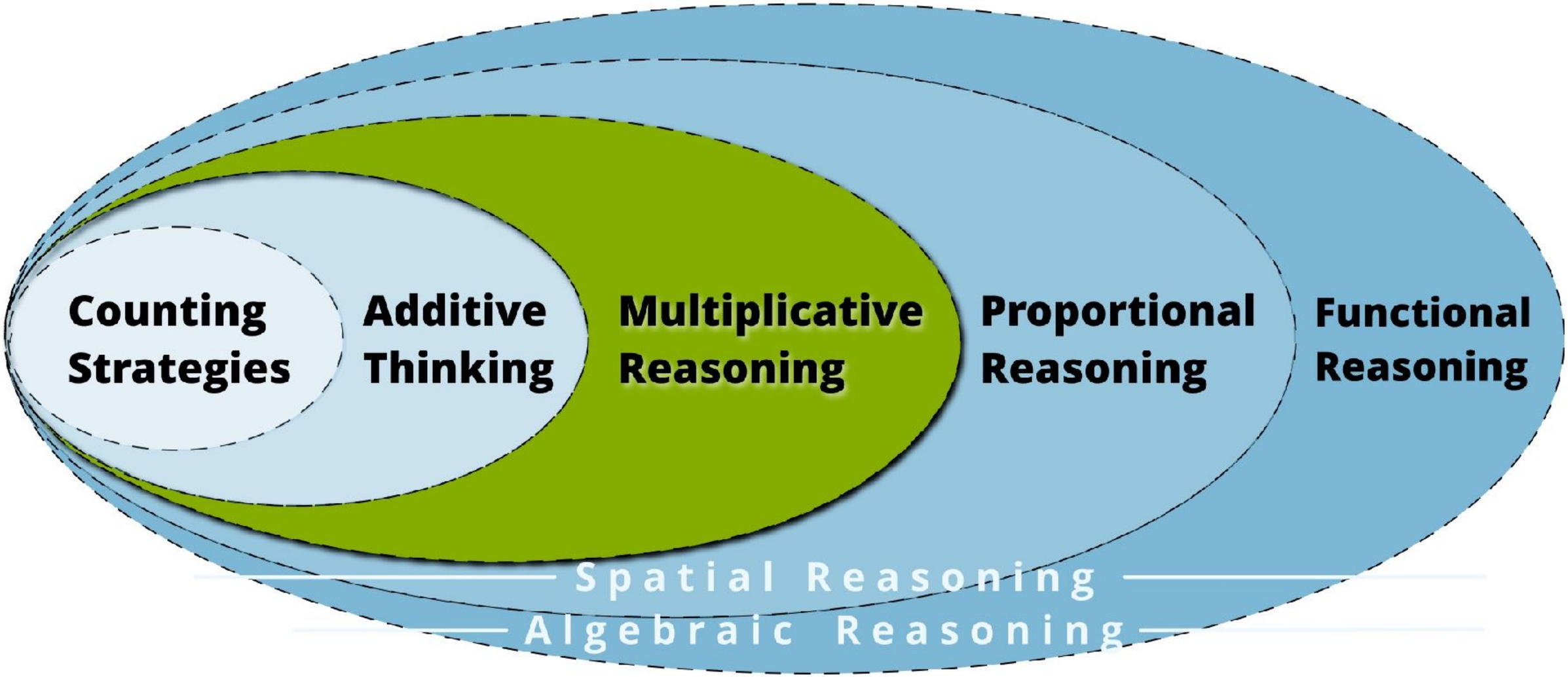
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

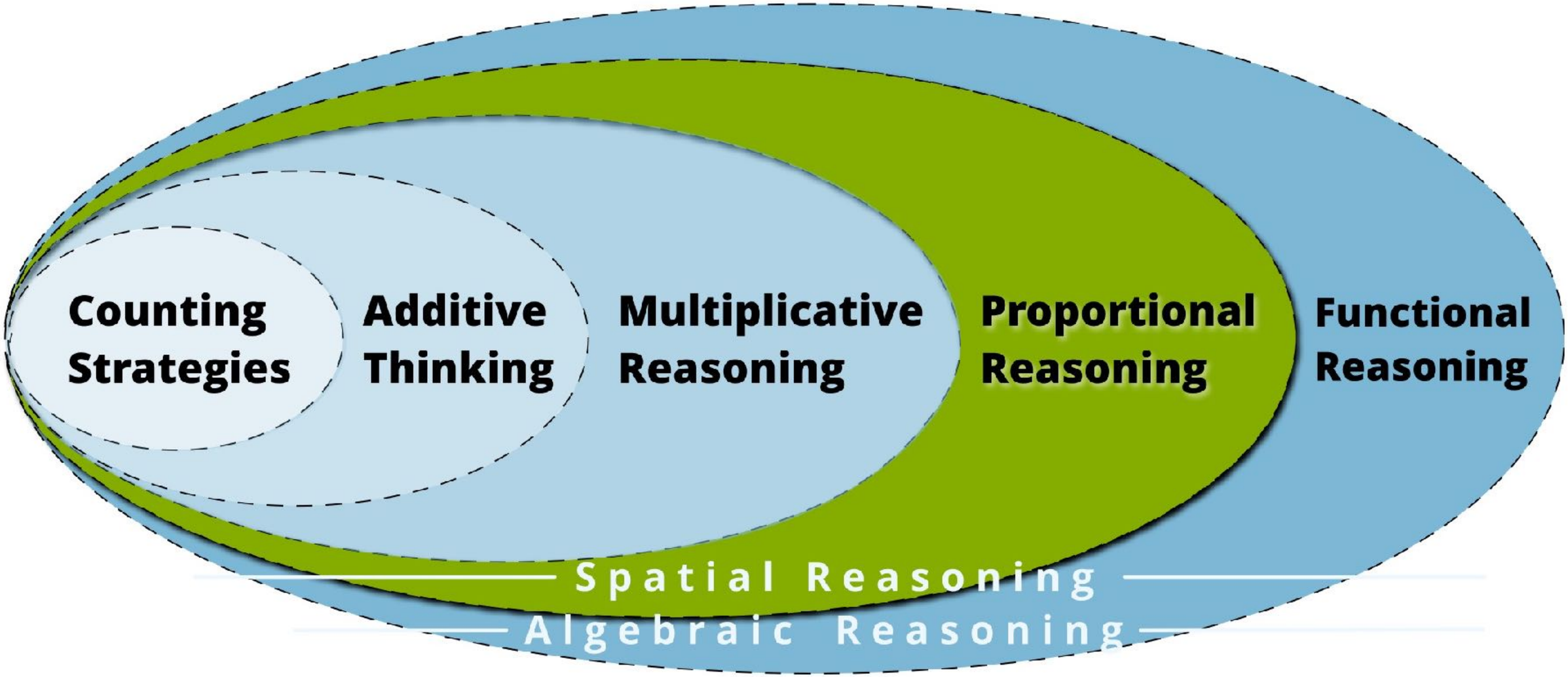
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

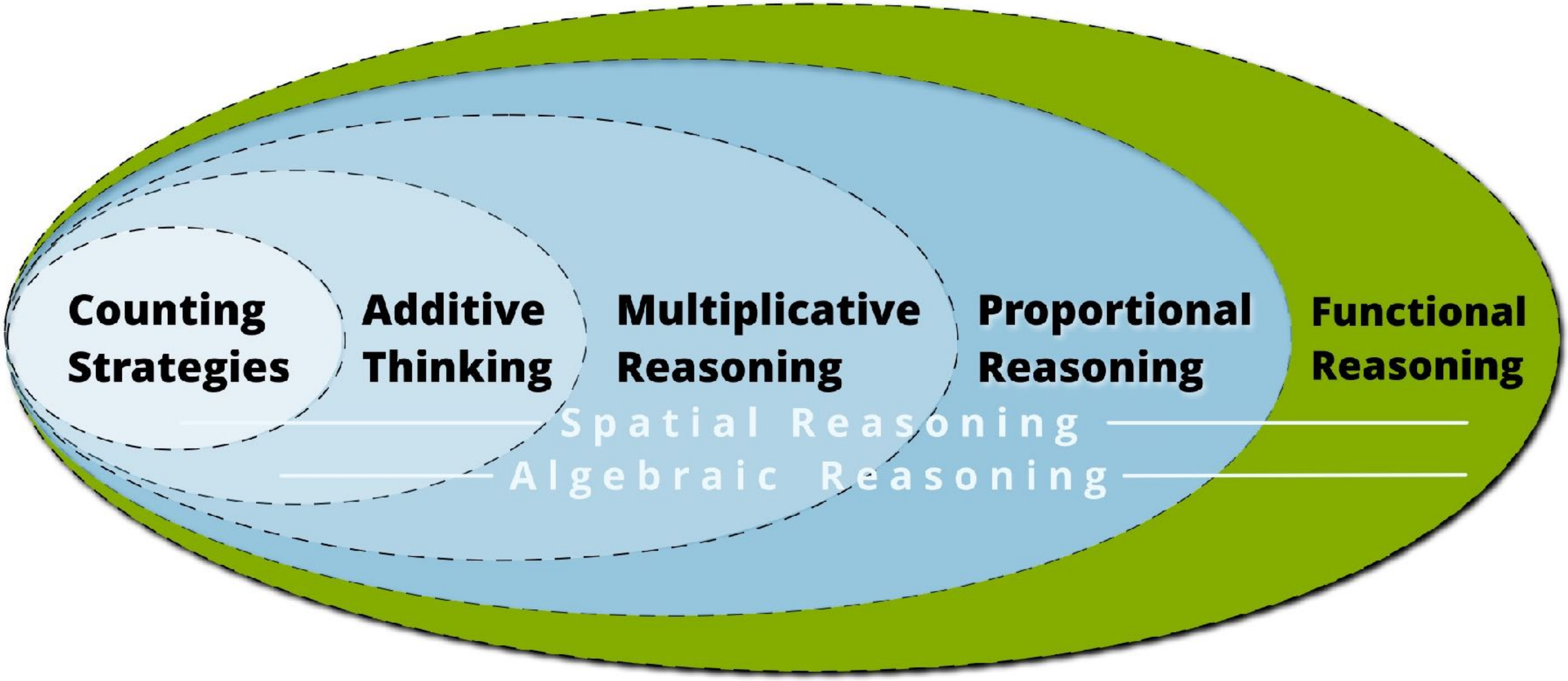
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

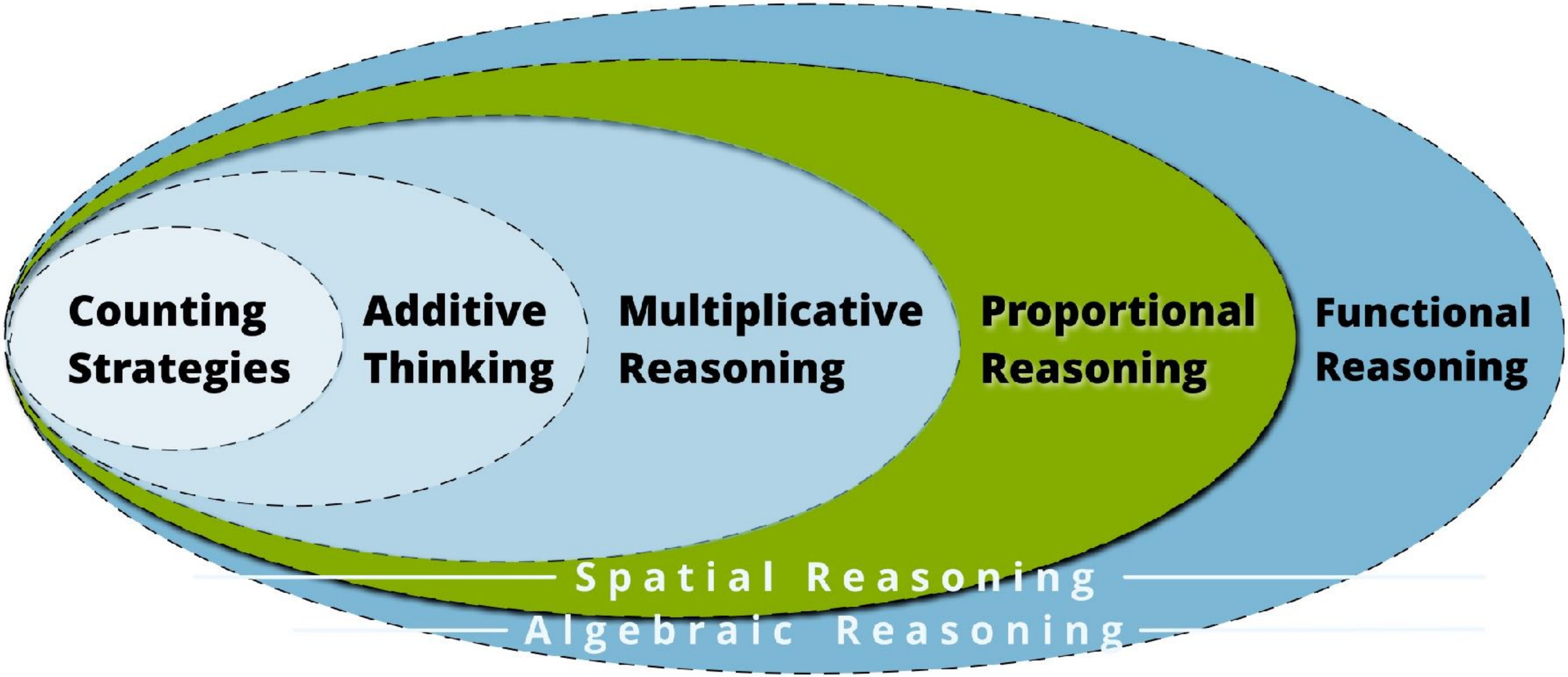
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

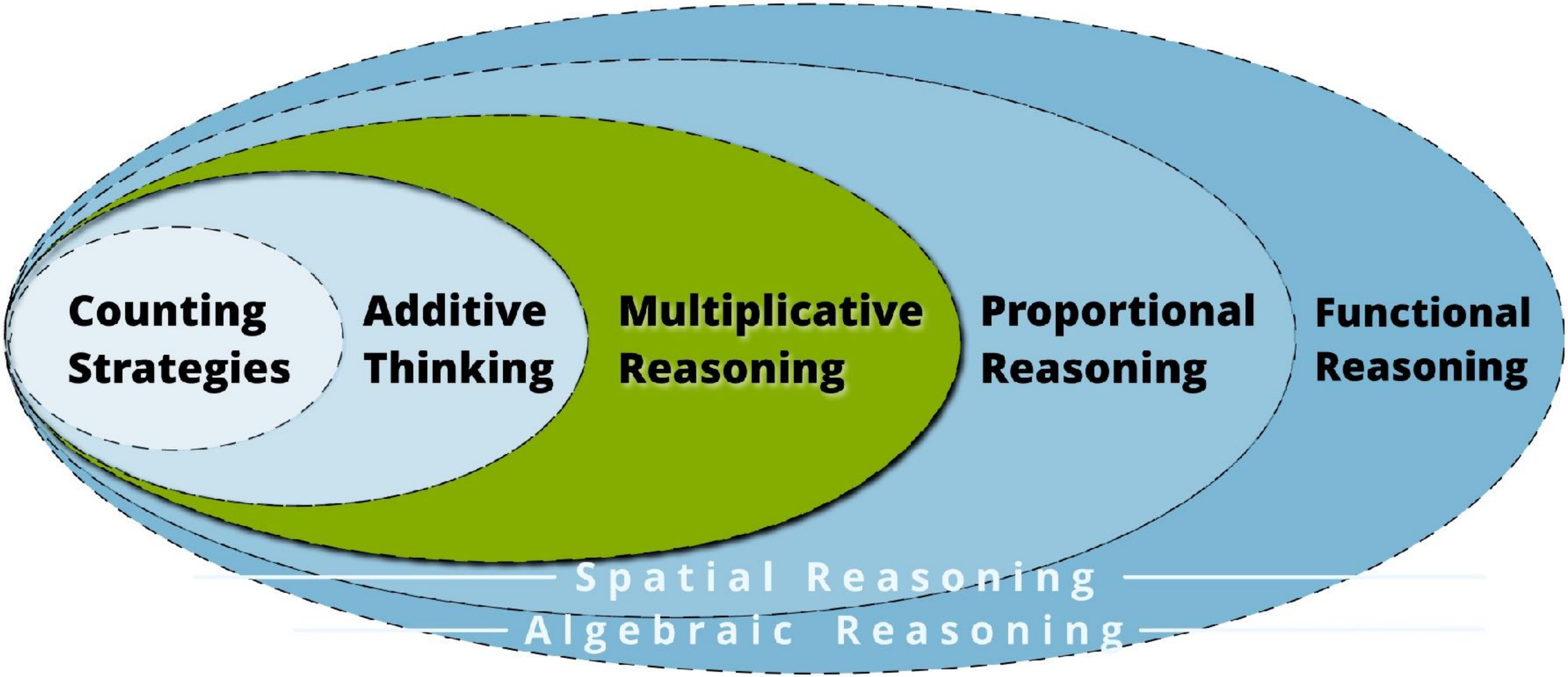
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

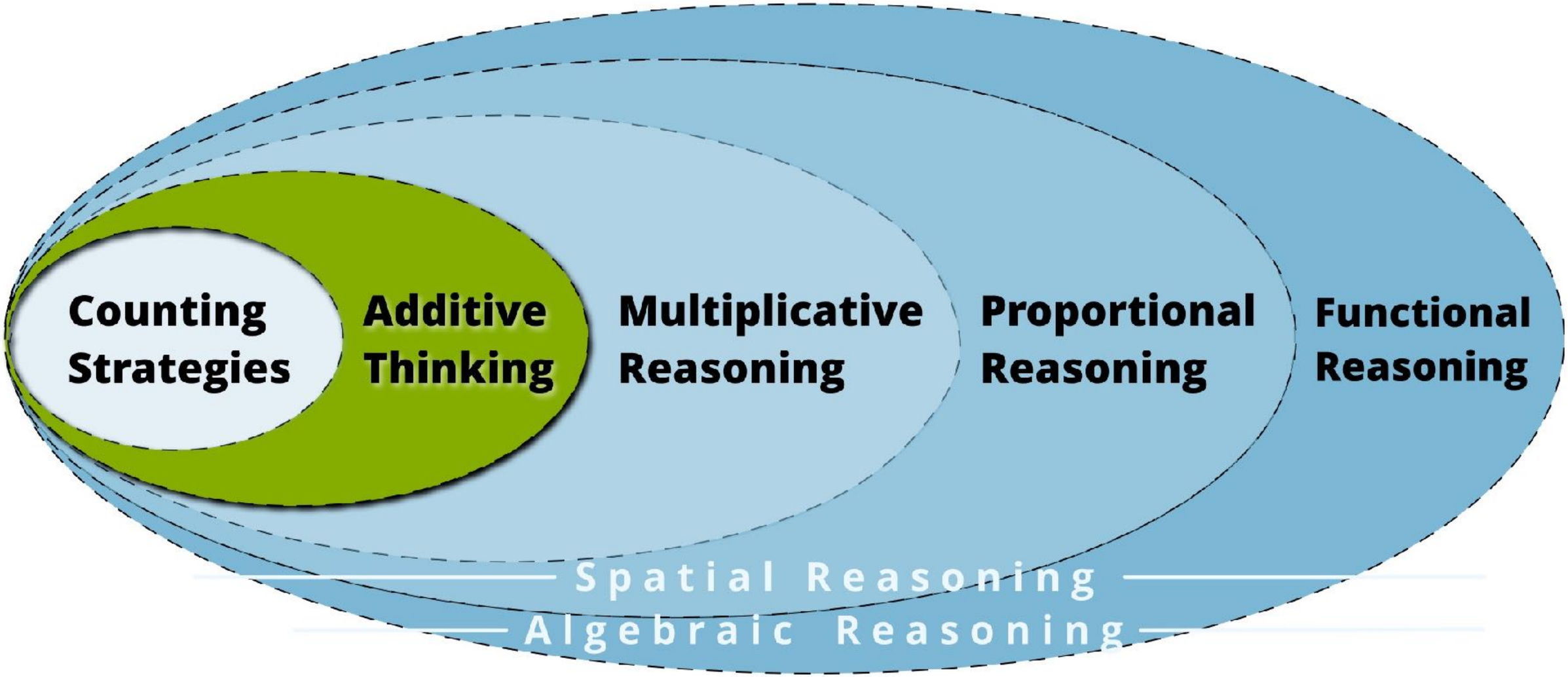
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

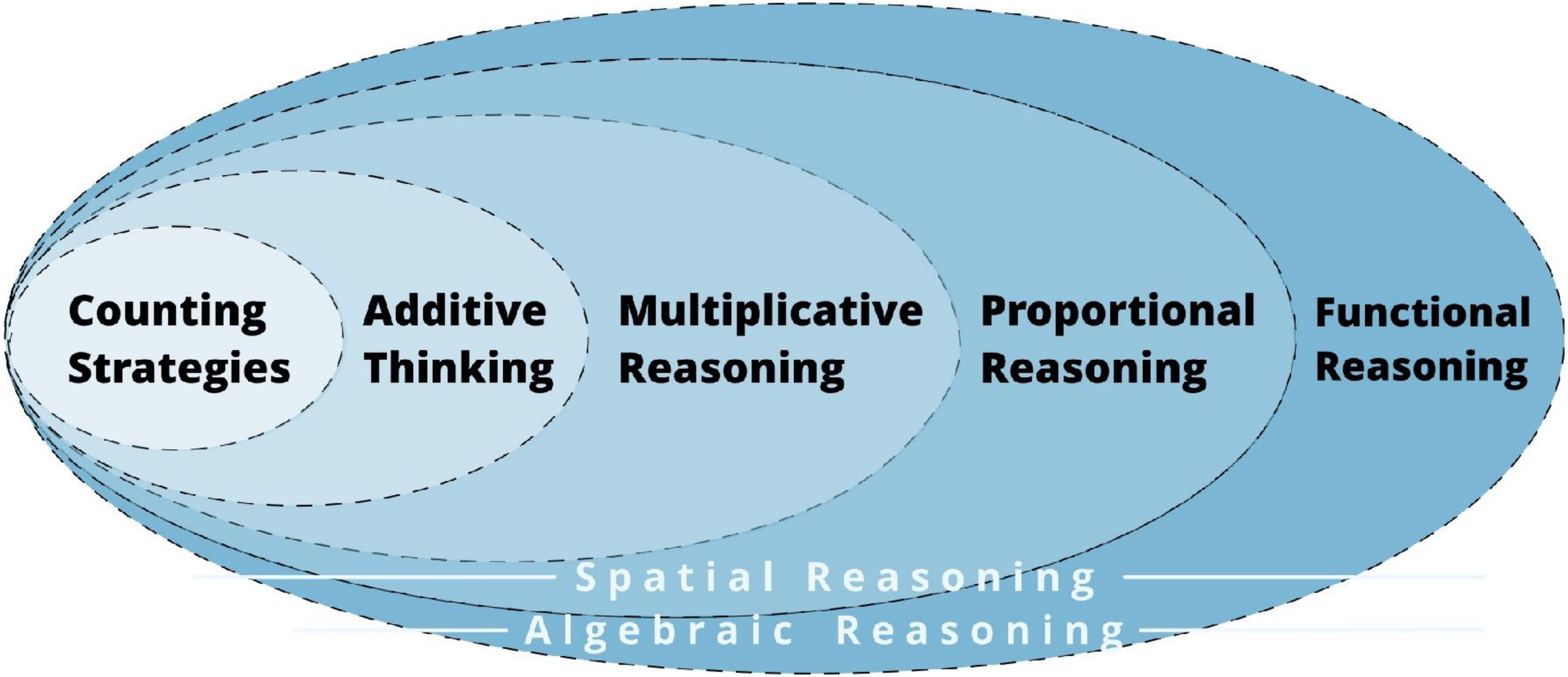
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

48 + 5

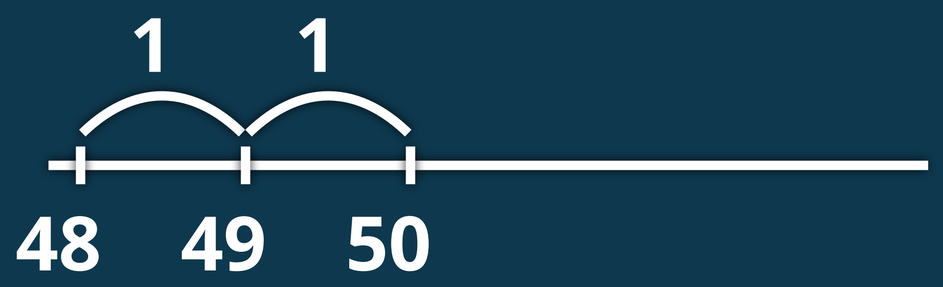
48 + 5

48 + 5

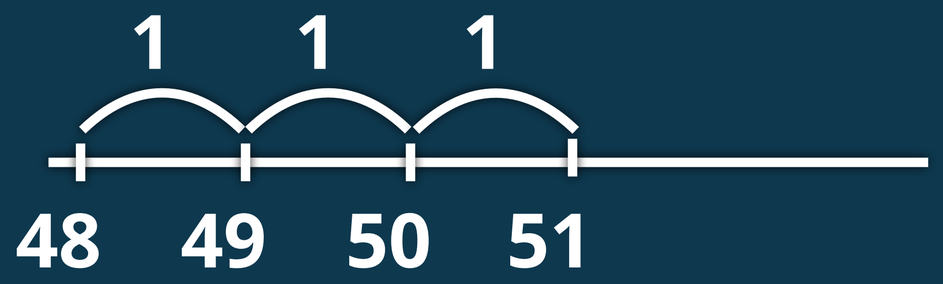
48 + 5

48 + 5

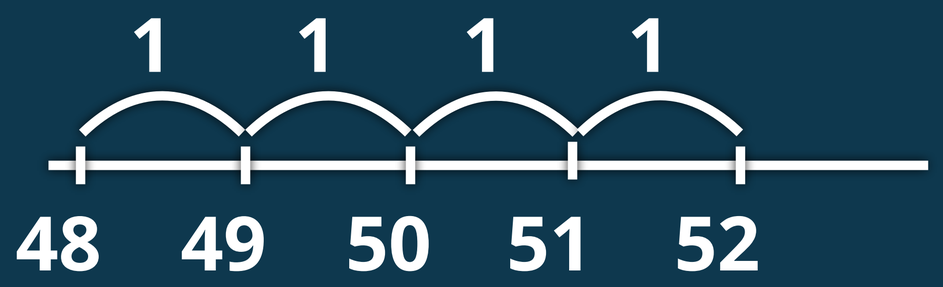
48 + 5



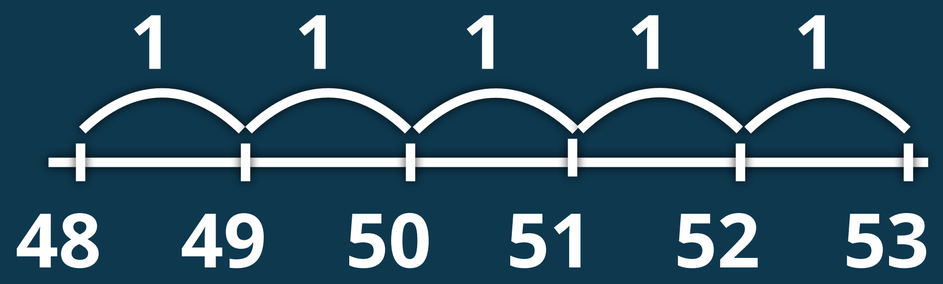
48 + 5



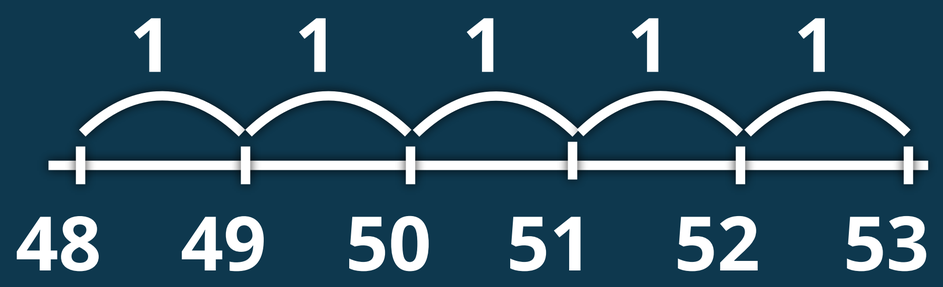
48 + 5



48 + 5

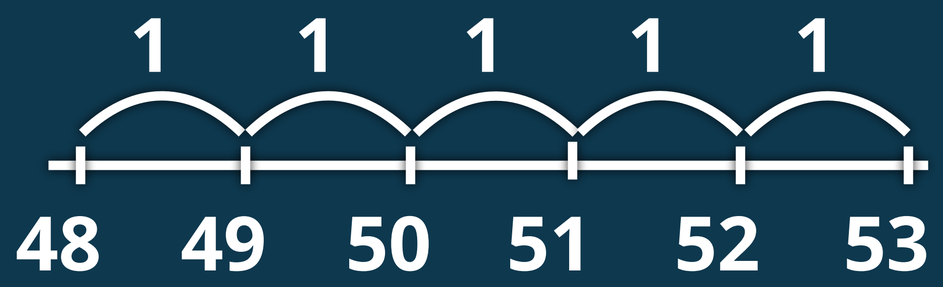


48 + 5



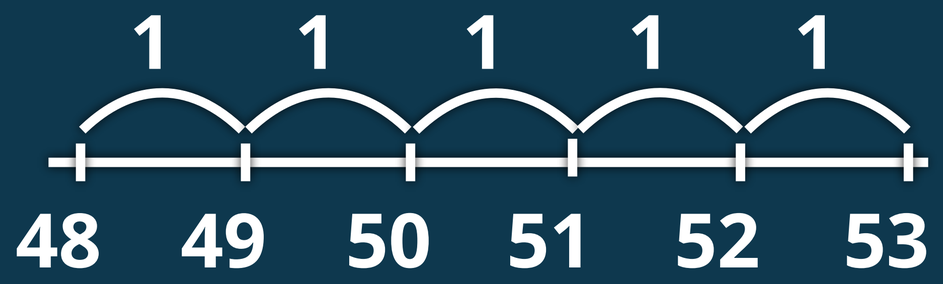
counting strategy

48 + 5



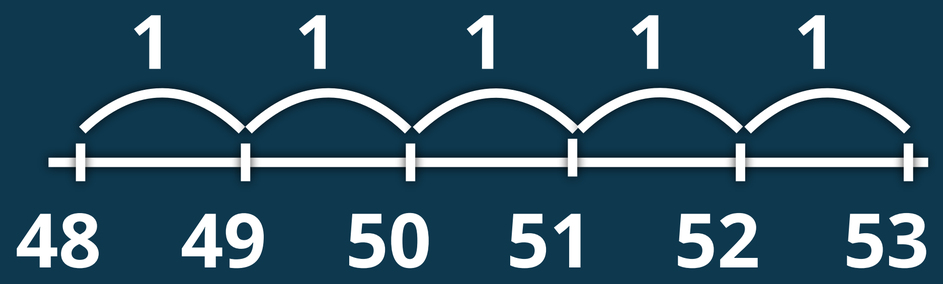
counting strategy

48 + 5



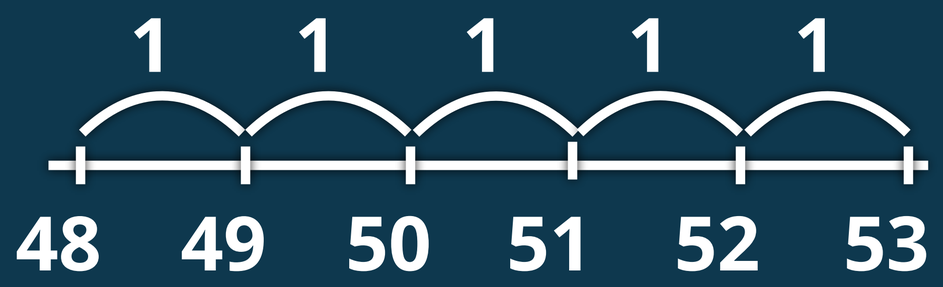
counting strategy

48 + 5

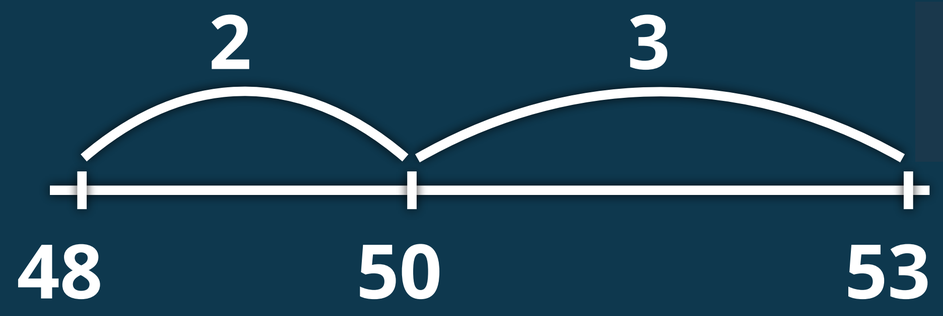


counting strategy

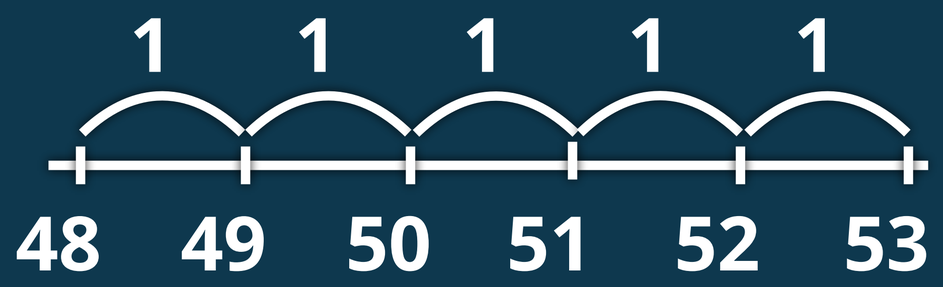
48 + 5



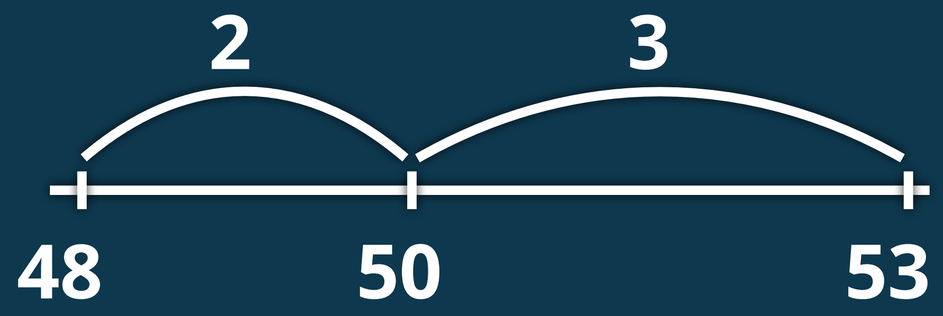
counting strategy



48 + 5



counting strategy



additive thinking

48 + 99

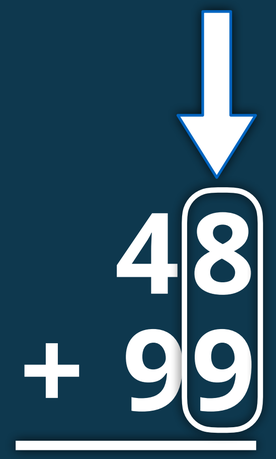
48 + 99

$$\begin{array}{r} 48 \\ +99 \\ \hline \end{array}$$

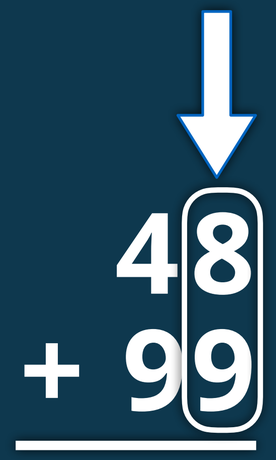
48 + 99

$$\begin{array}{r} 48 \\ +\ 99 \\ \hline \end{array}$$

48 + 99



48 + 99



48 + 99

48 + 99

```
  1
  48
+ 99
----
   7
```

48 + 99

```
  1
  48
+ 99
----
😀 7
```

48 + 99

maybe counting strategy

48 + 99

$$\begin{array}{r} 1 \\ 48 \\ +\ 99 \\ \hline \text{😀}\ 7 \end{array}$$

maybe counting strategy ☹️

48 + 99

$$\begin{array}{r} 1 \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

99

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

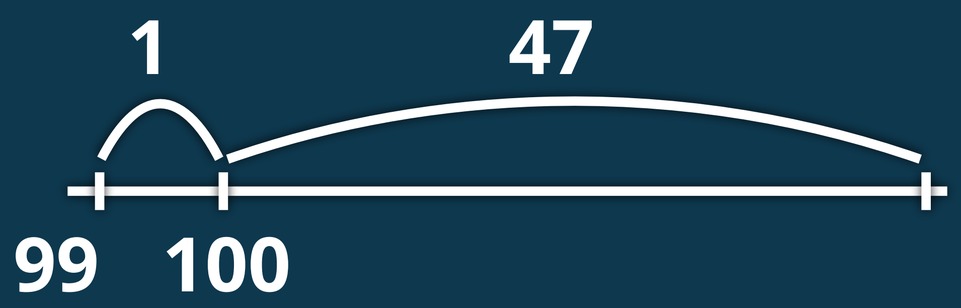
maybe additive thinking but only with single digits 😐

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

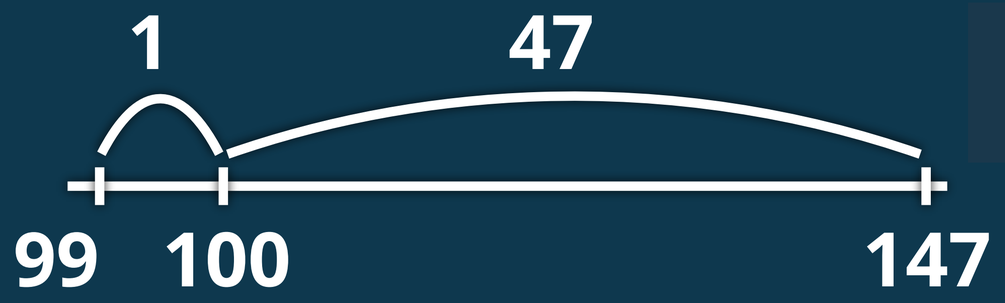


48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐

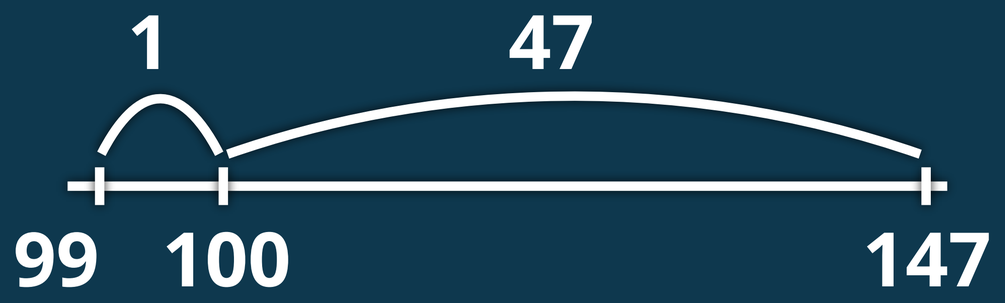


48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐



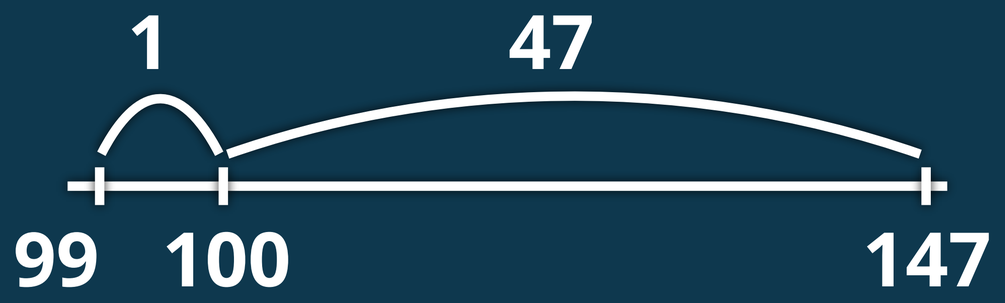
additive thinking

48 + 99

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \\ +\ 99 \\ \hline 7 \end{array}$$

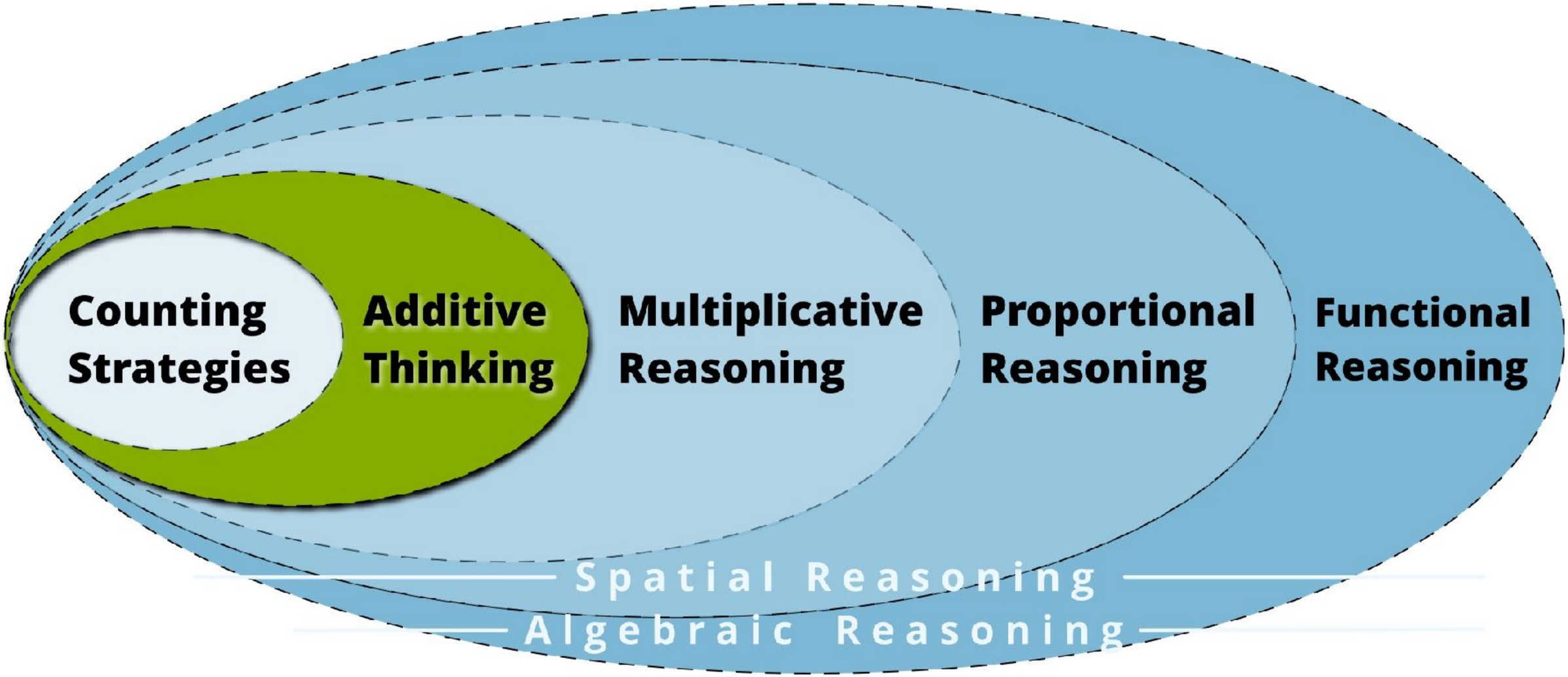
maybe counting strategy ☹️

maybe additive thinking but only with single digits 😐



additive thinking 😀

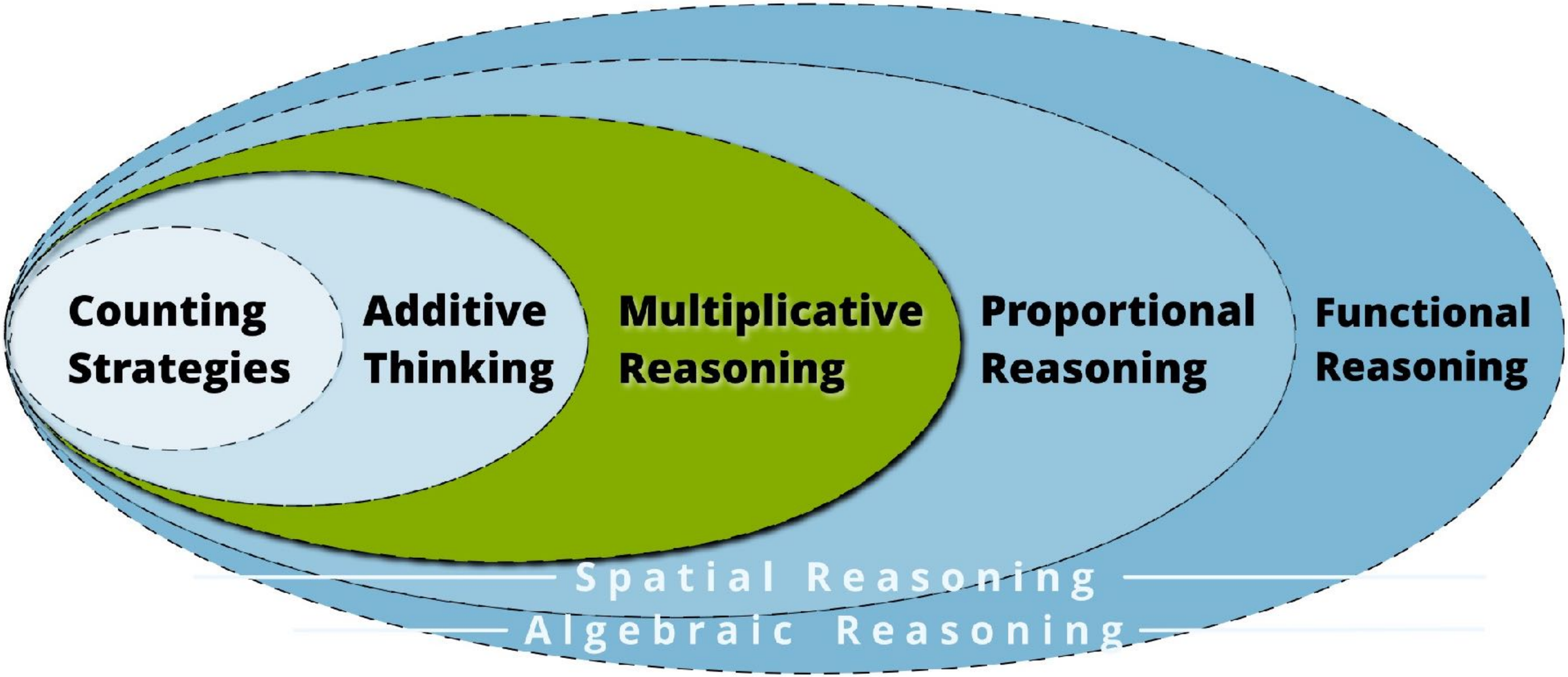
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

$$7 \times 8$$

7×8

rhyme

7×8

rhyme

fake math

7×8

~~rhyme~~

~~fake math~~

7×8

~~rhyme~~

~~fake math~~

7×8

~~rhyme~~

~~fake math~~

7×8

~~rhyme~~

~~fake math~~

7×8

~~rhyme~~

~~fake math~~

counting strategy

7×8

~~rhyme~~

~~fake math~~

counting strategy

8, 16, 24, 32, 40, 48, 56

7×8

~~rhyme~~

~~fake math~~

counting strategy

8, 16, 24, 32, 40, 48, 56

additive thinking

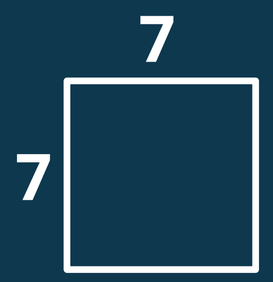
$$7 \times 8$$

7×8

7 x 7

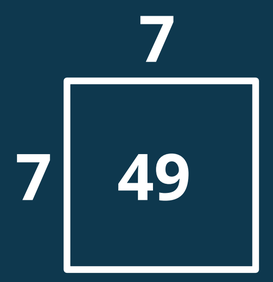
7×8

7 x 7



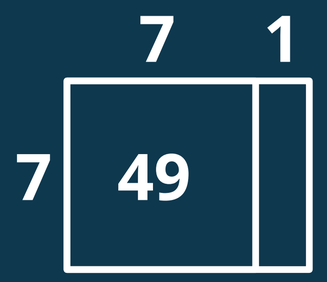
7×8

7 x 7



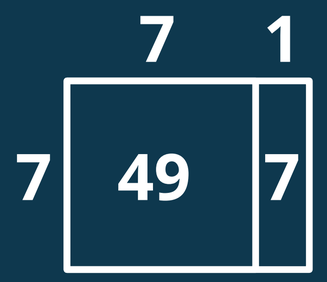
7×8

7 x 7



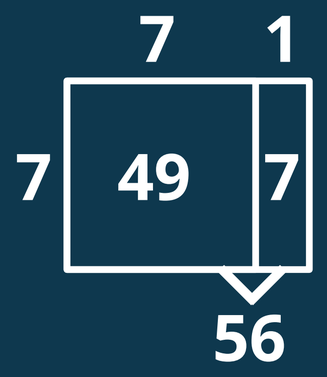
7×8

7 x 7



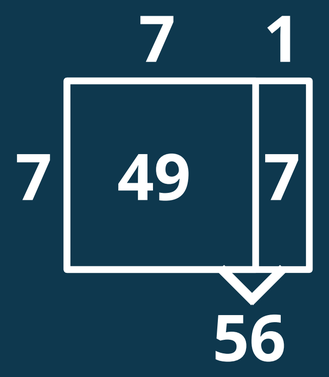
7×8

7 x 7

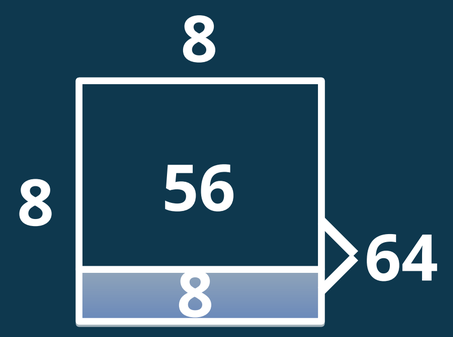


7 × 8

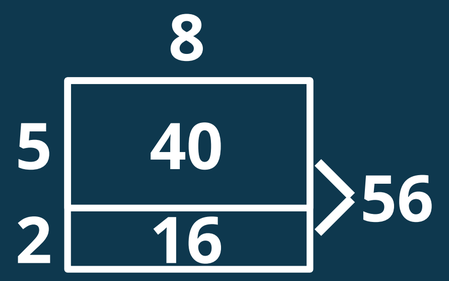
7 x 7



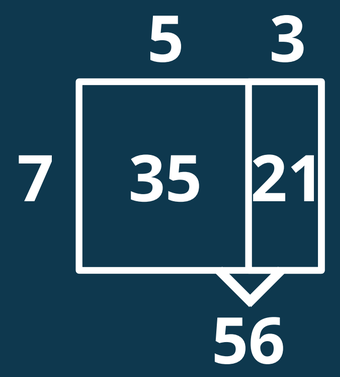
8 x 8



5 x 8

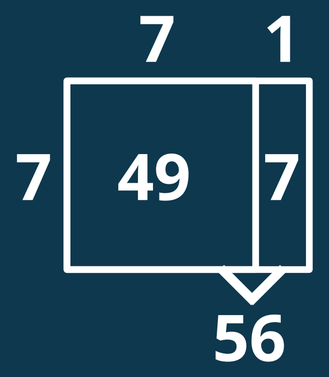


5 x 7

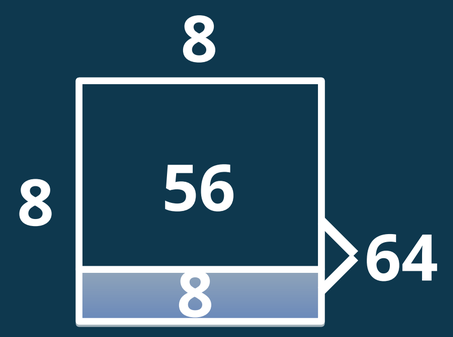


7 × 8

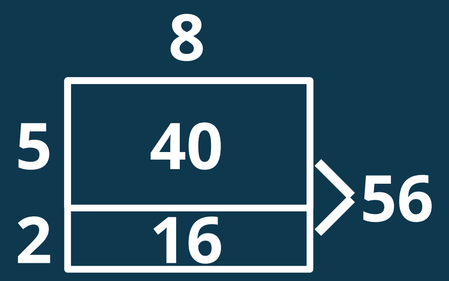
7 x 7



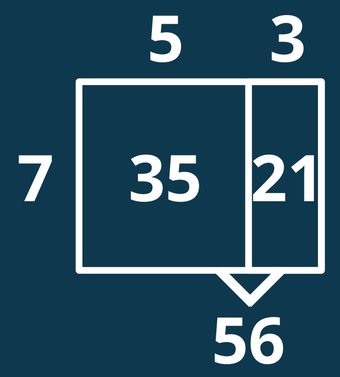
8 x 8



5 x 8



5 x 7



multiplicative reasoning 😀

@pwharris

PAM HARRIS

48 x 99

48 x 99

$$\begin{array}{r} 48 \\ \times\, 99 \\ \hline \end{array}$$

48 x 99

$$\begin{array}{r} 48 \\ \times\ 99 \\ \hline \end{array}$$

48 x 99

$$\begin{array}{r} 48 \\ \times\ 99 \\ \hline 2 \end{array}$$

48 x 99

$$\begin{array}{r} {\scriptstyle 7} \\ 48 \\ \times\ 99 \\ \hline 2 \end{array}$$

48 x 99

$$\begin{array}{r} 48 \\ \times\ 99 \\ \hline 2 \end{array}$$

48 x 99

$$\begin{array}{r} 48 \\ \times\ 99 \\ \hline 432 \end{array}$$

48 x 99

```
  48
x 99
----
432
```

48 x 99

```
  48
x 99
----
432
  0
```

48 x 99

```
  48
x 99
----
432
  0
```

48 x 99

```
  7
  x
 48
x 99
----
432
   0
```

48 x 99

```
   7
   x
  48
x 99
----
 432
xxx0
----
xxxxx
```

48 x 99

```
   7
   x
  48
x 99
----
 432
xxx0
----
😀xxxx
```

48 x 99

```
   7
   x
  48
x 99
----
 432
xxx0
----
xxxxx
```

additive thinking ☹️

48 x 99

```
   7
   x
  48
x 99
----
 432
xxx0
----
xxxxx
```

additive thinking

maybe mulitiplicative thinking but only with single digits

48 x 99

48 x 99

99 x 48

48 x 99

99 x 48

100 x 48 = 4800

99 x 48

$$48 \times 99$$

$$99 \times 48$$

$$100 \times 48 = 4800$$

$$99 \times 48 = 4800 - 48$$

48 x 99

99 x 48

100 x 48 = 4800

99 x 48 = 4752

48 x 99

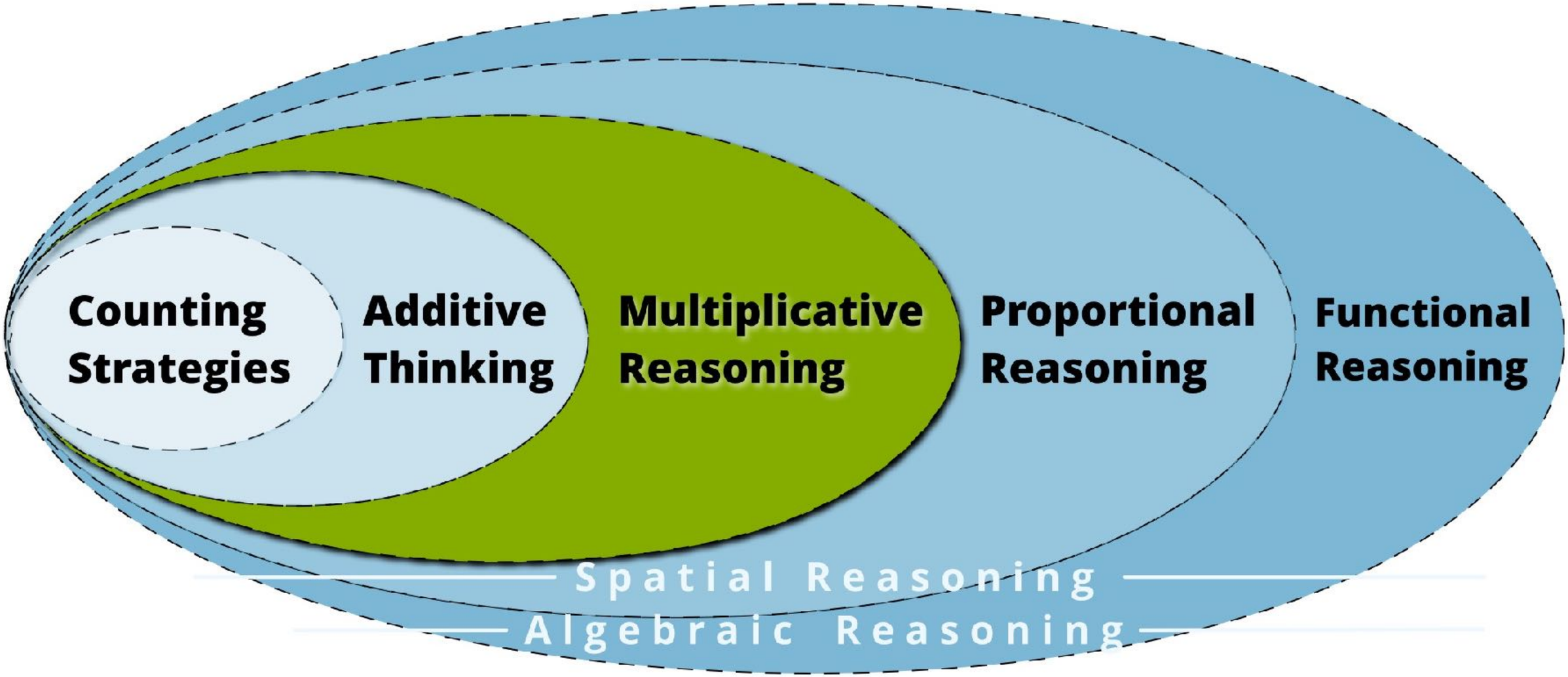
99 x 48

100 x 48 = 4800

99 x 48 = 4752

multiplicative reasoning 😀

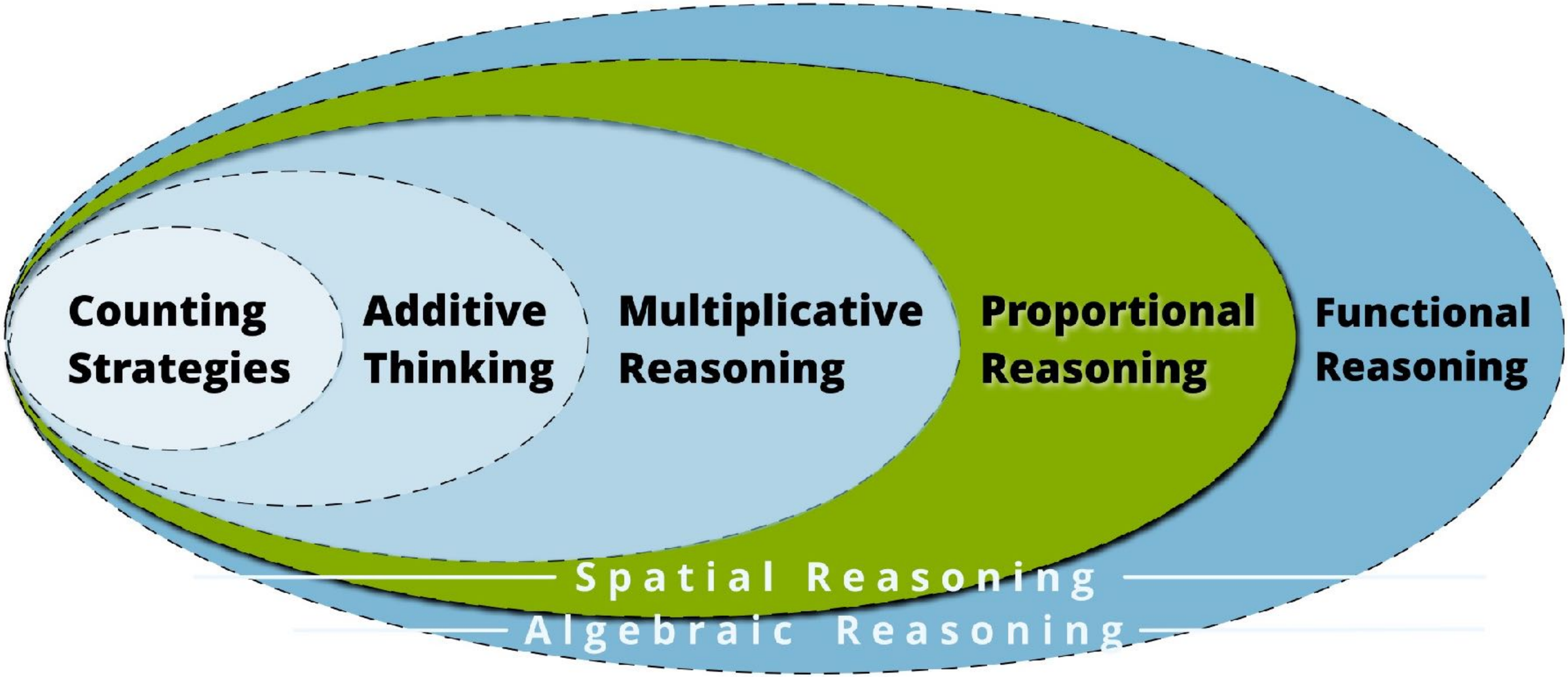
The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

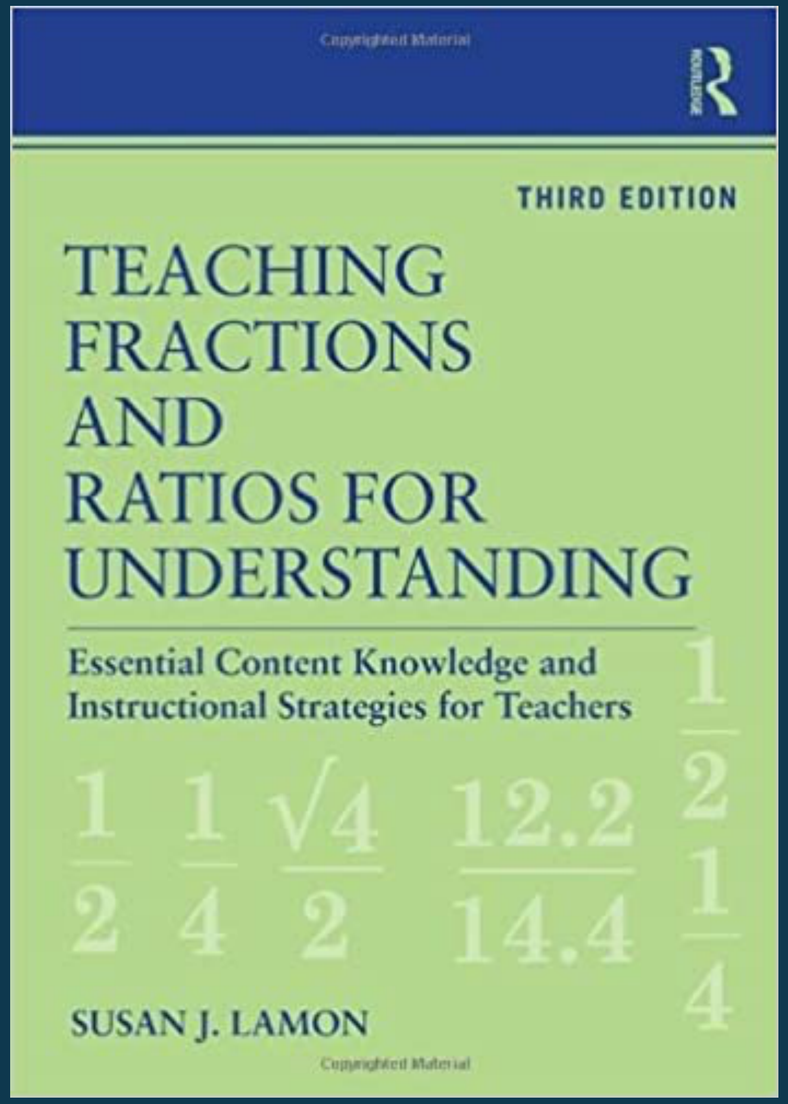
The Development of Mathematical Reasoning



@pwharris

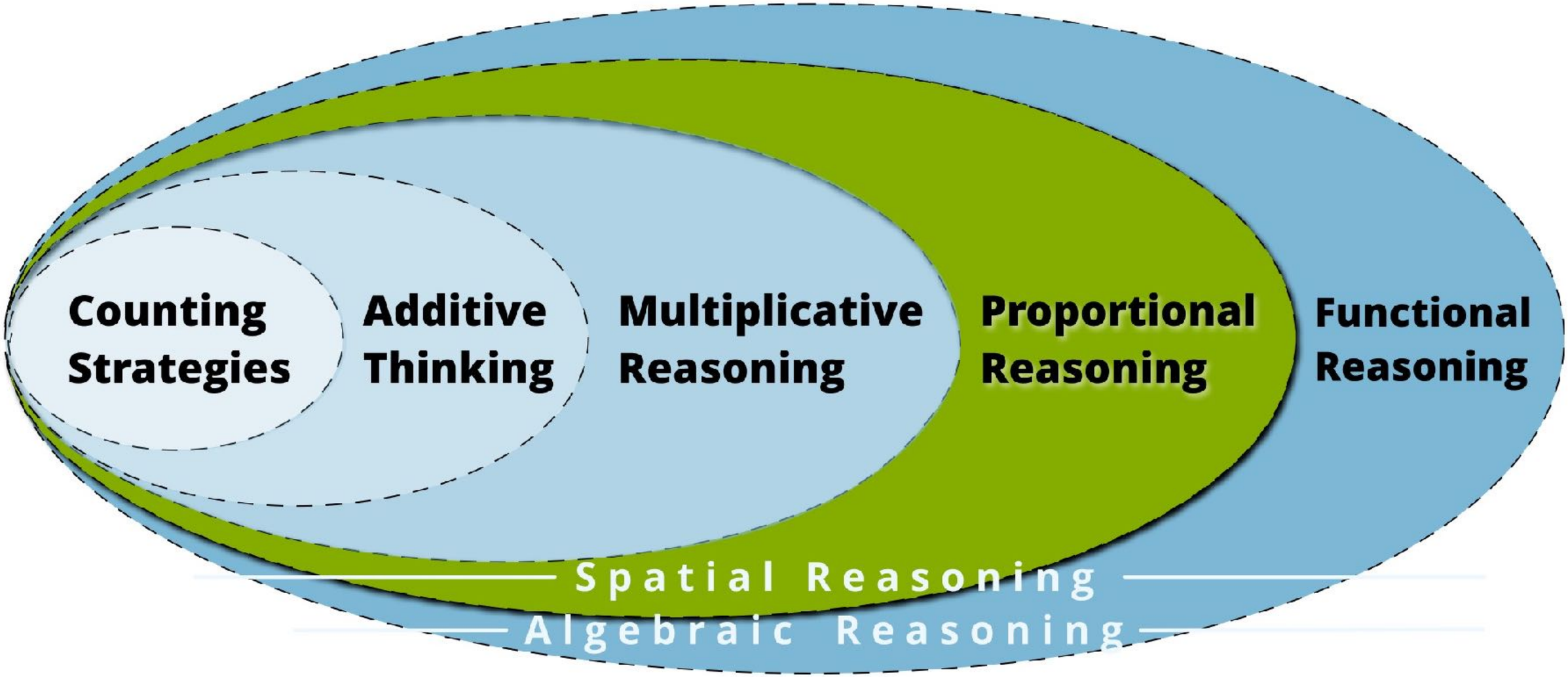
PAM HARRIS

90% OF ADULTS DO NOT REASON PROPORTIONALLY


Copyrighted Material
THIRD EDITION
TEACHING FRACTIONS AND RATIOS FOR UNDERSTANDING
Essential Content Knowledge and Instructional Strategies for Teachers
SUSAN J. LAMON
Copyrighted Material

Susan Lamon

The Development of Mathematical Reasoning

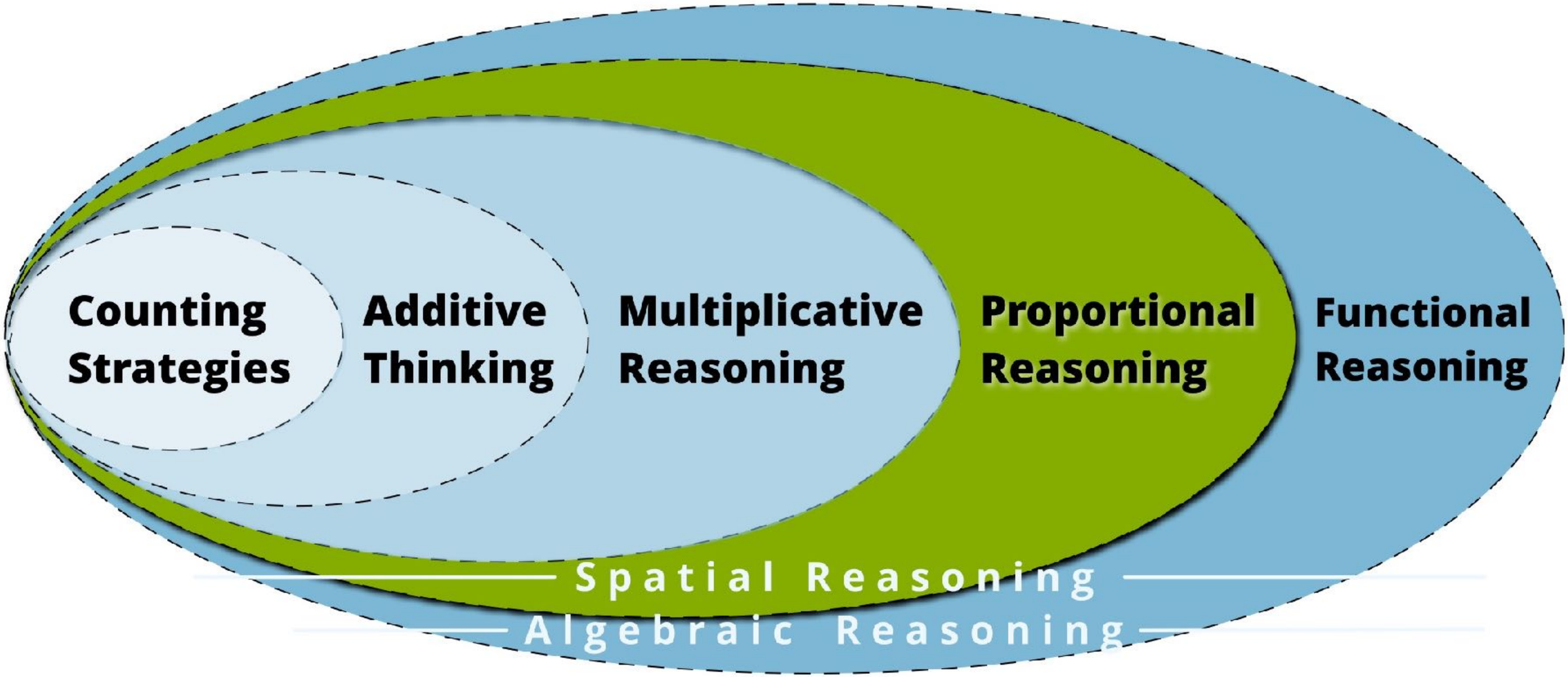


@pwharris

PAM HARRIS

Cross Multiply and Divide?

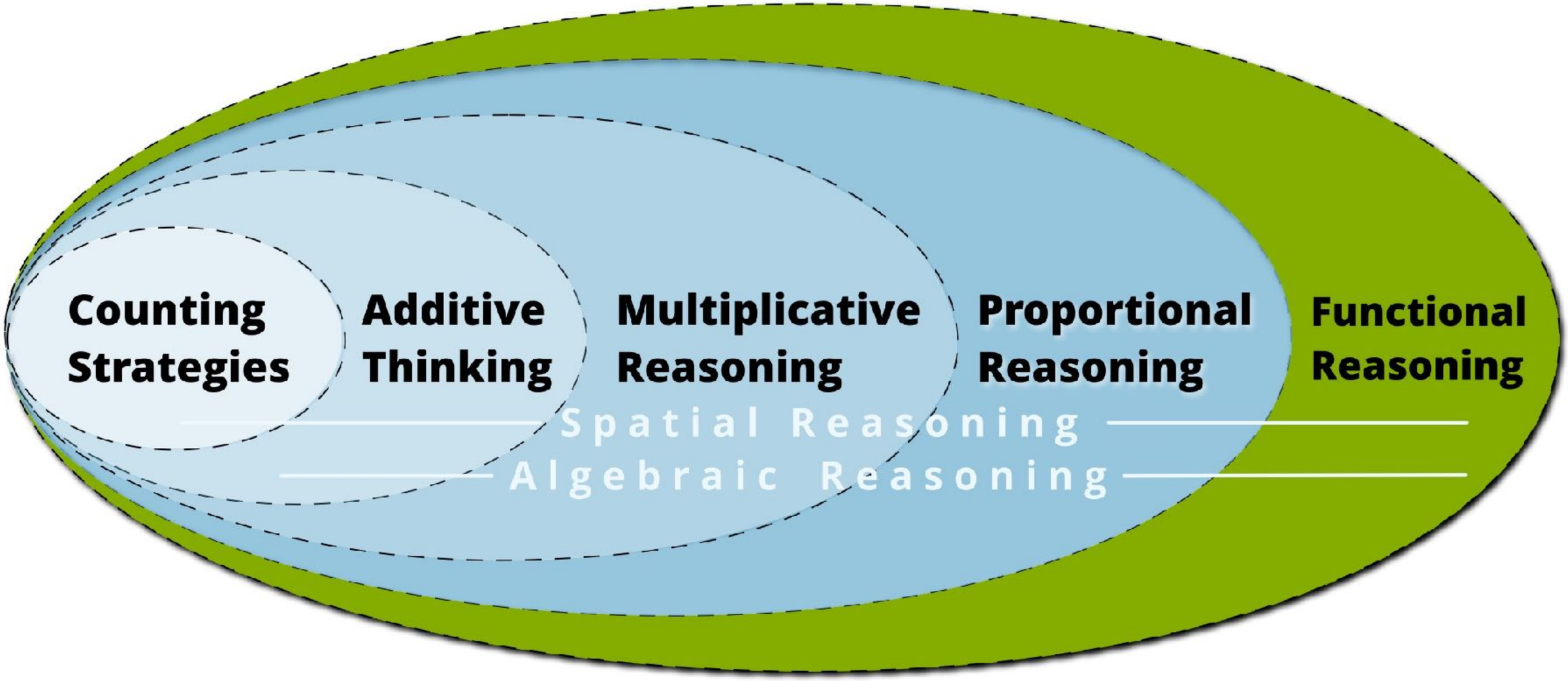
The Development of Mathematical Reasoning



@pwharris

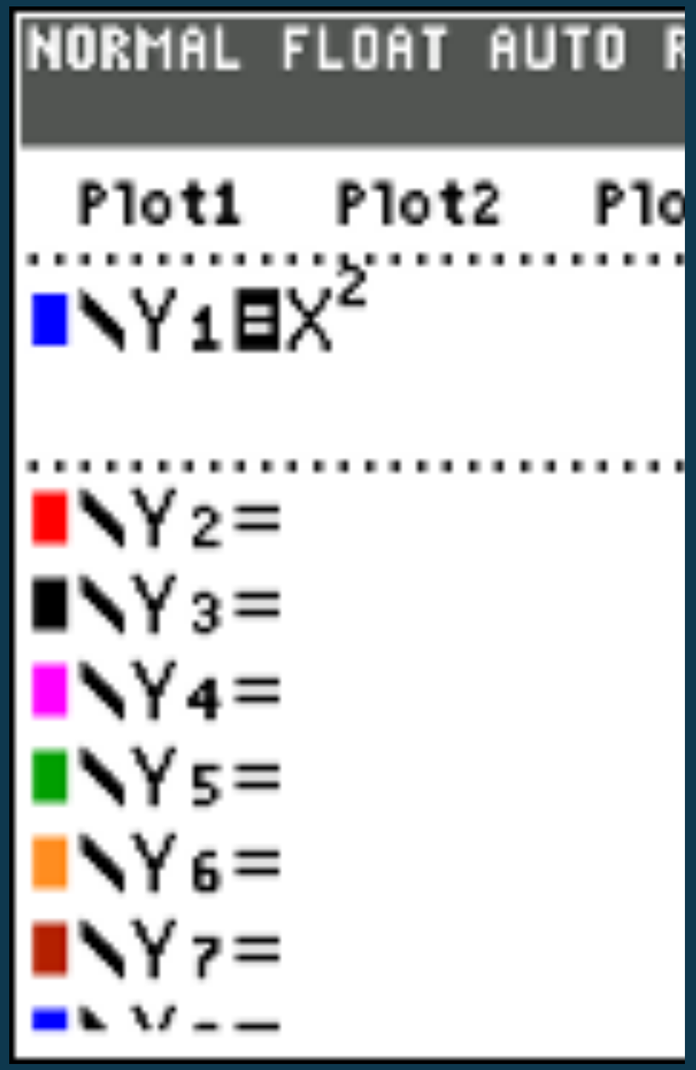
PAM HARRIS

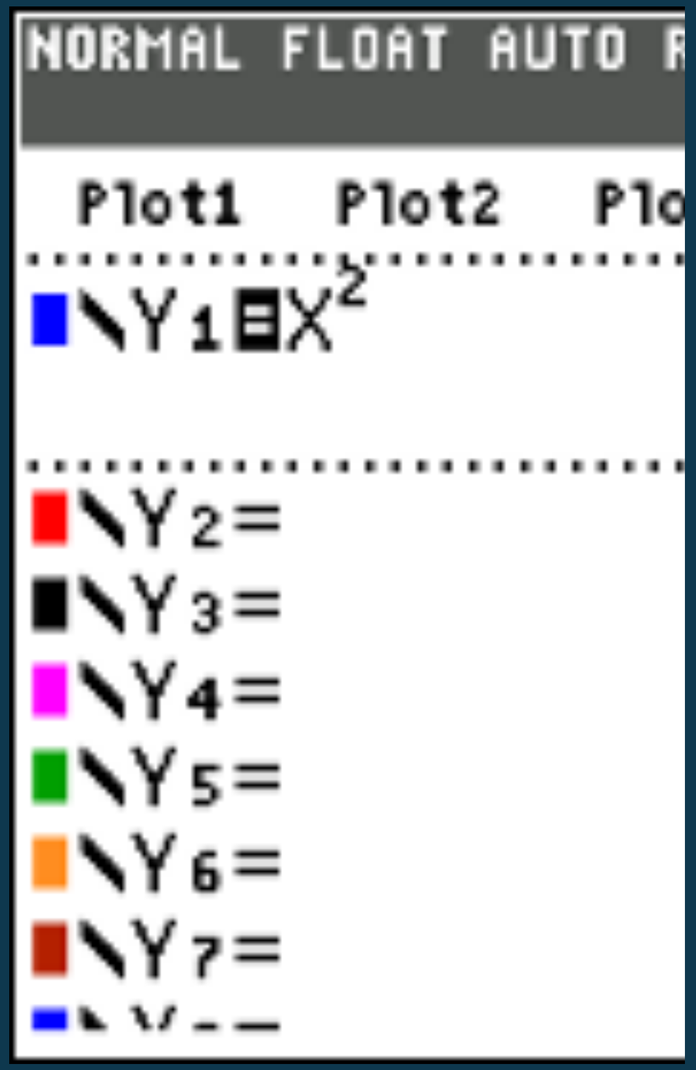
The Development of Mathematical Reasoning

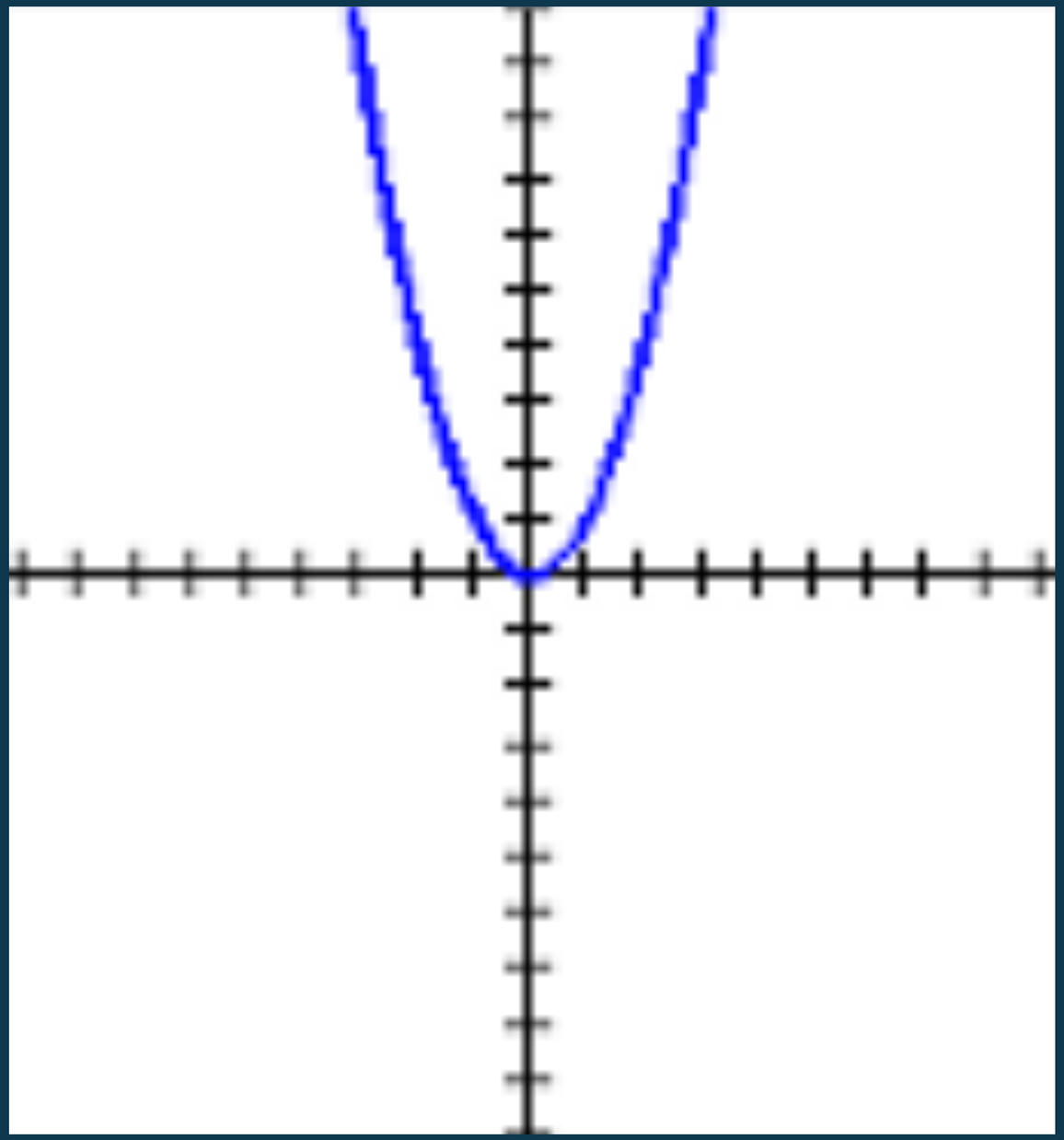


@pwharris

PAM HARRIS

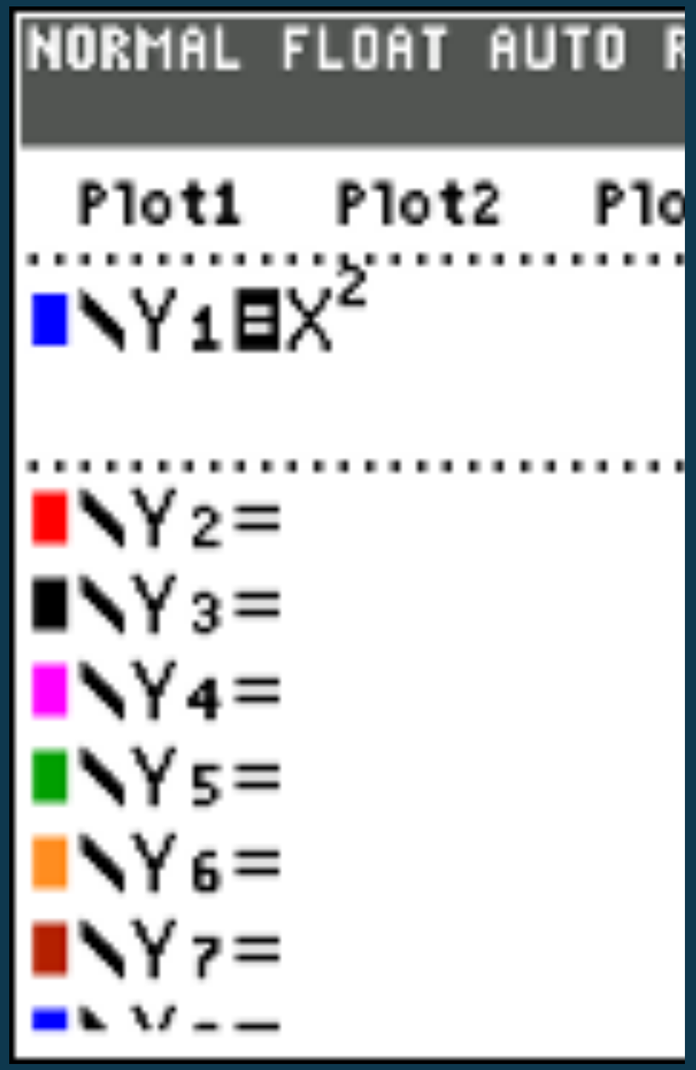

NORMAL FLOAT AUTO
Plot1 Plot2 Plo
Y1=X2
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

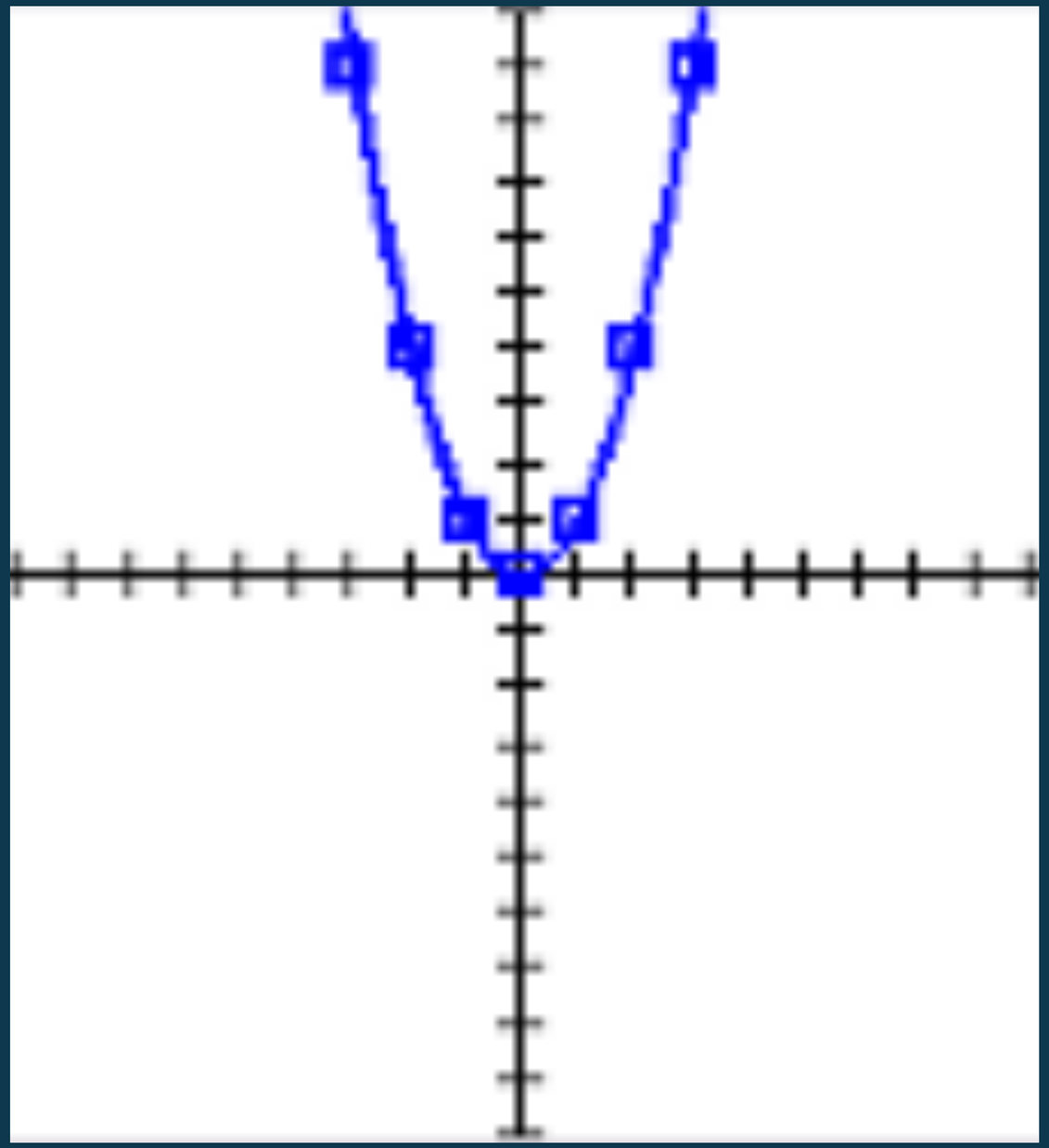




@pwharris

PAM HARRIS

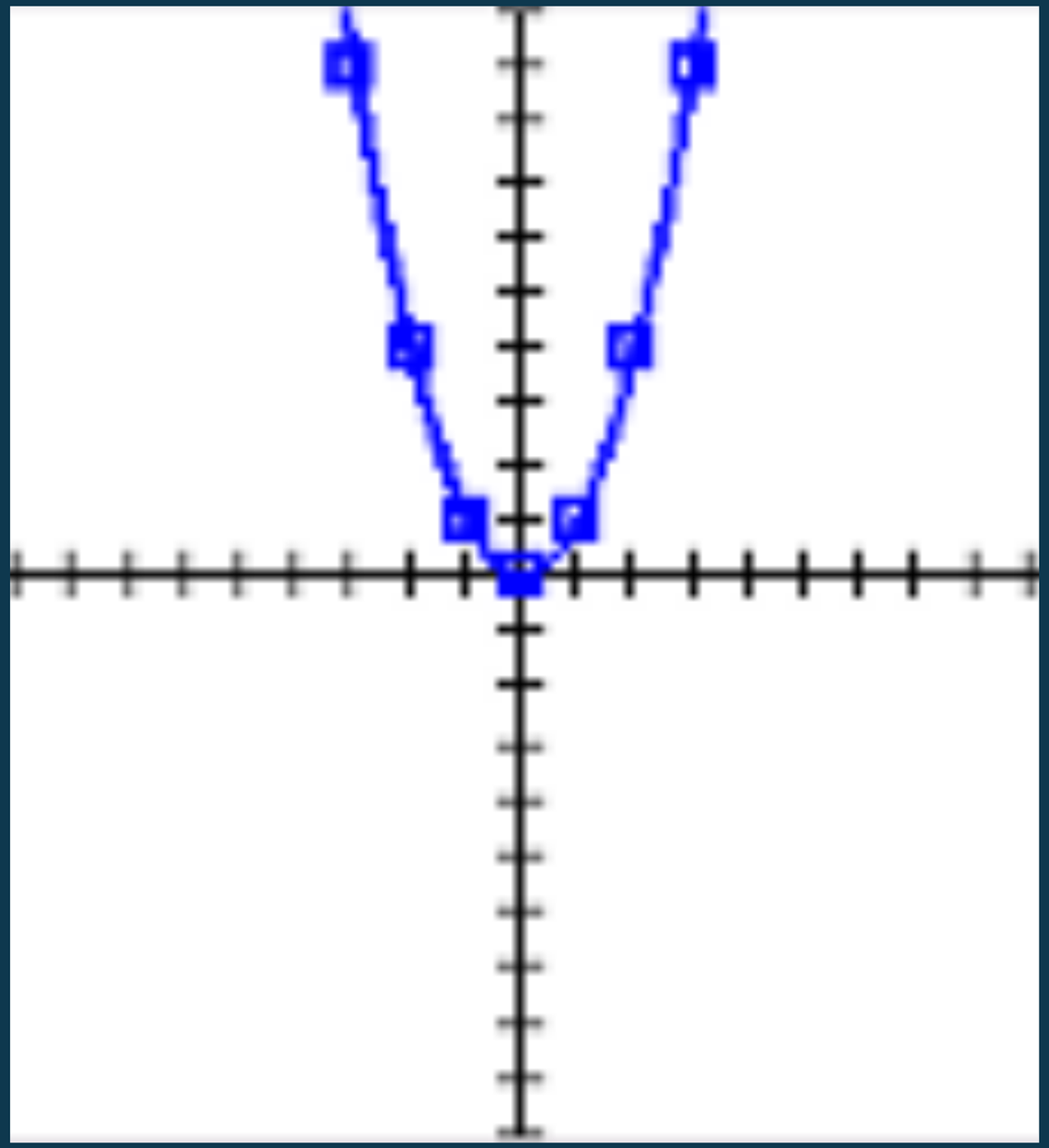




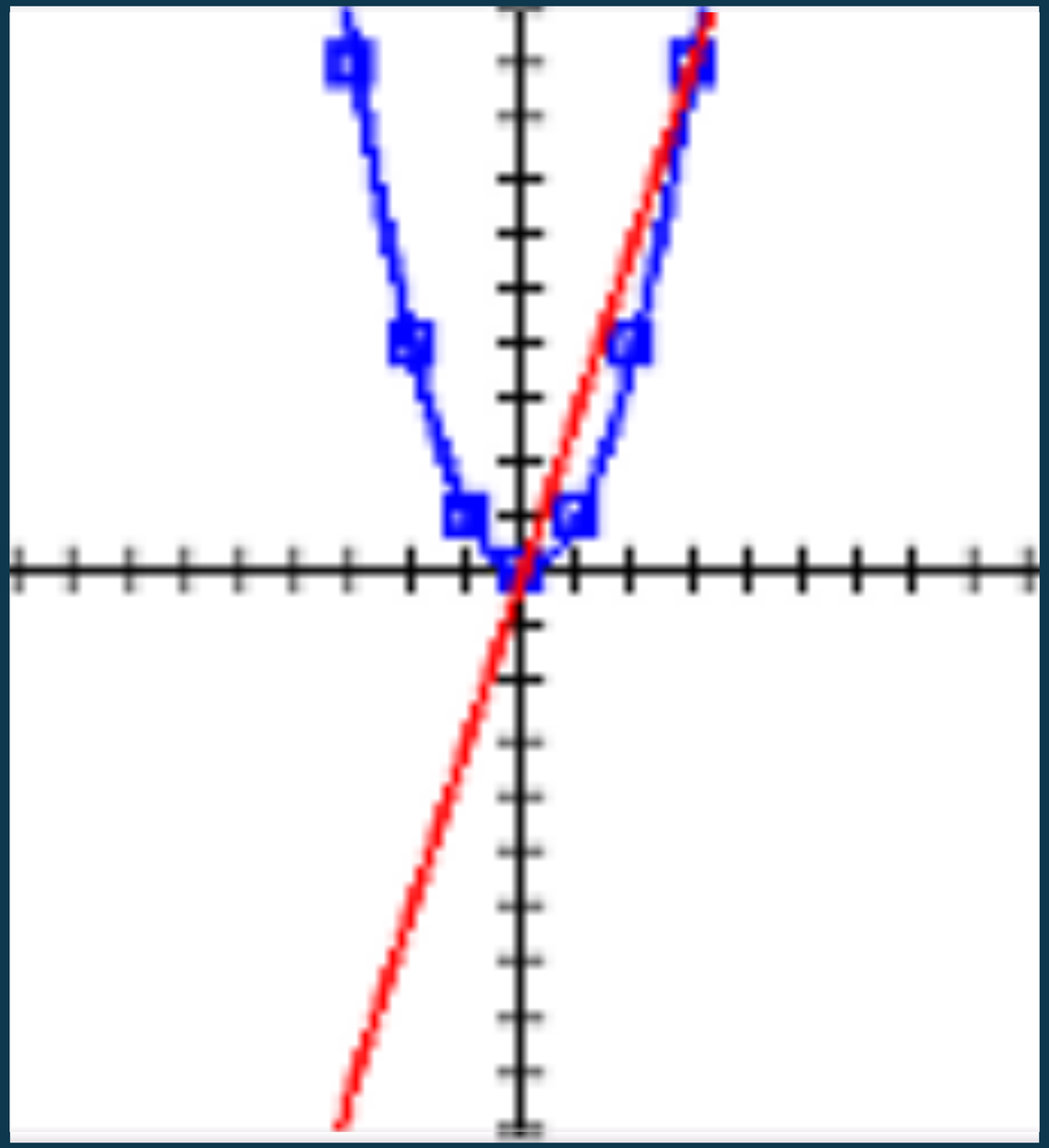
@pwharris

PAM HARRIS

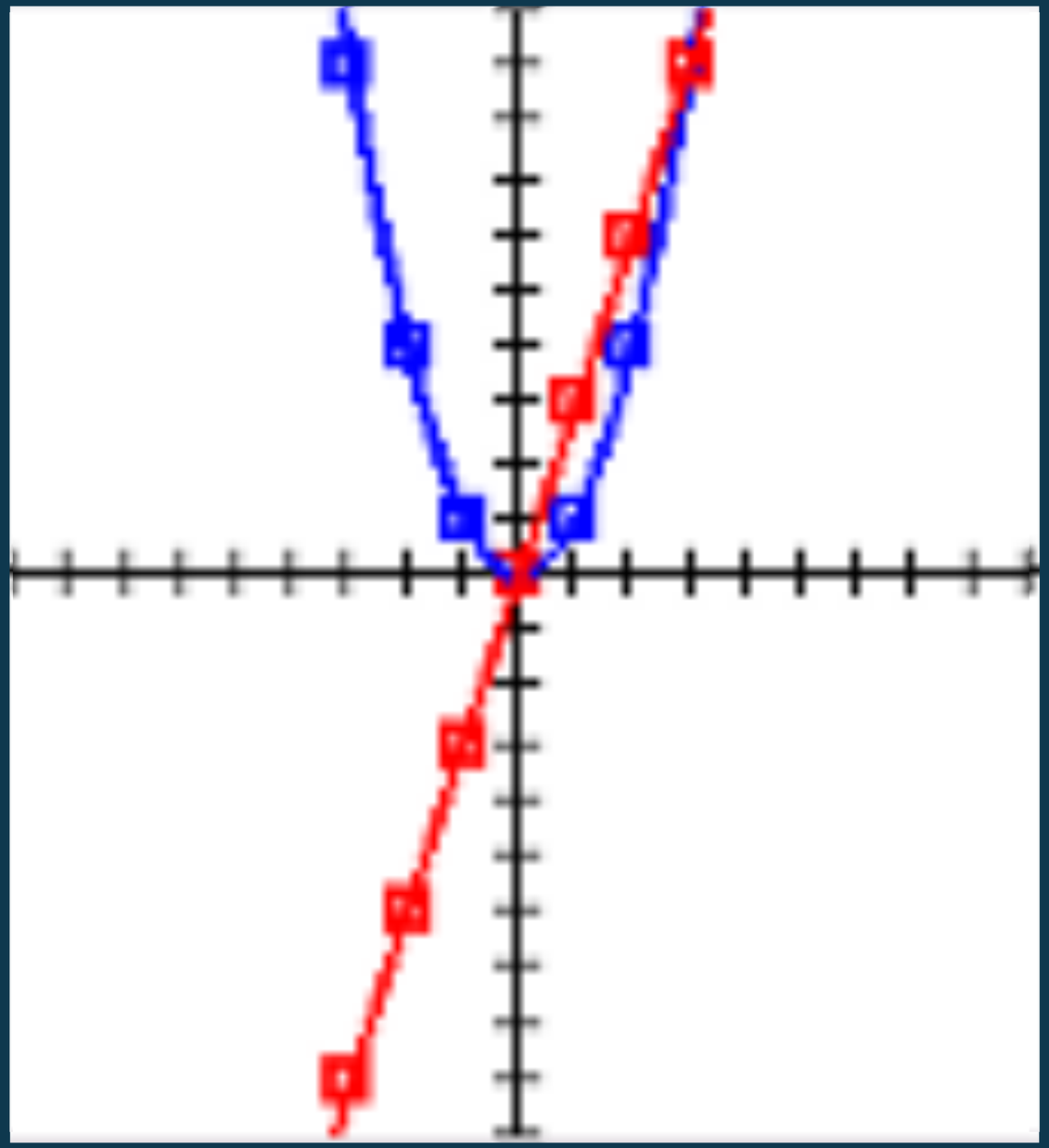
NORMAL FLOAT AUT
Plot1
Plot2
Y1=X2
Y2=3X
Y3=
Y4=
Y5=
Y6=
Y7=

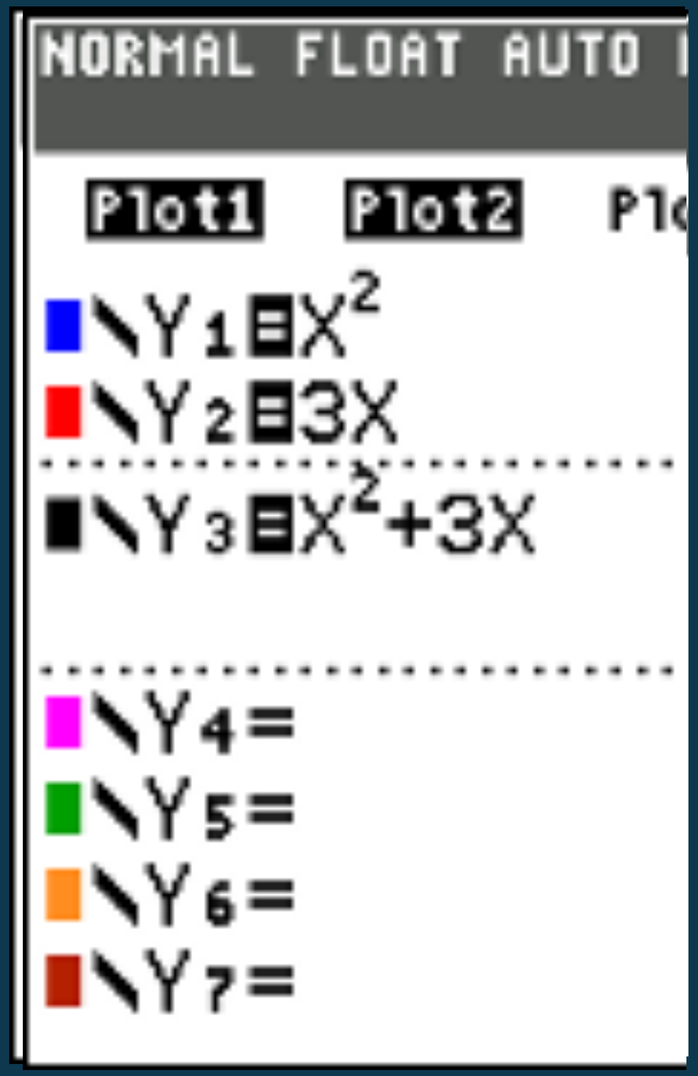


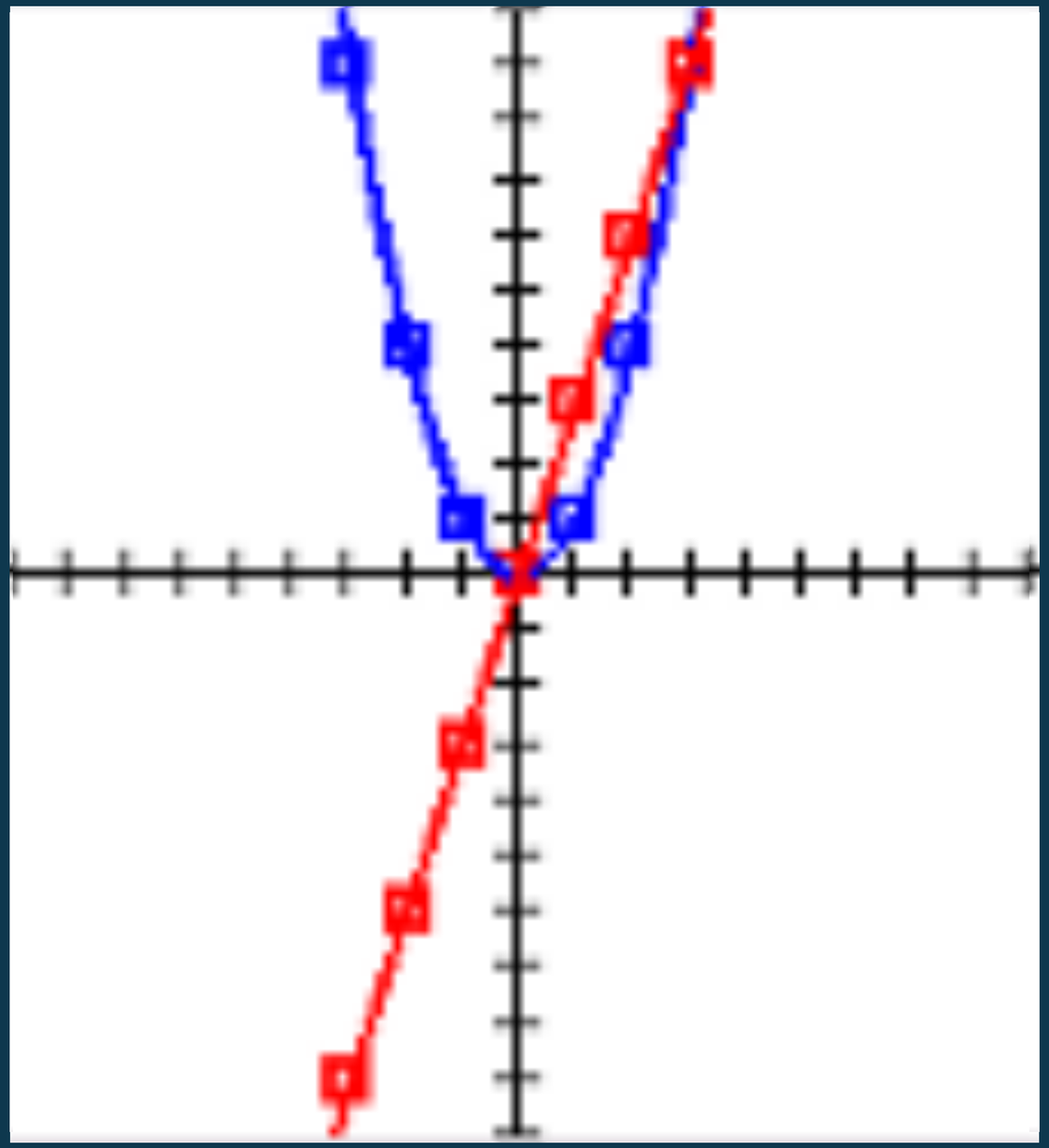
NORMAL FLOAT AUT
Plot1
Plot2
Y1=X2
Y2=3X
Y3=
Y4=
Y5=
Y6=
Y7=



NORMAL FLOAT AUT
Plot1
Plot2
Y1=X2
Y2=3X
Y3=
Y4=
Y5=
Y6=
Y7=

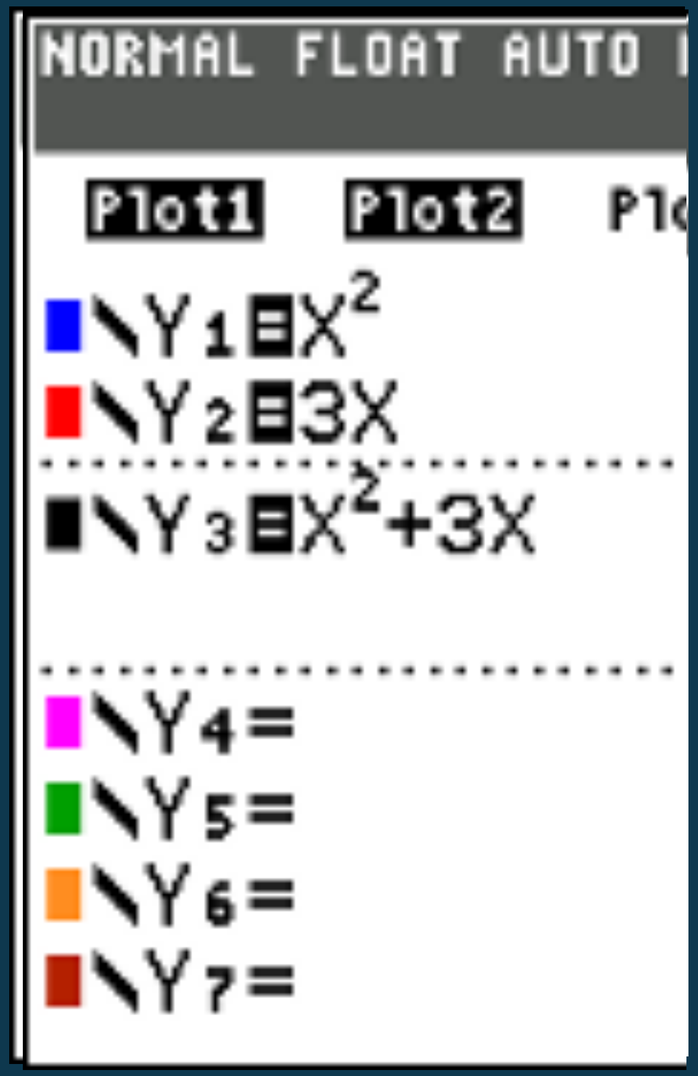


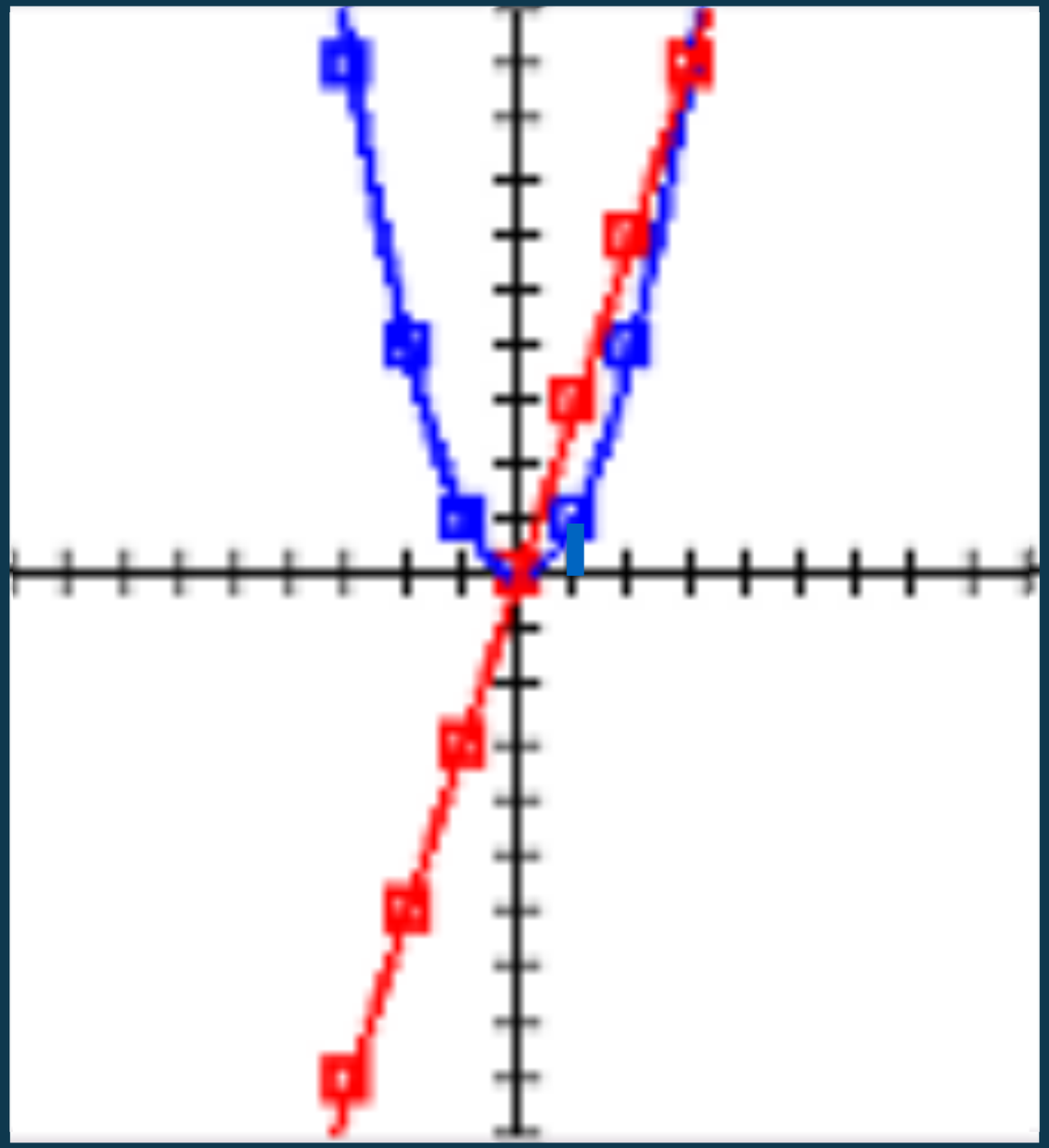


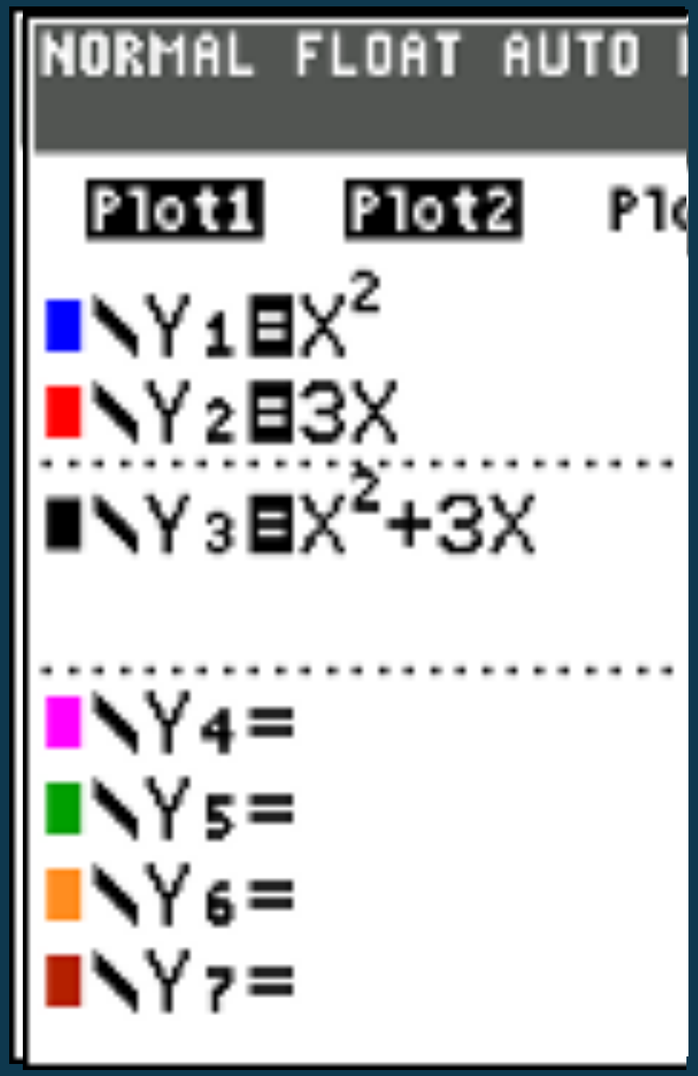


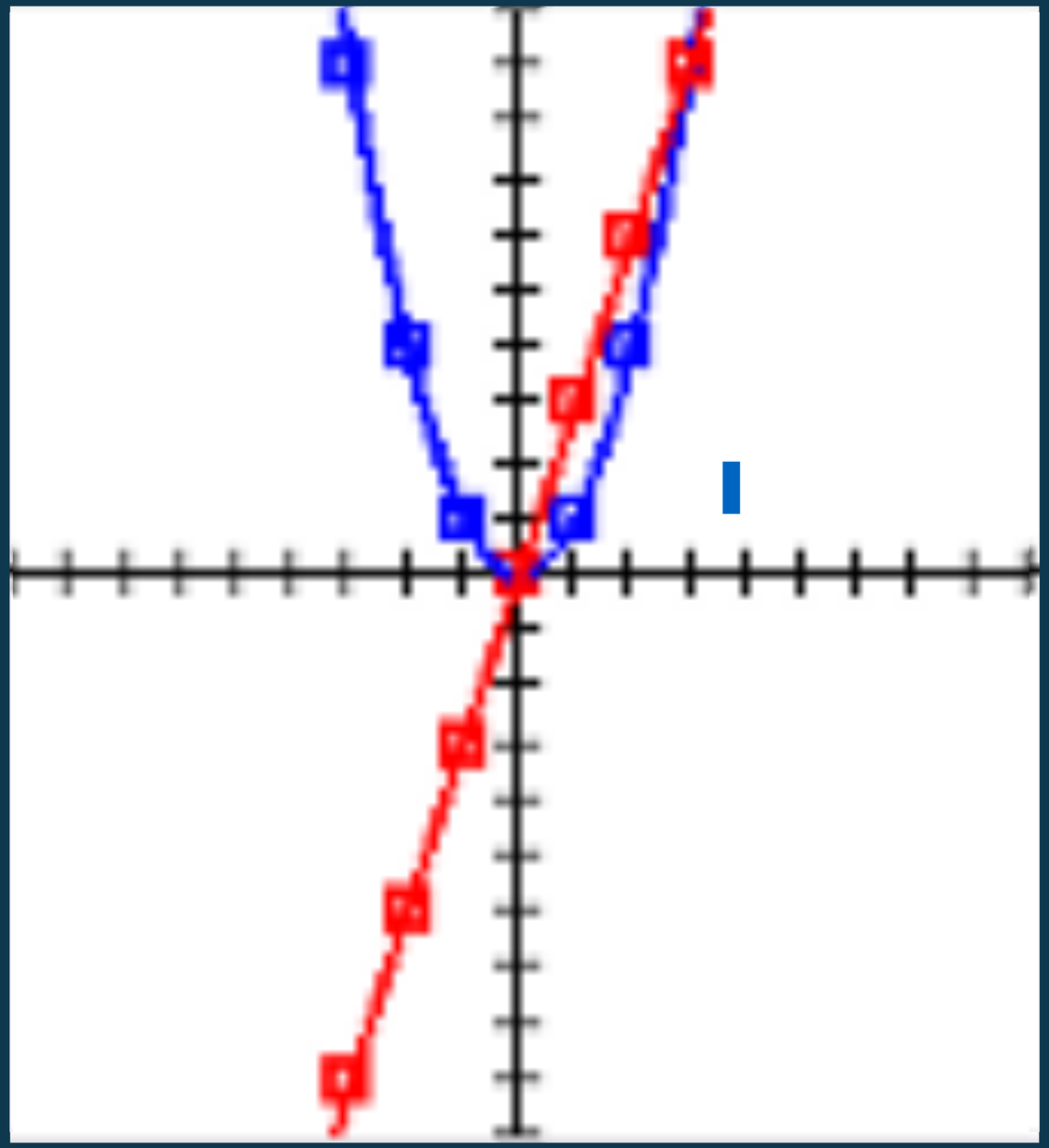
@pwharris

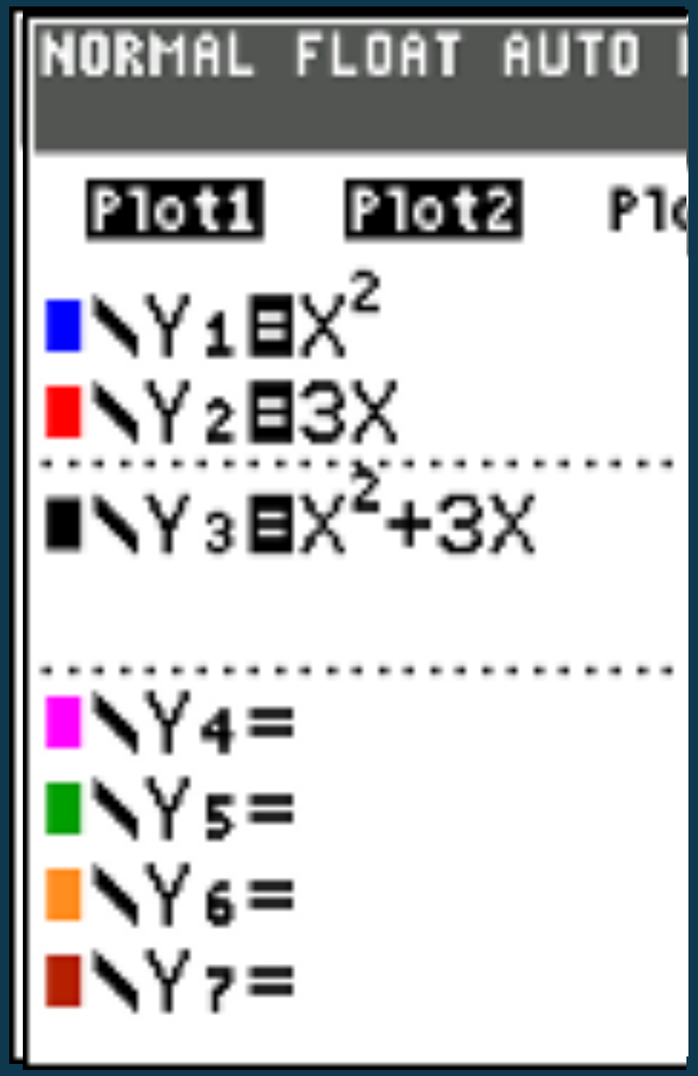
PAM HARRIS

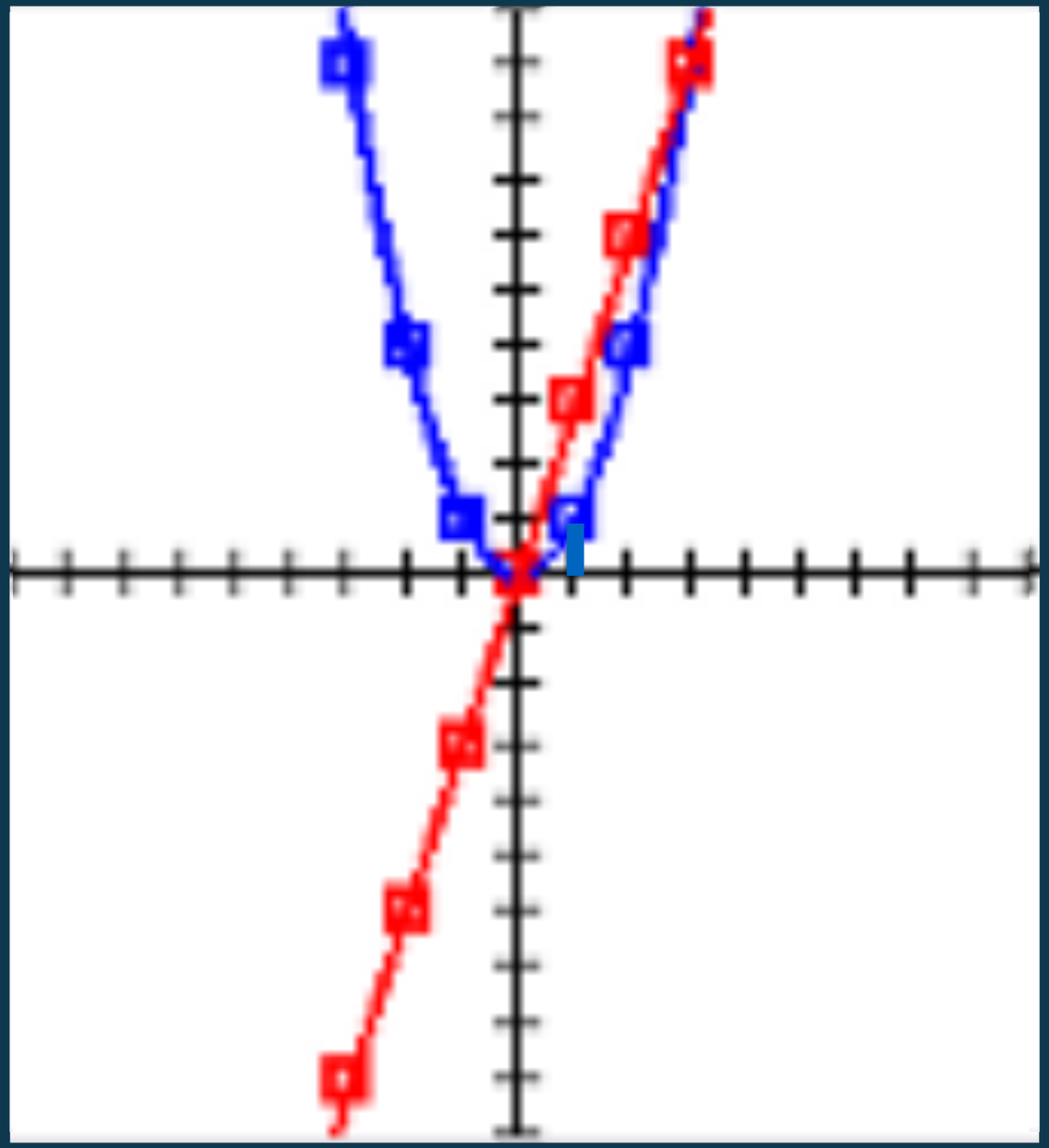

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

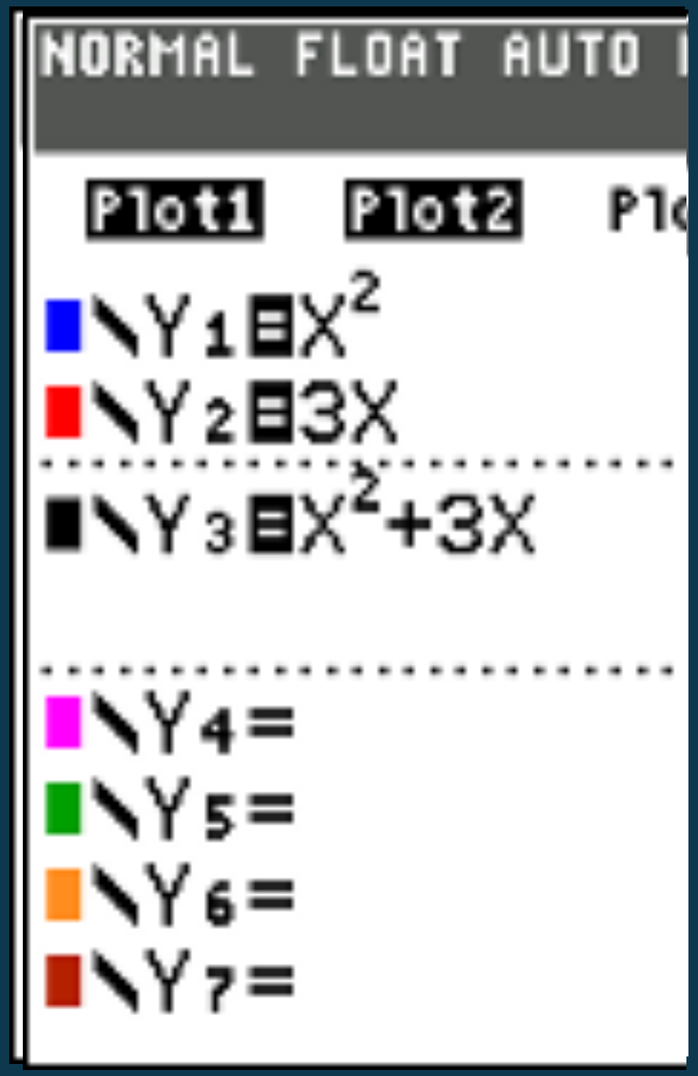


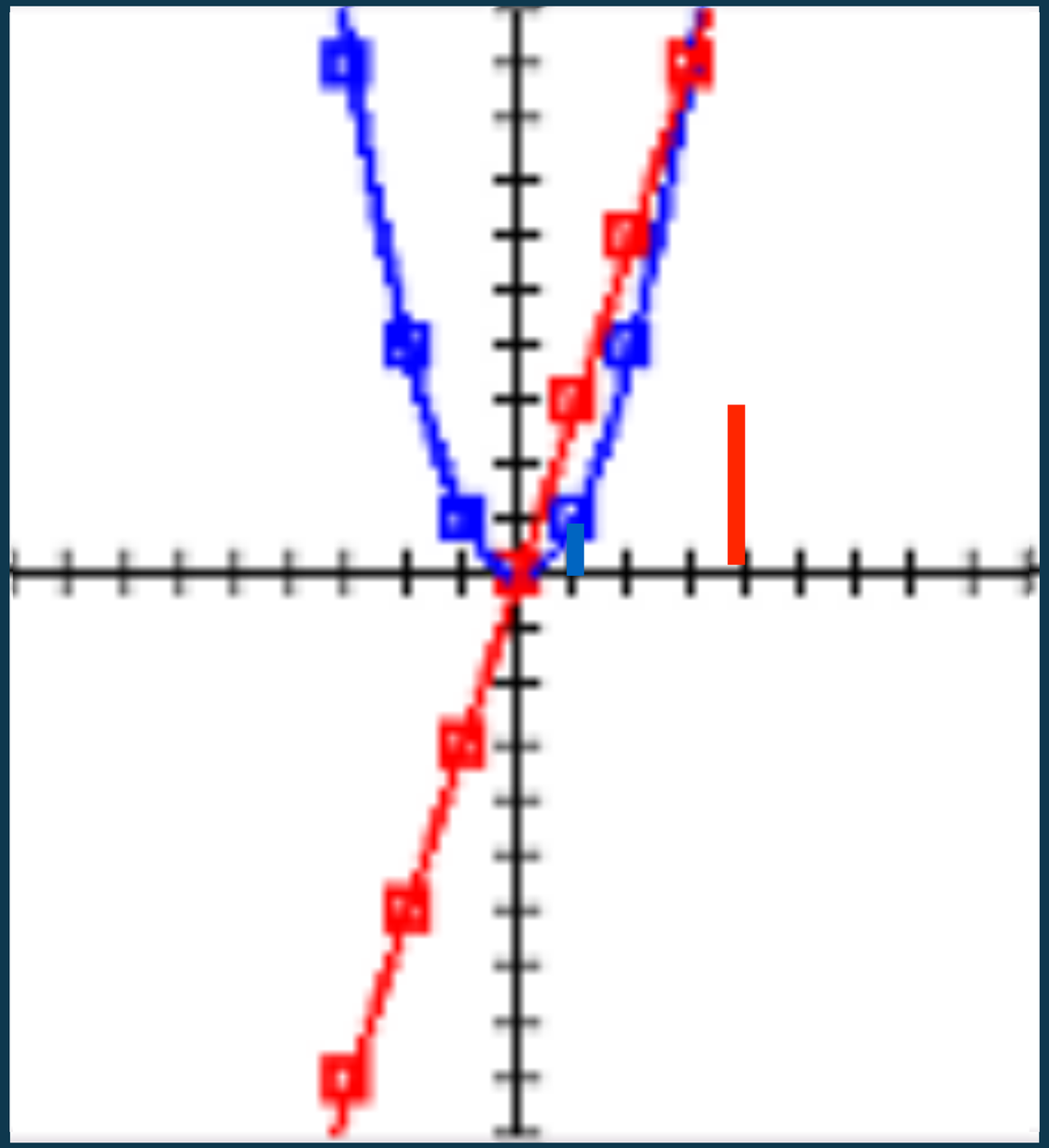

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

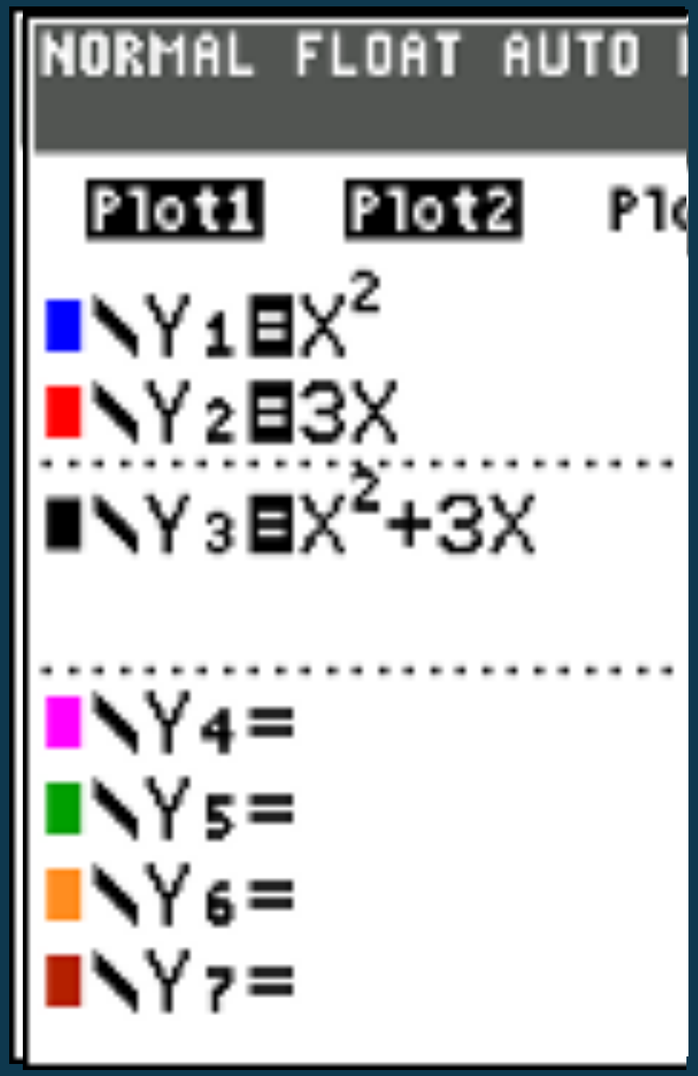


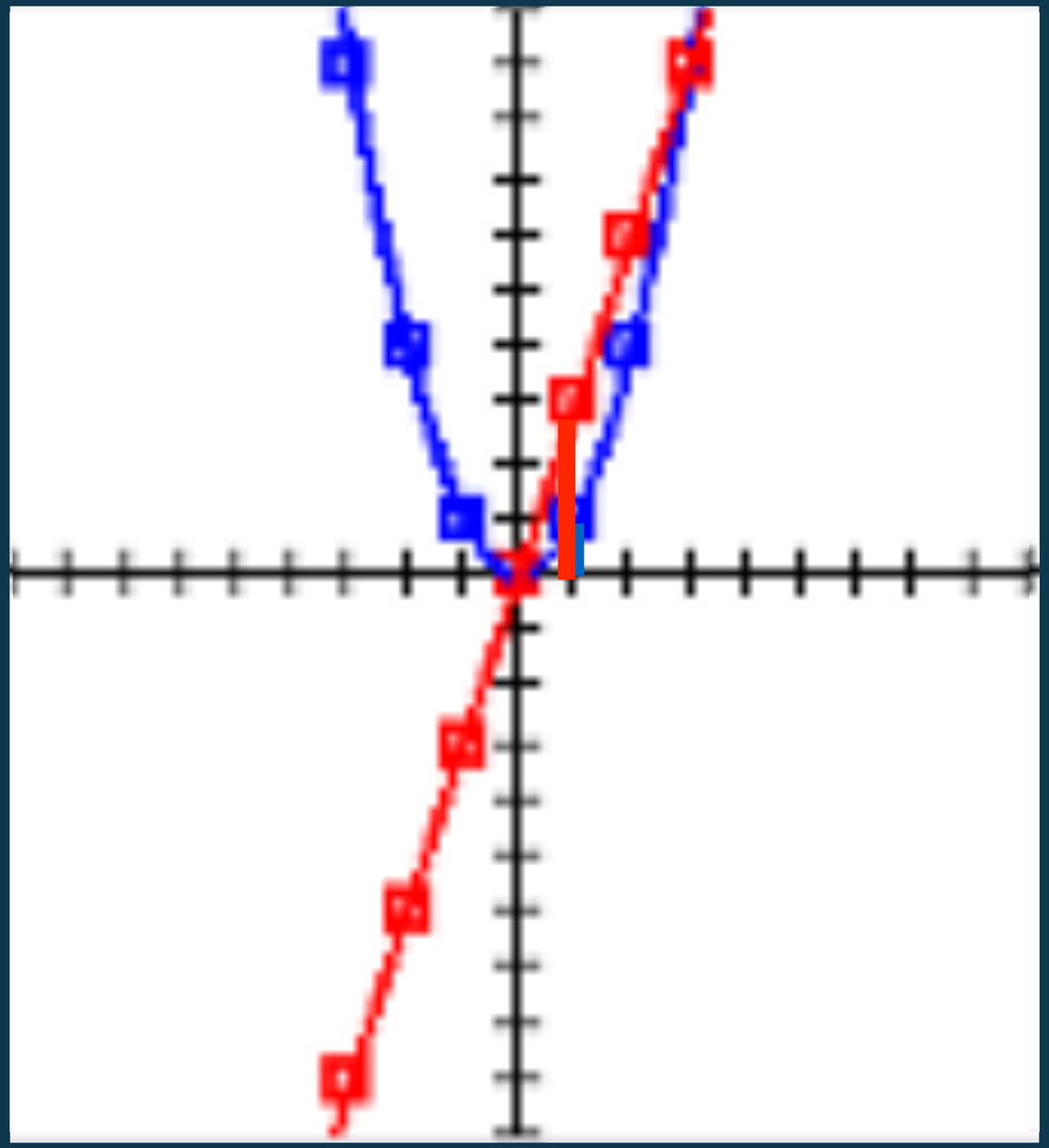

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

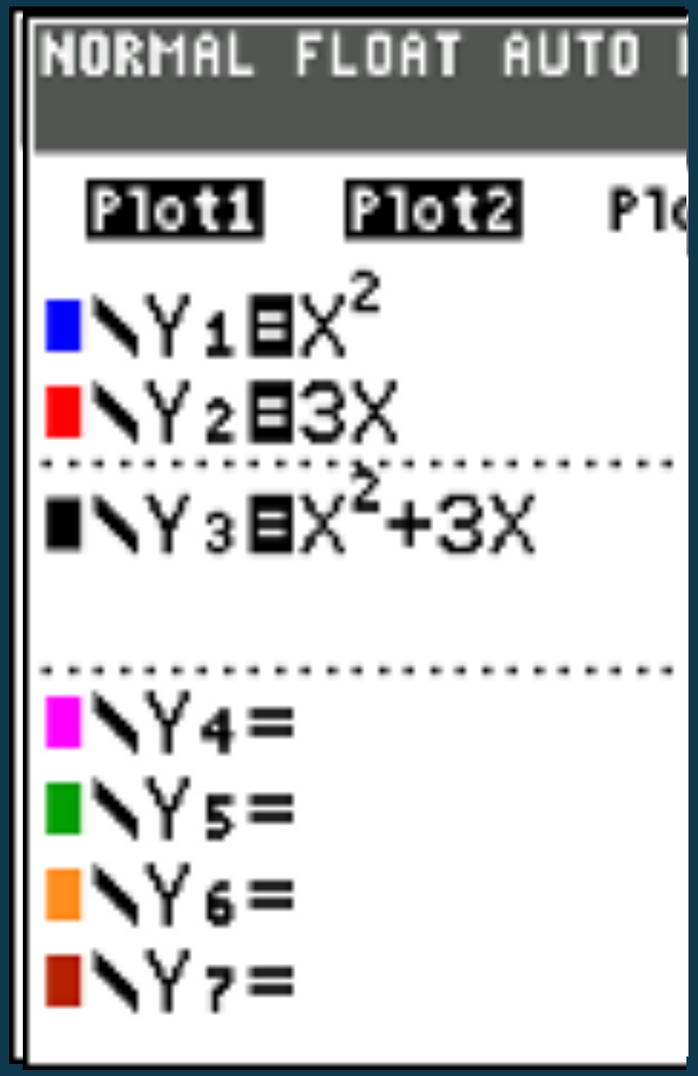


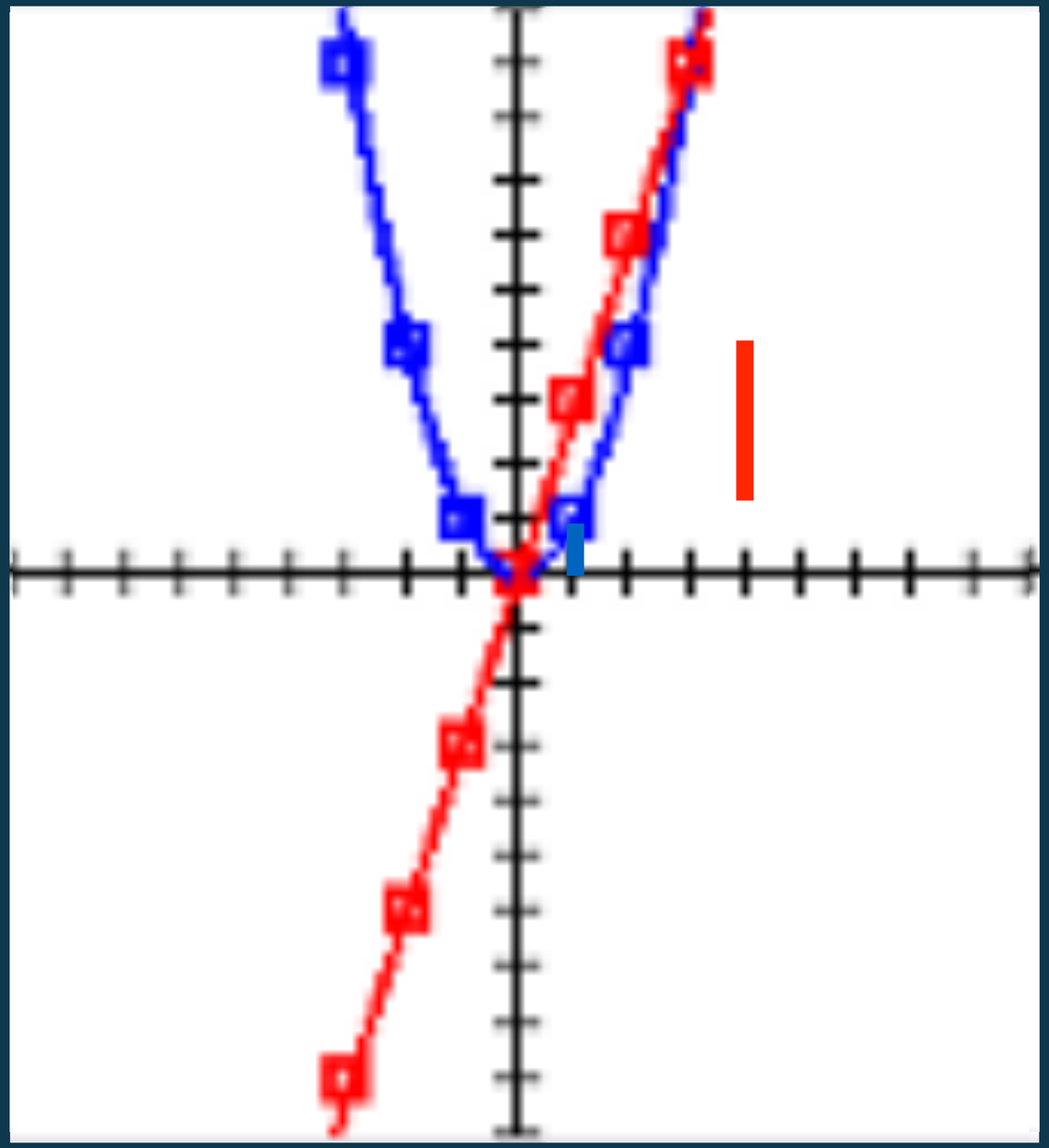

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

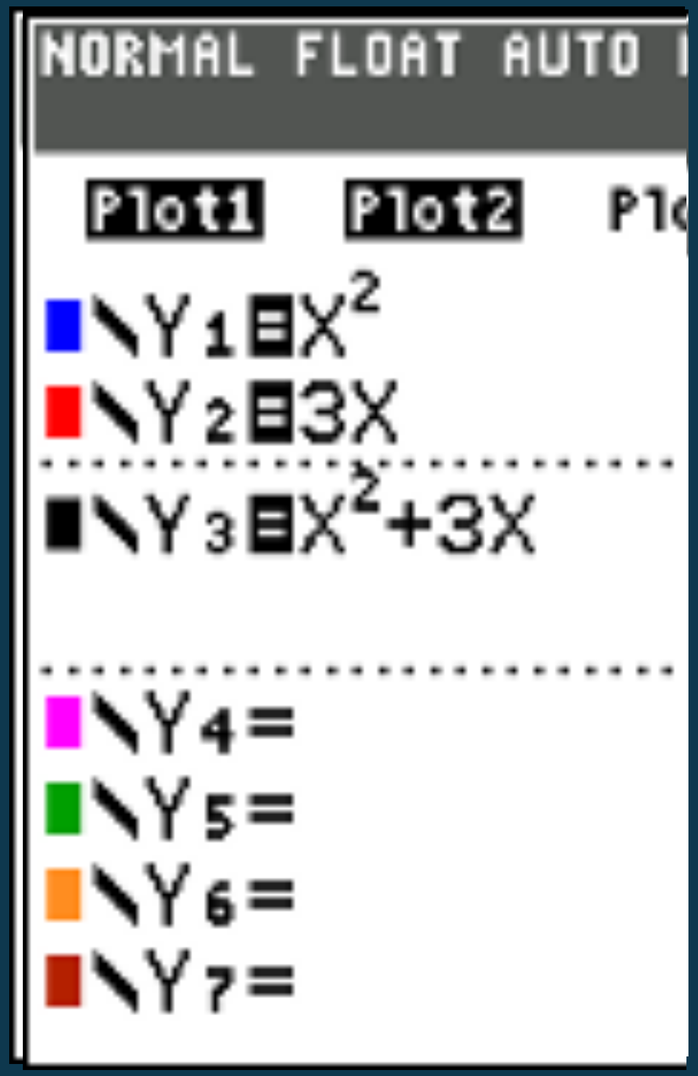


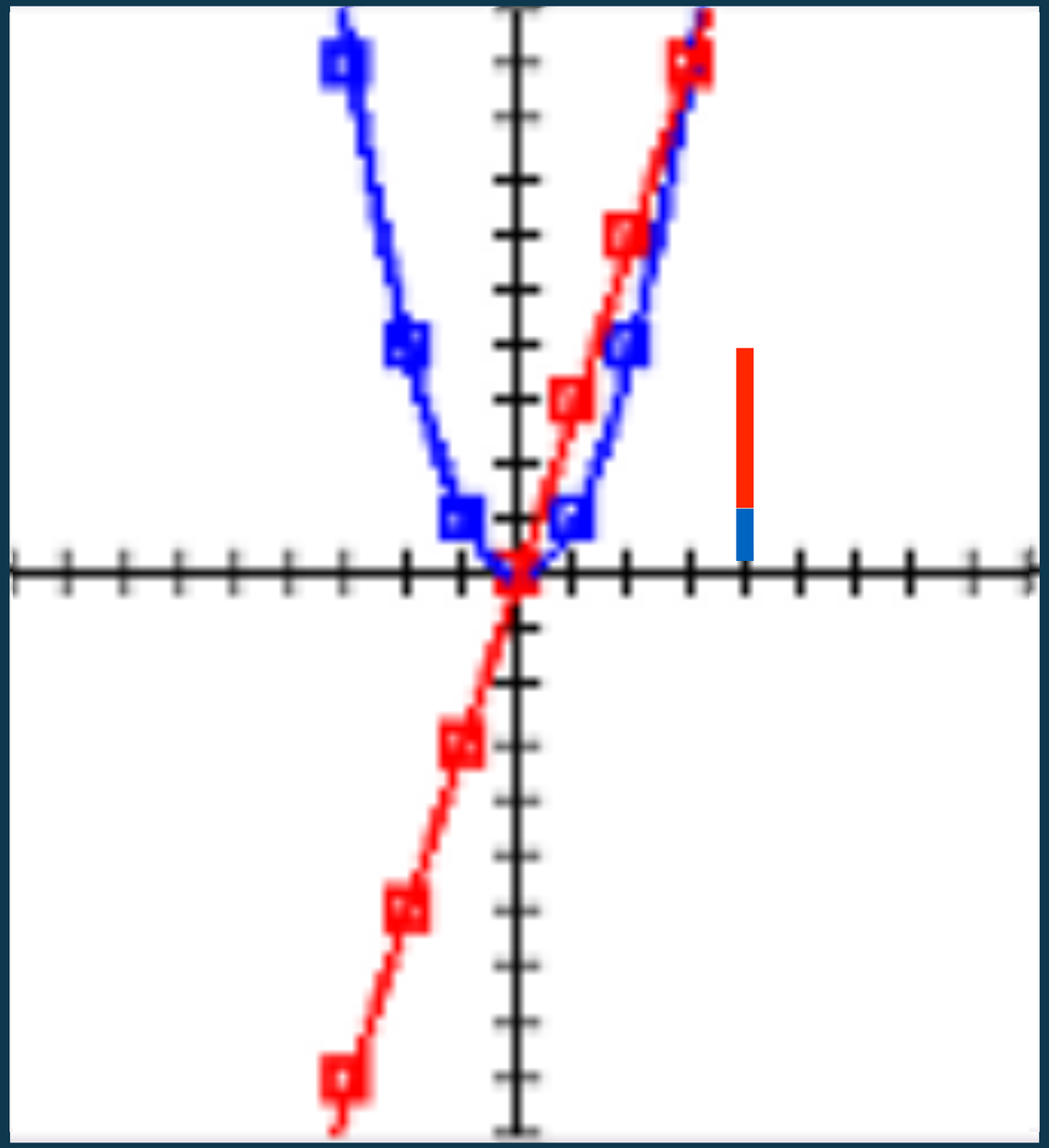

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=




NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

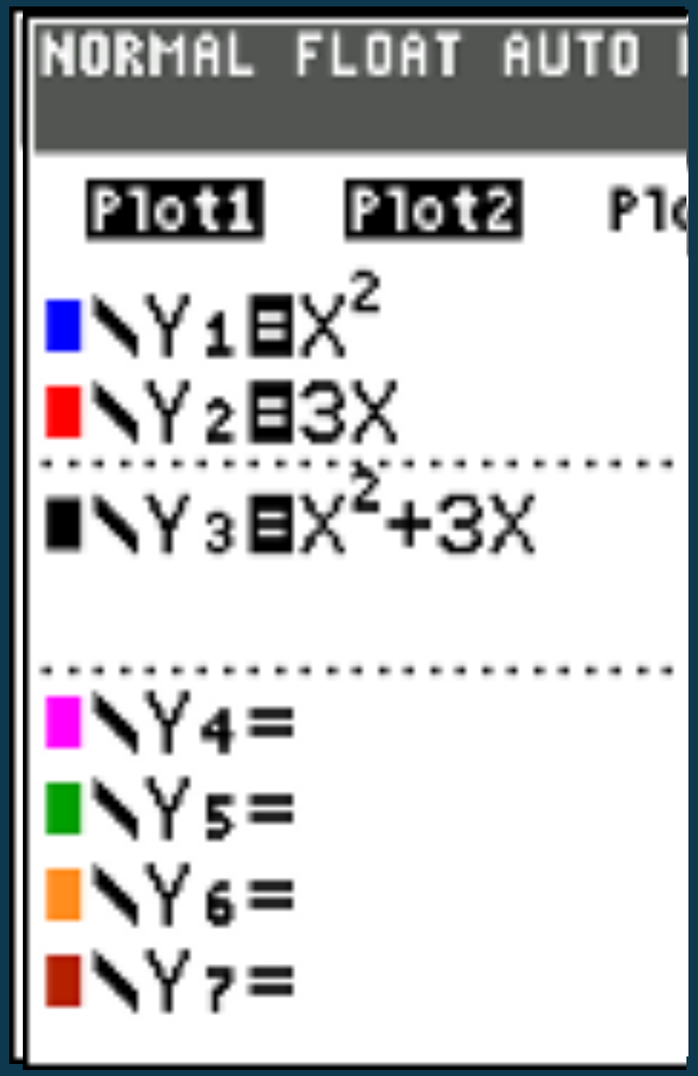


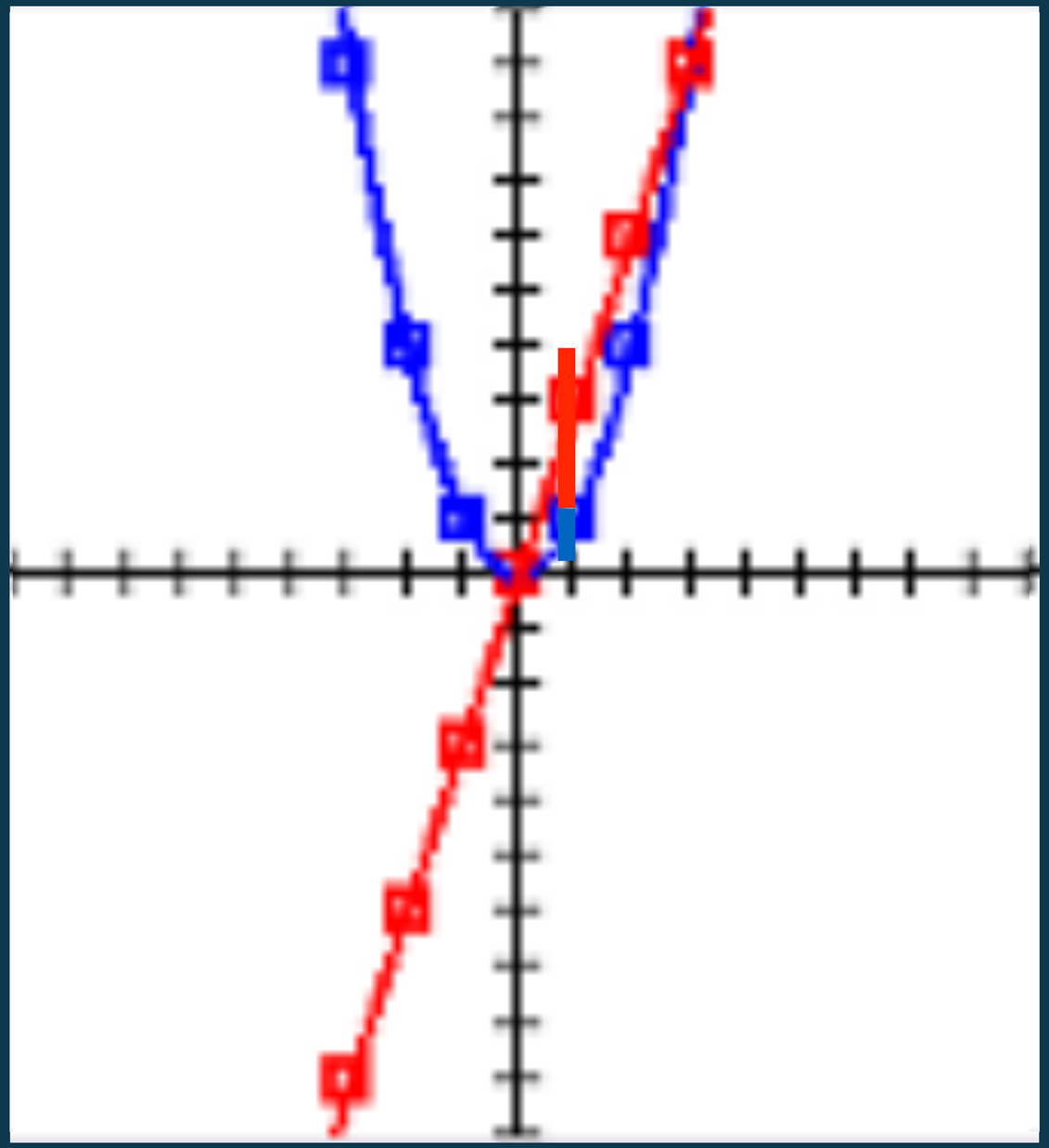


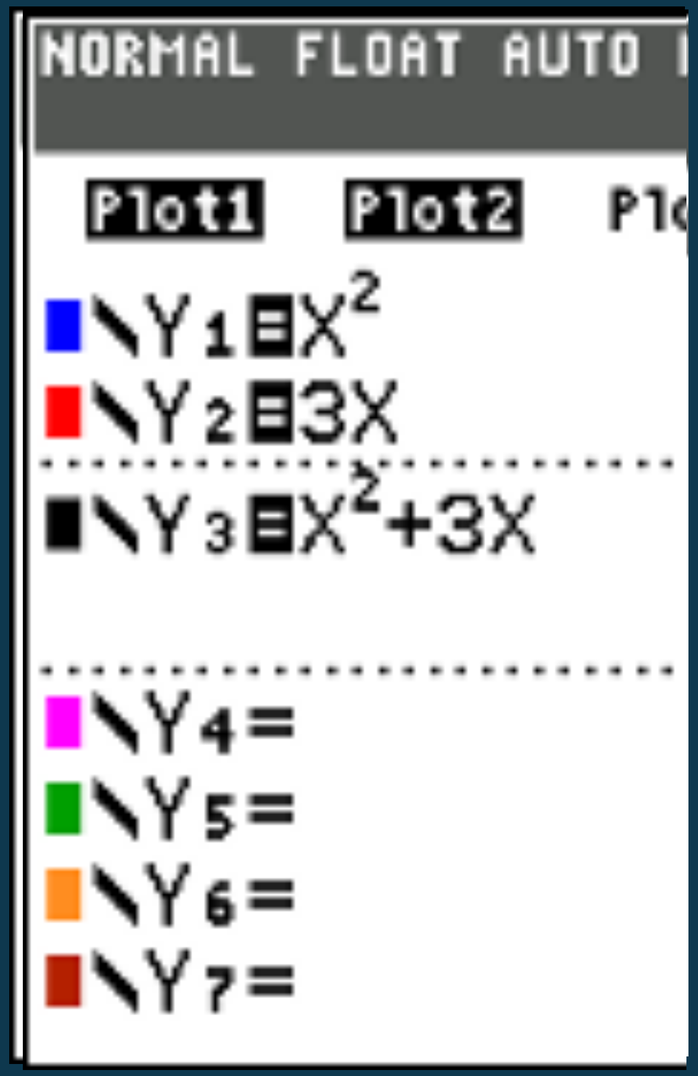


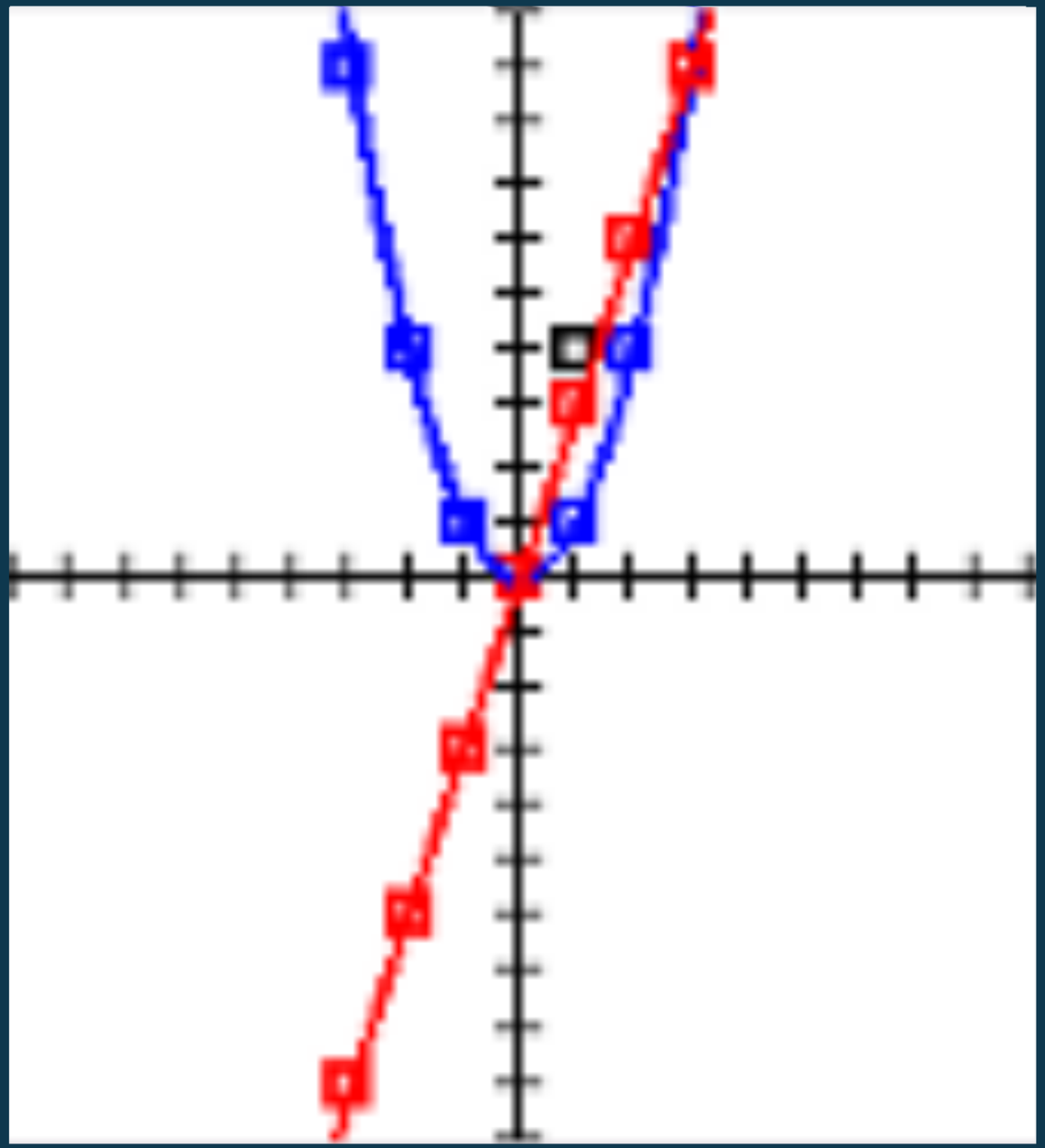
@pwharris

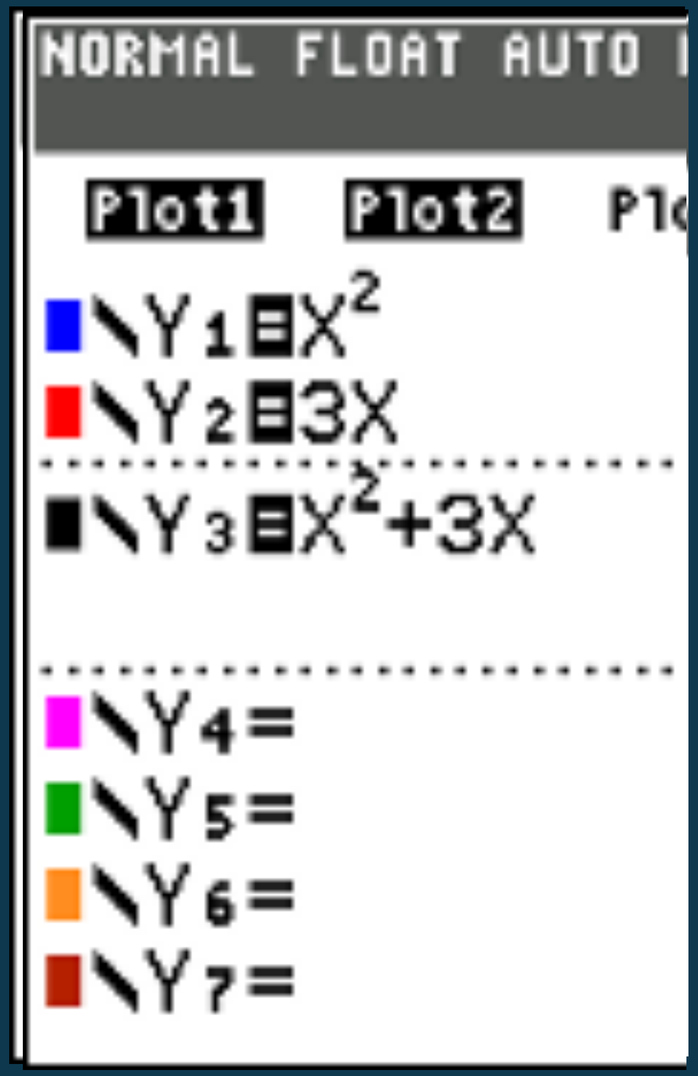
PAM HARRIS

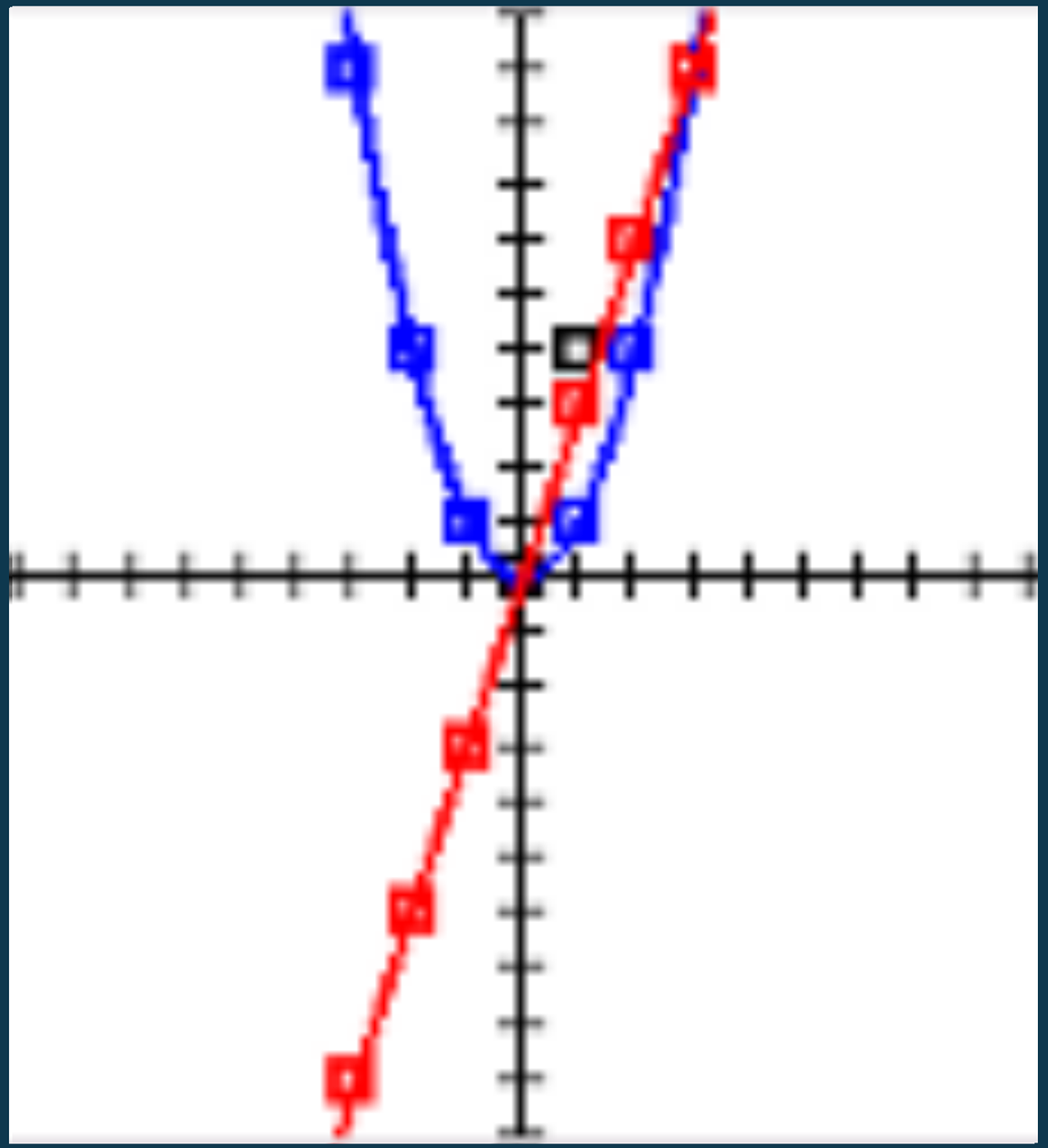

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

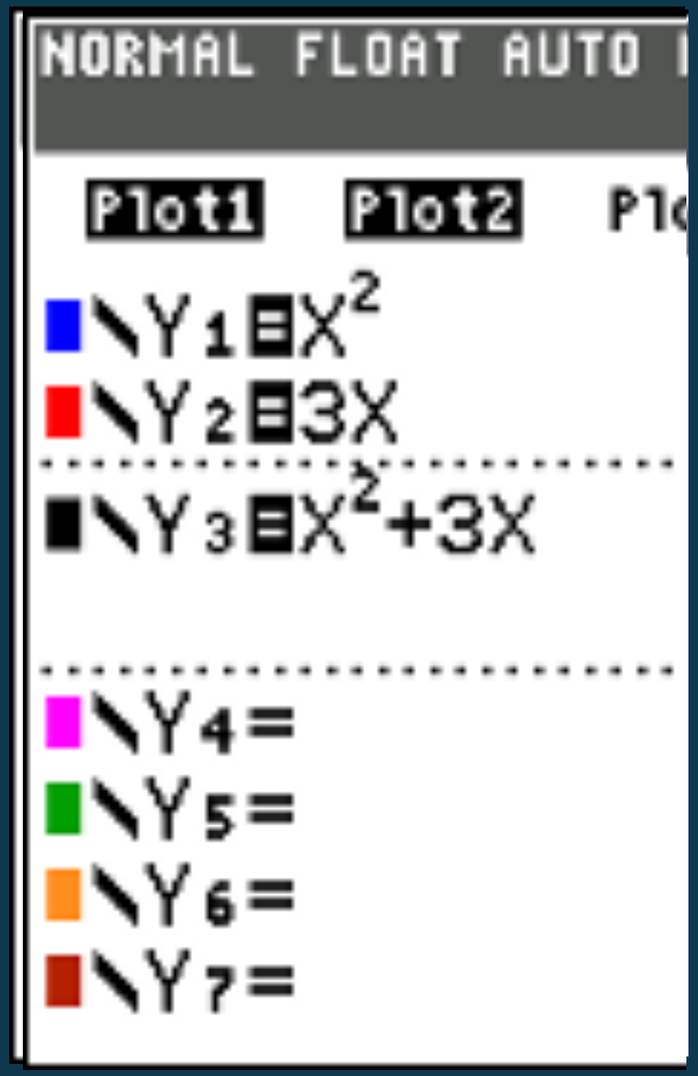


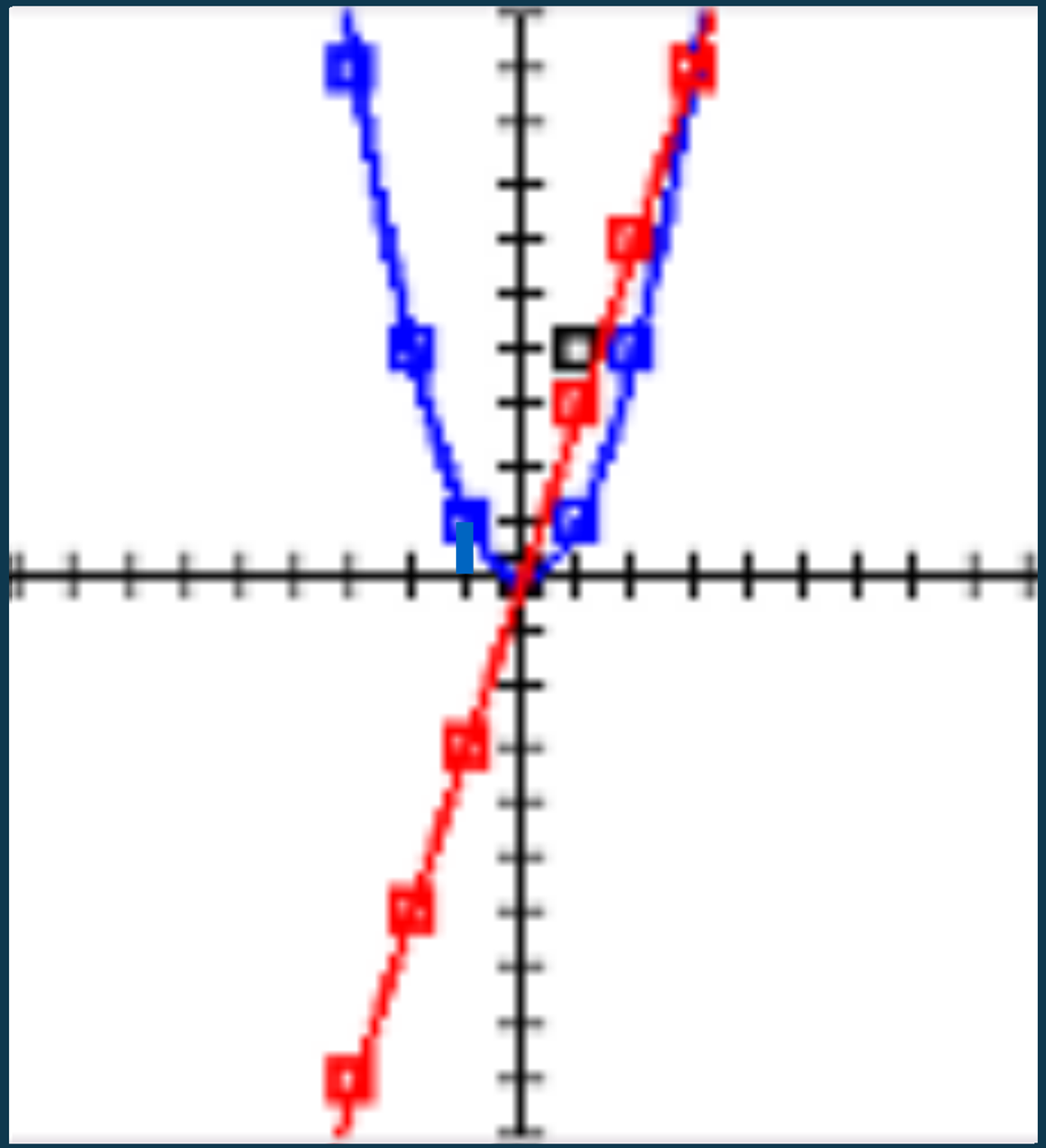

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

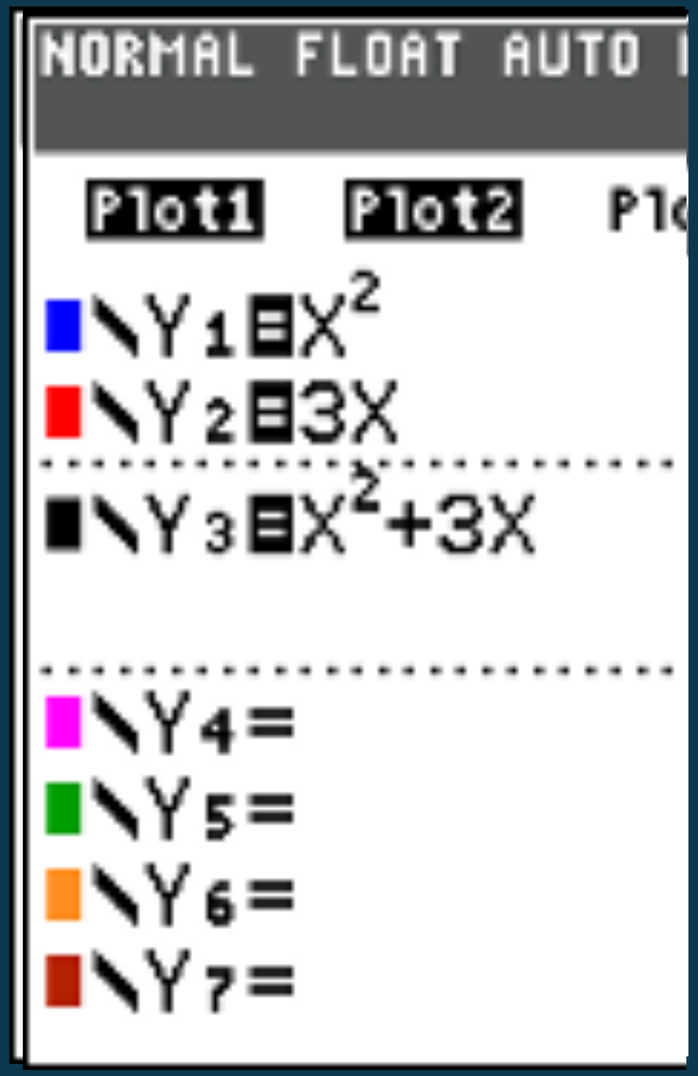


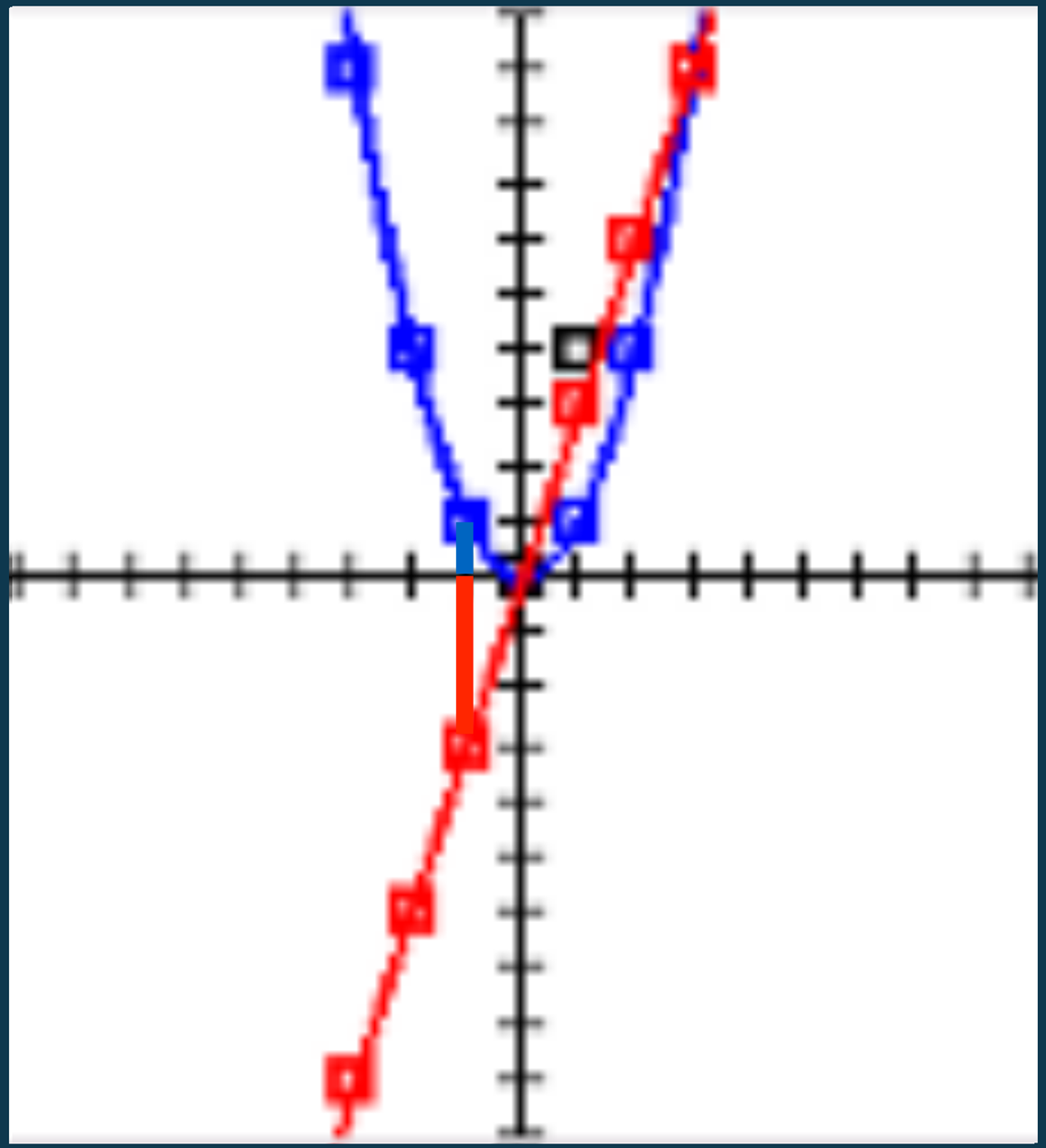

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

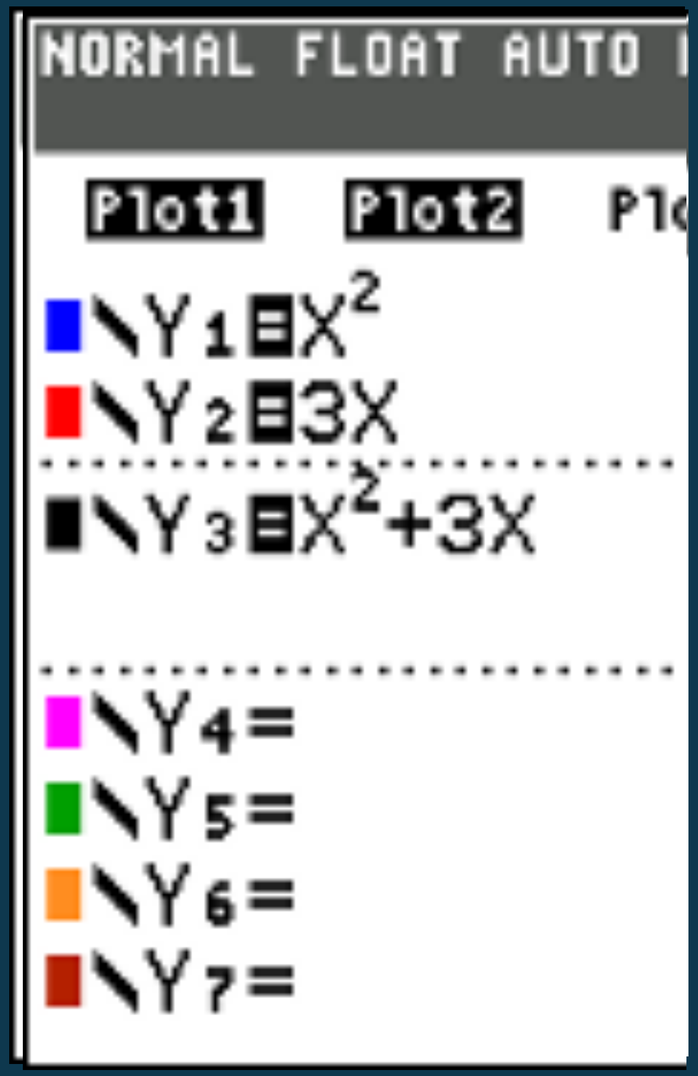


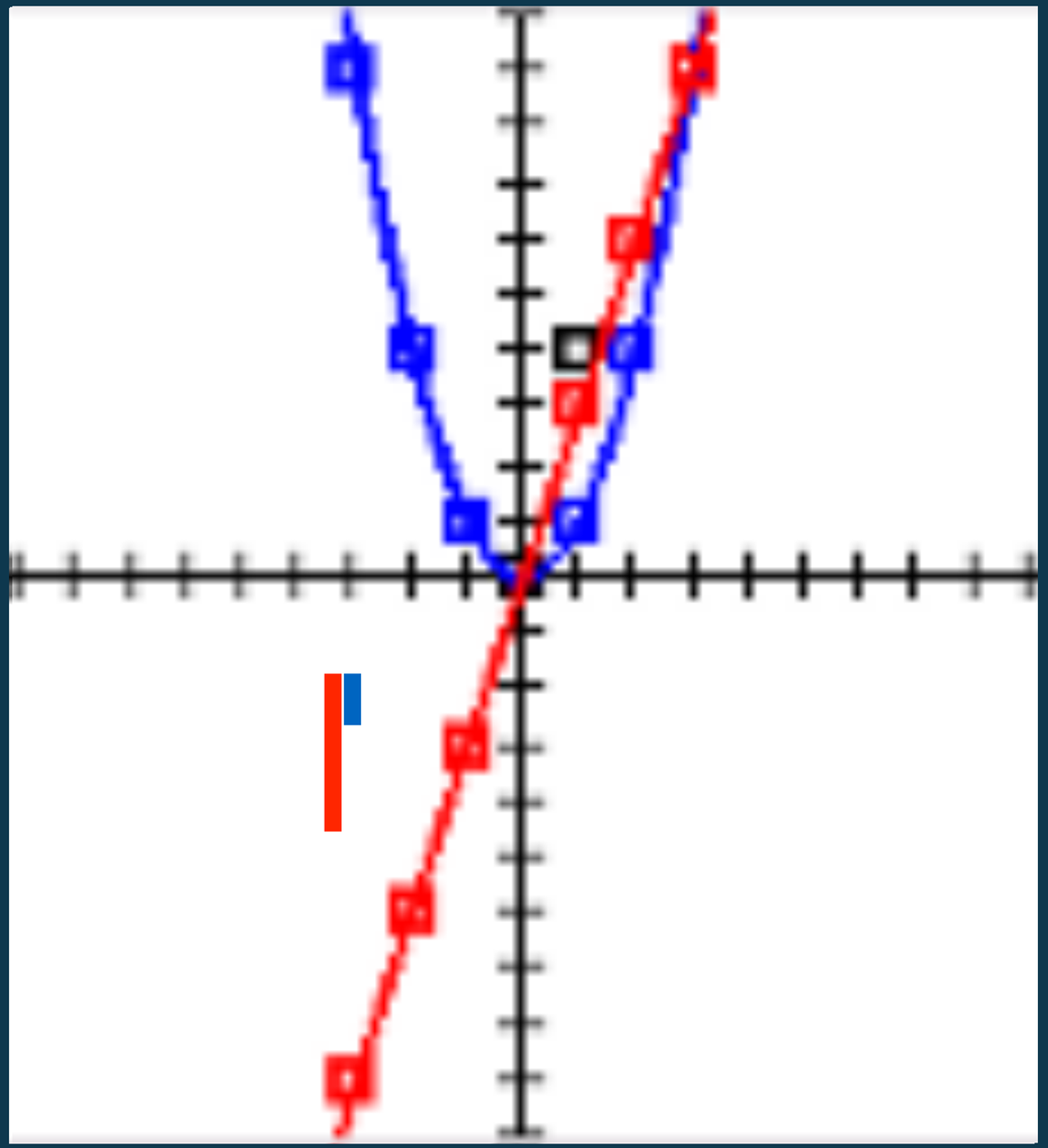

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

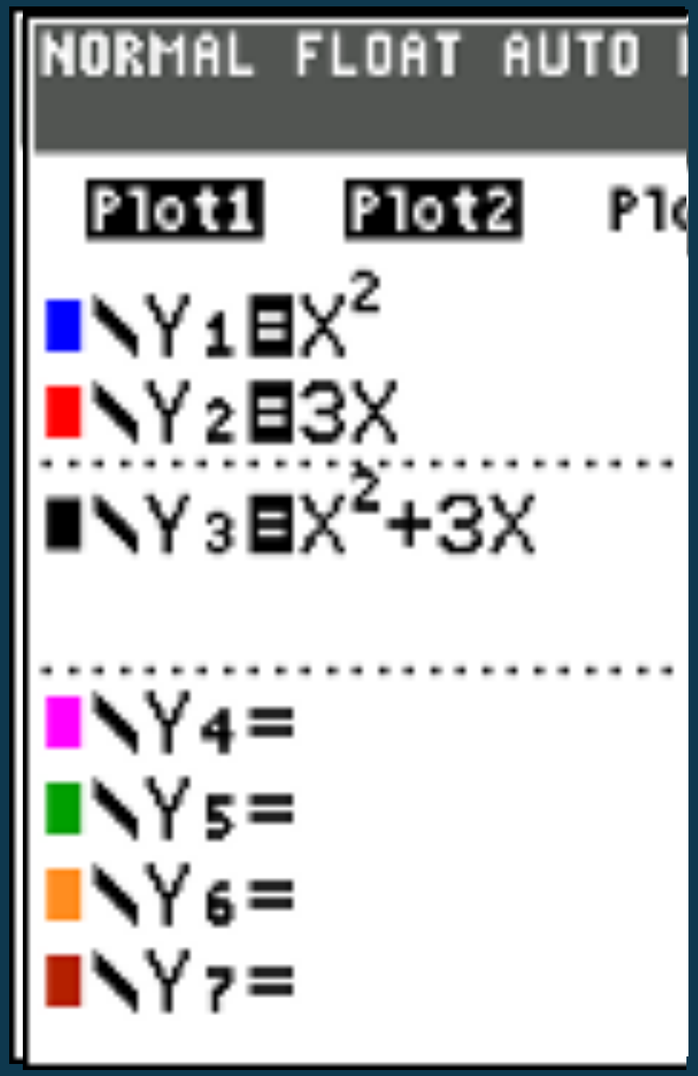


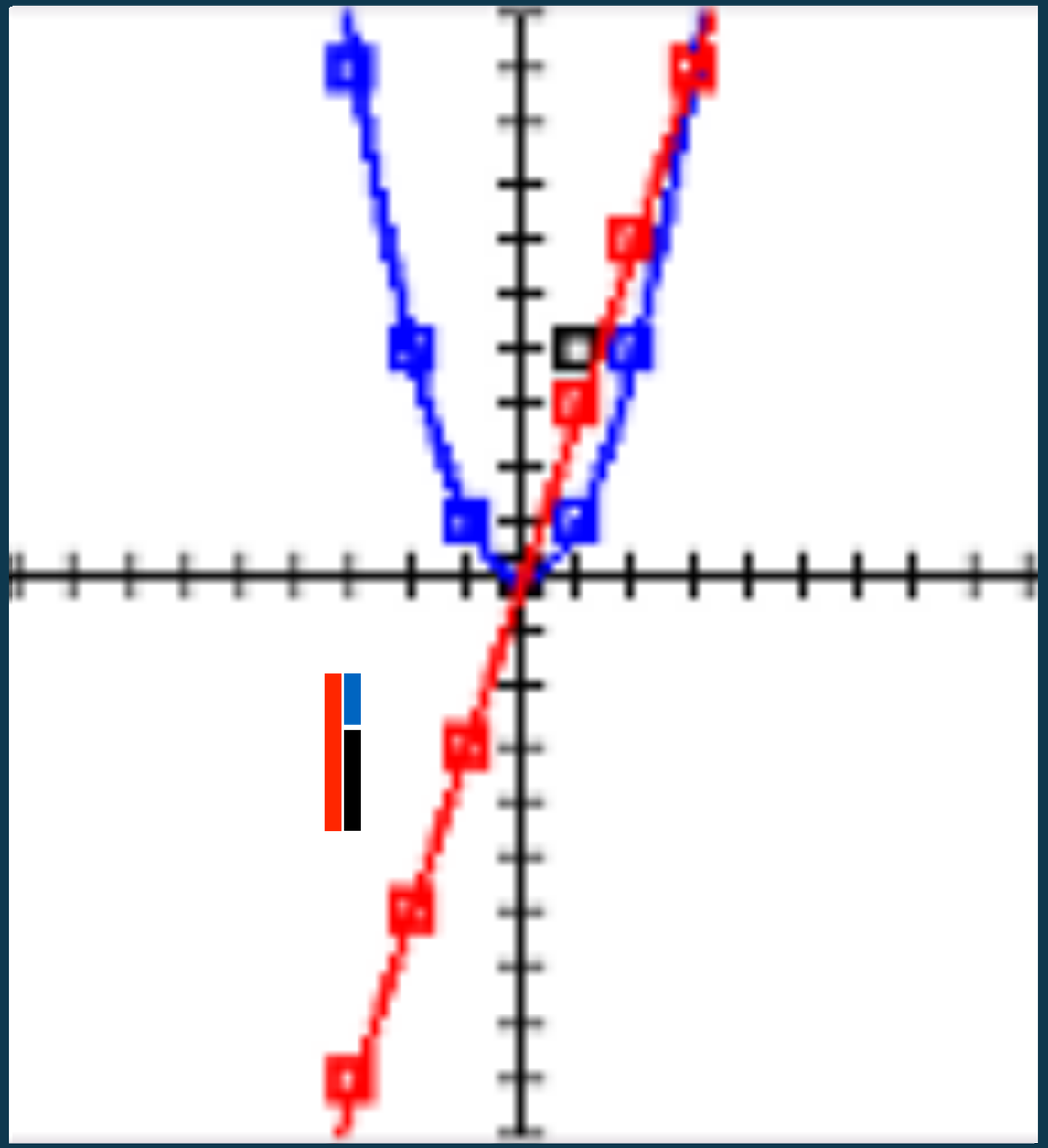

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

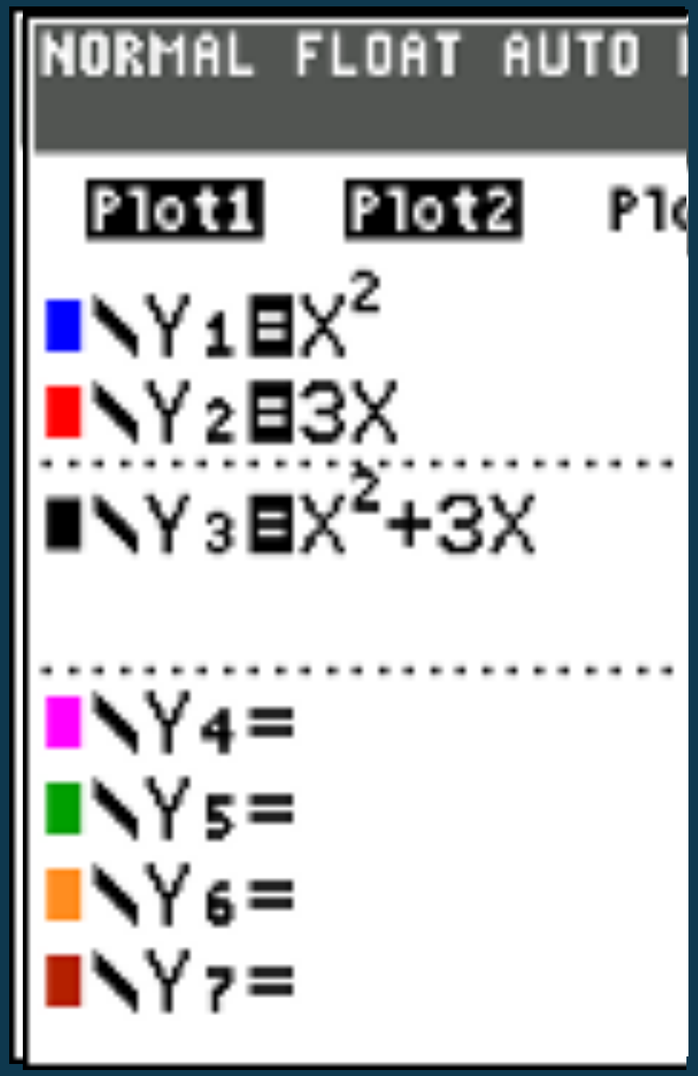


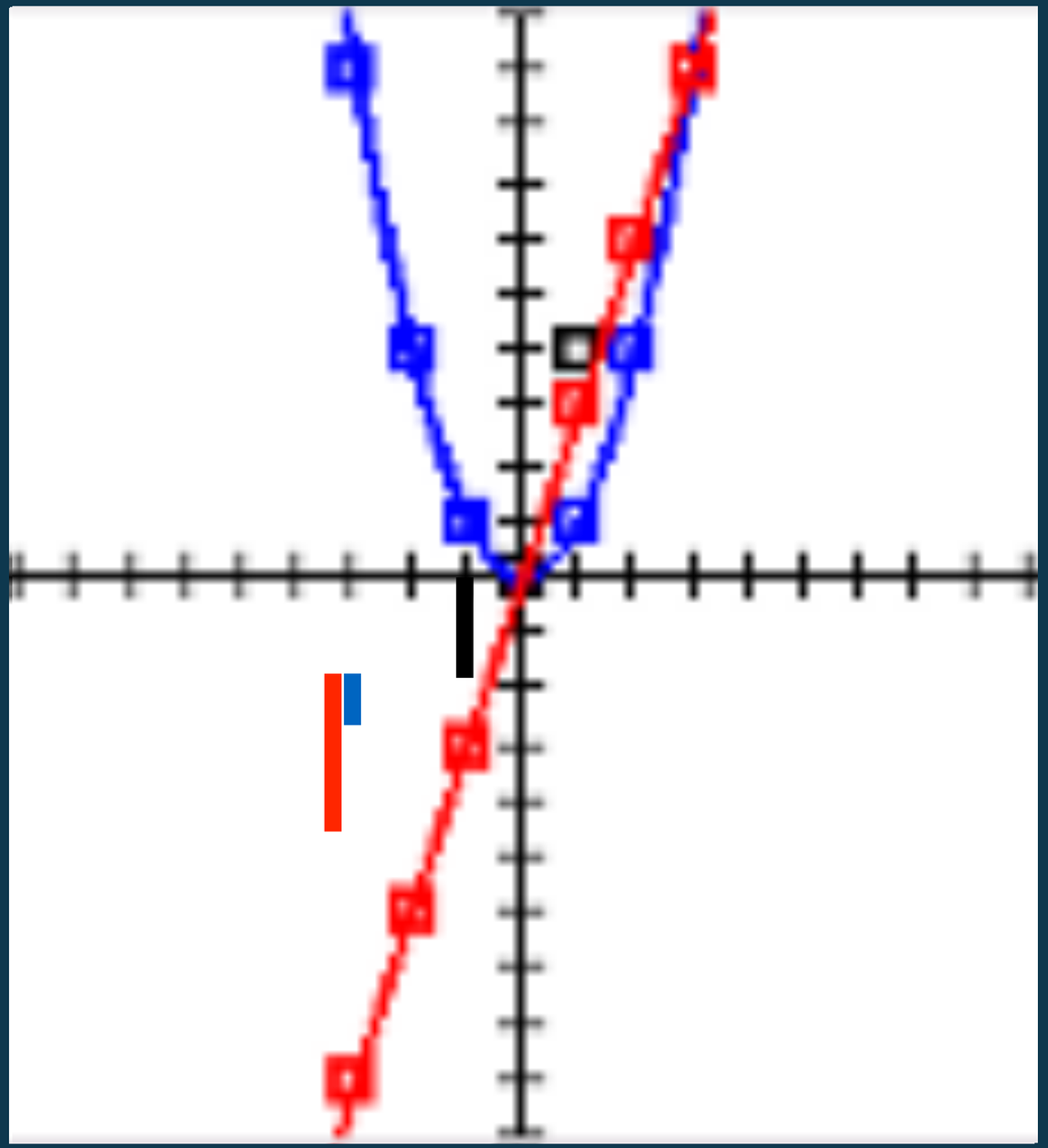

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=




NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=

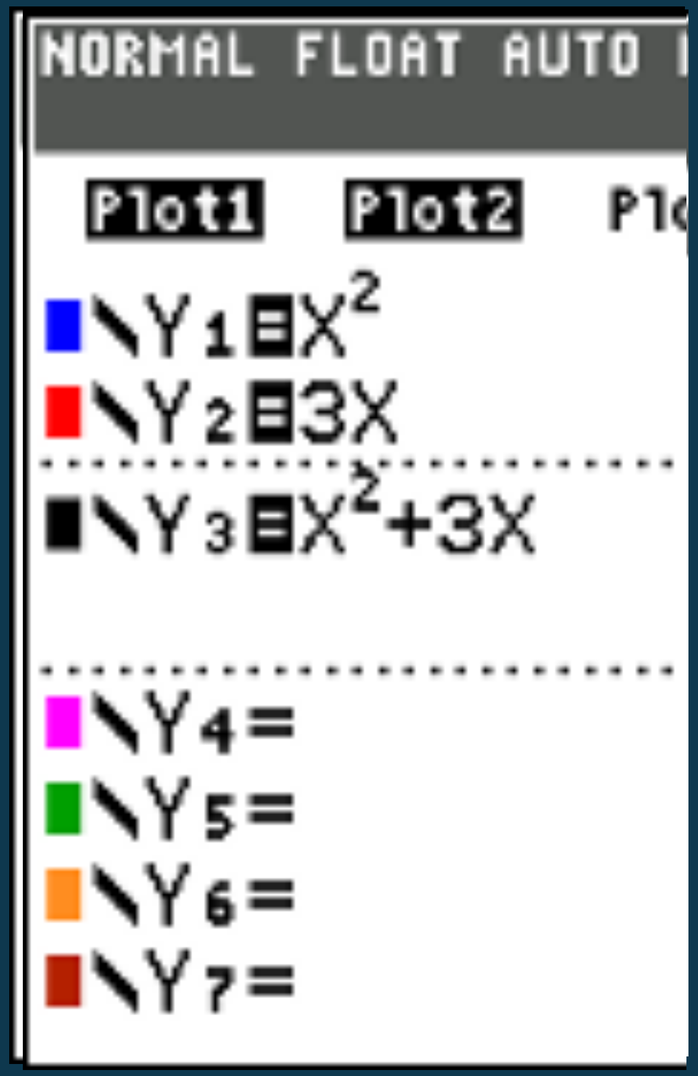


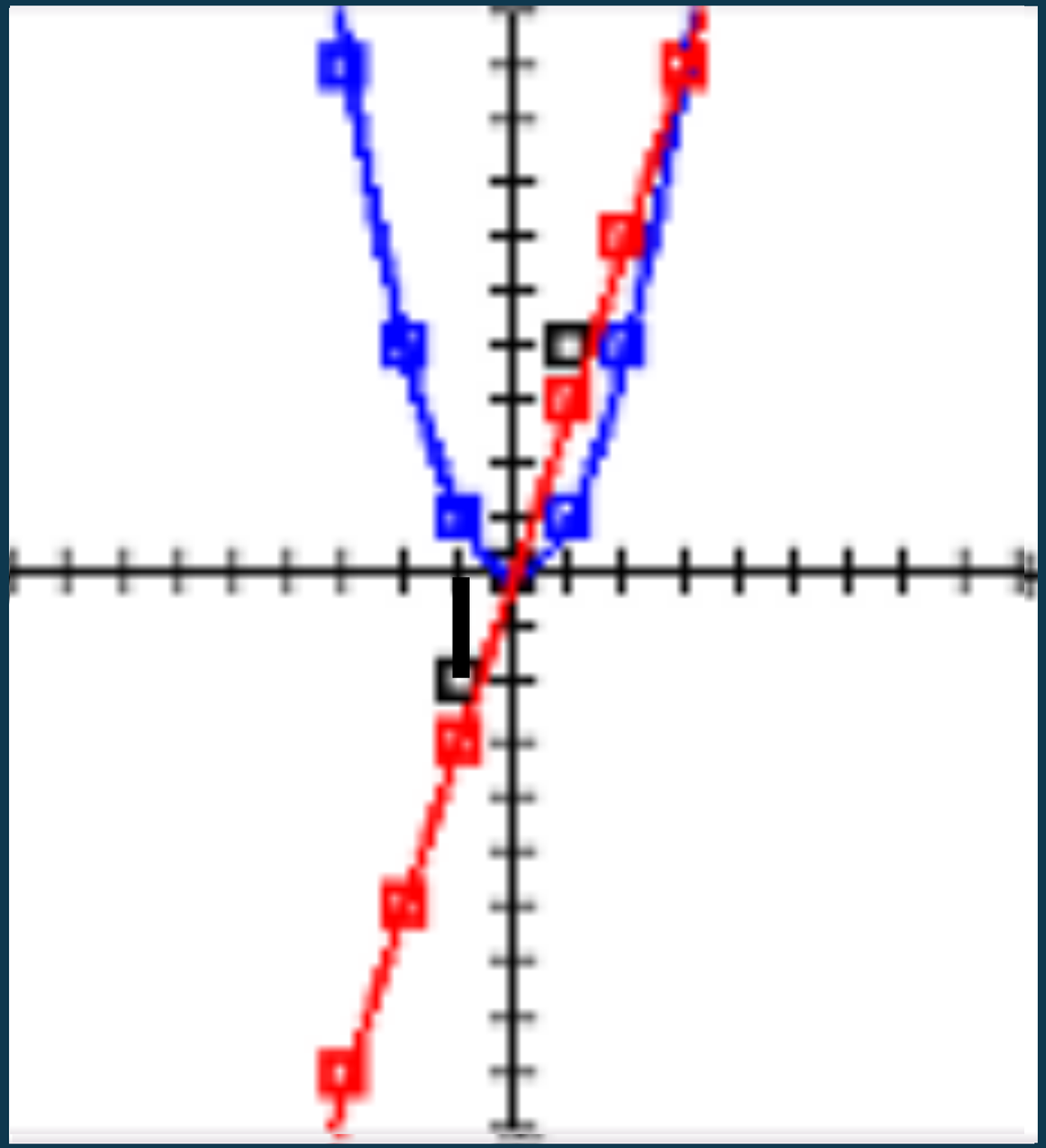


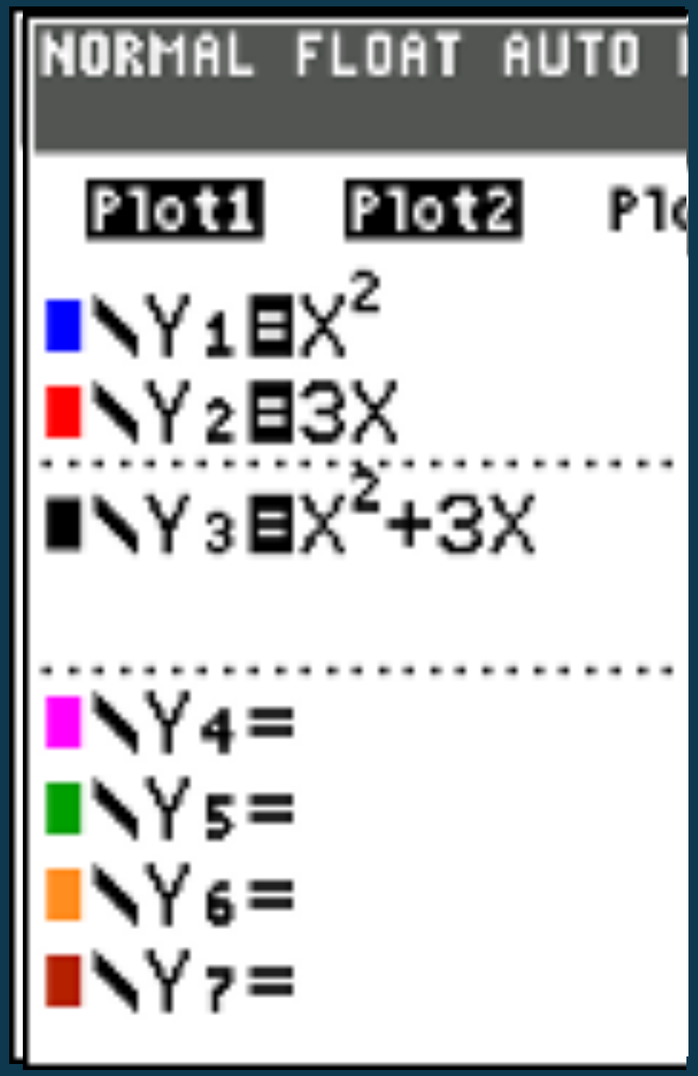


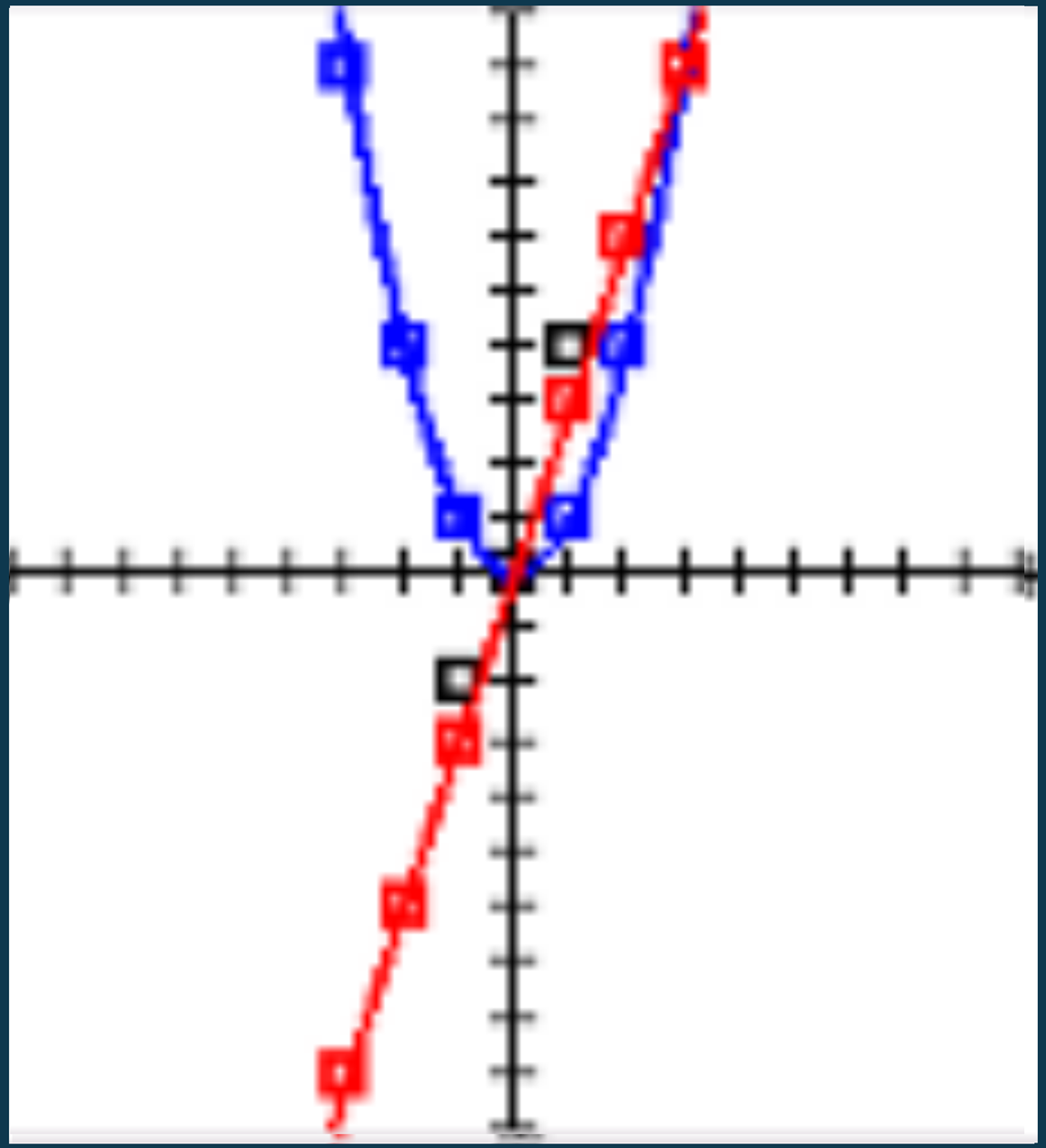
@pwharris

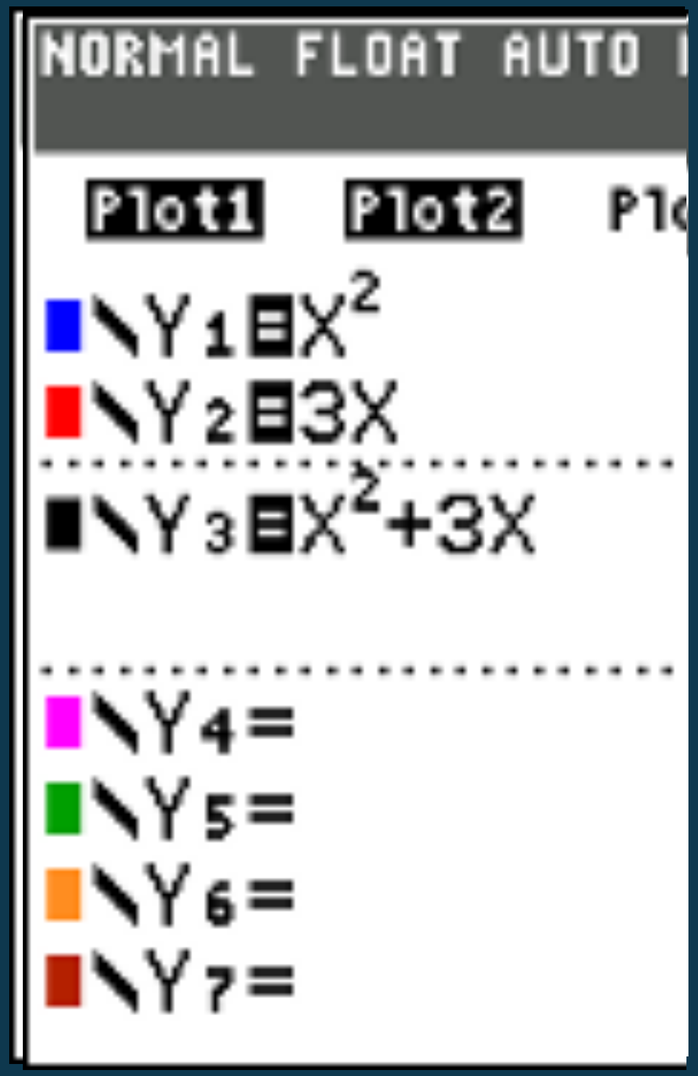
PAM HARRIS

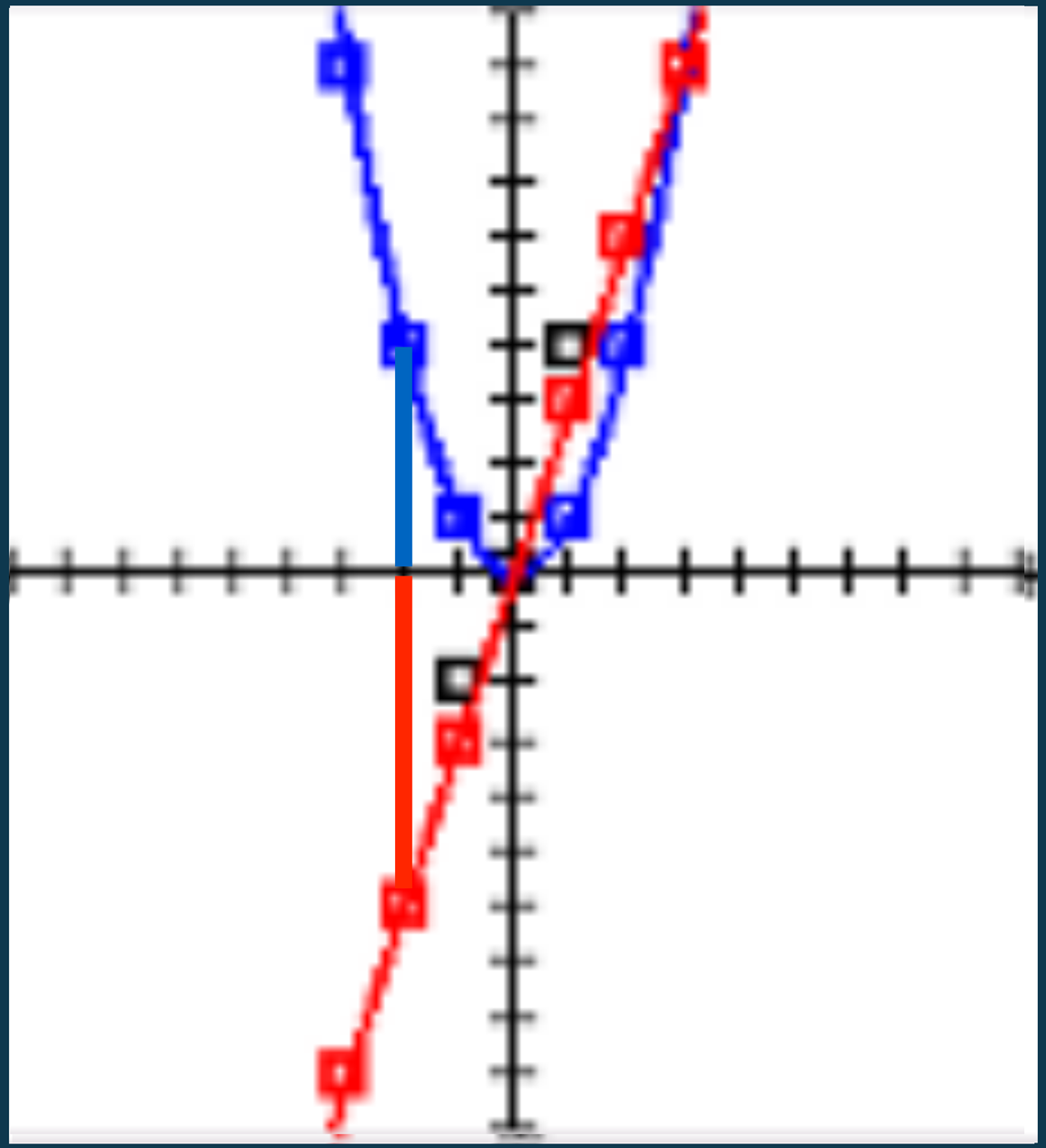

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

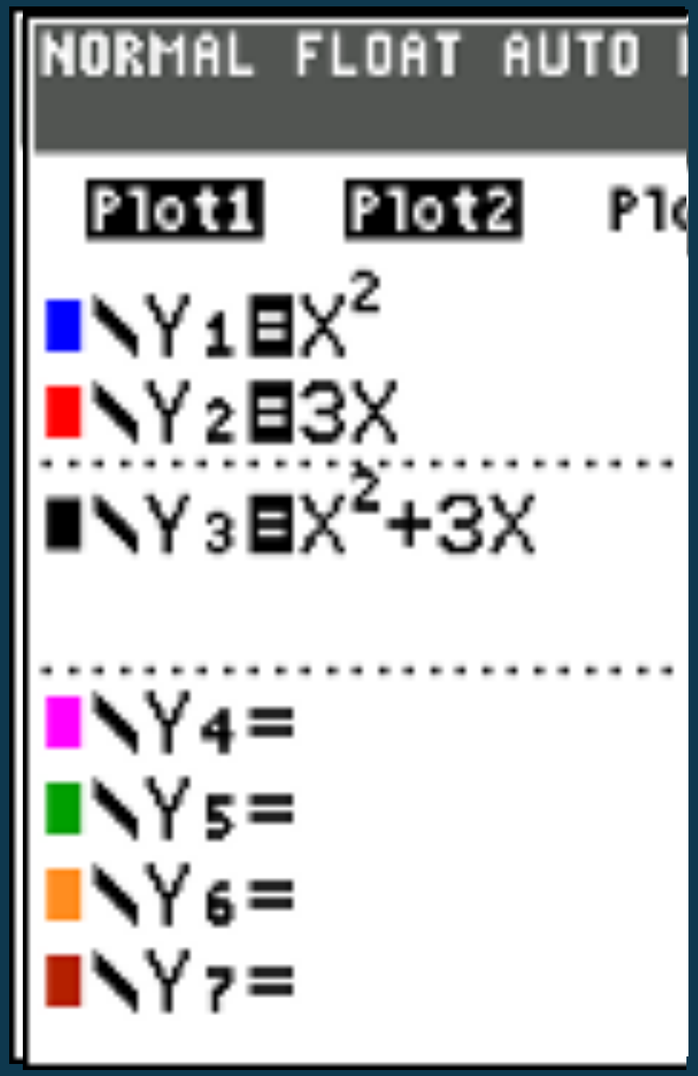


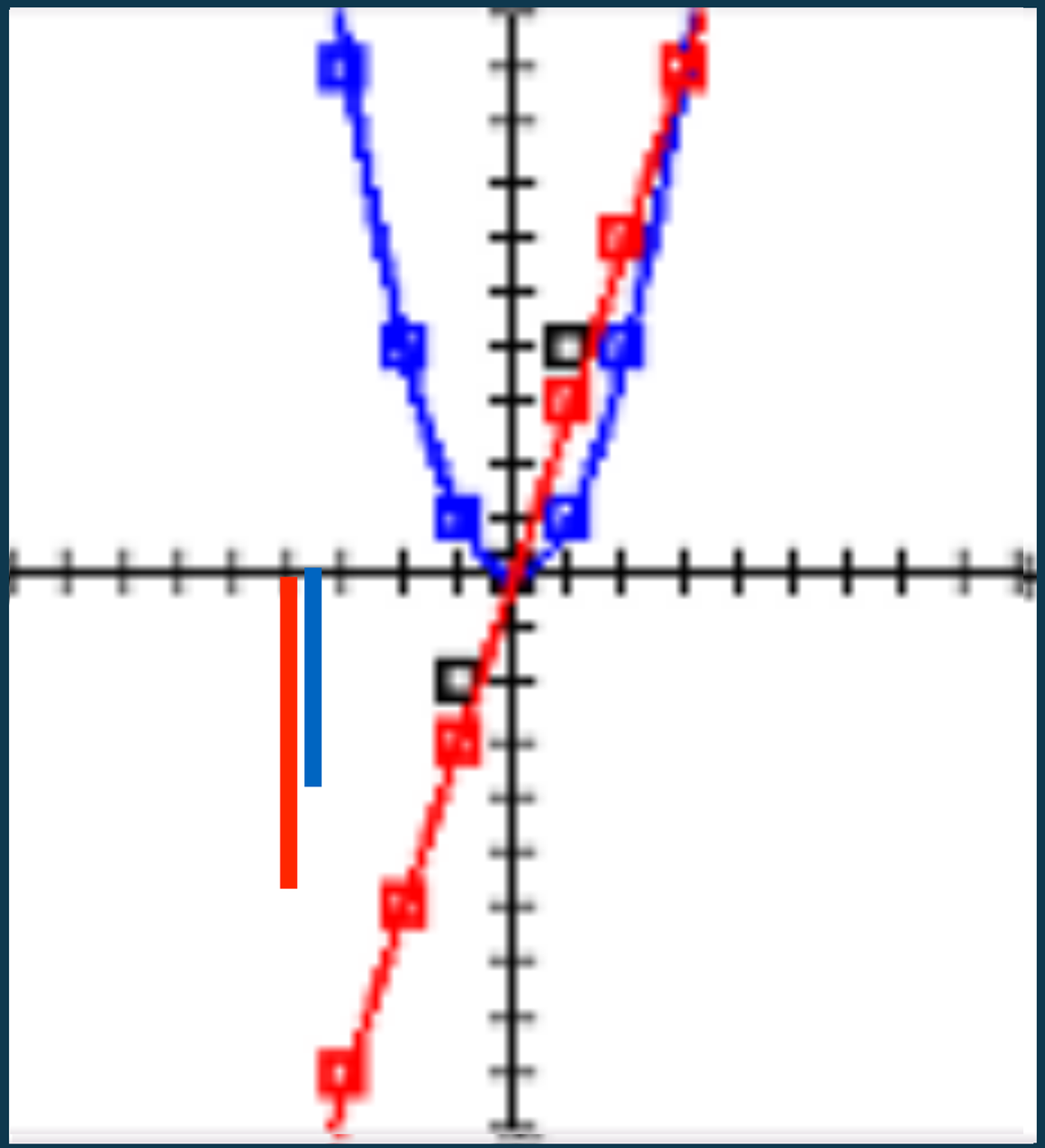

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

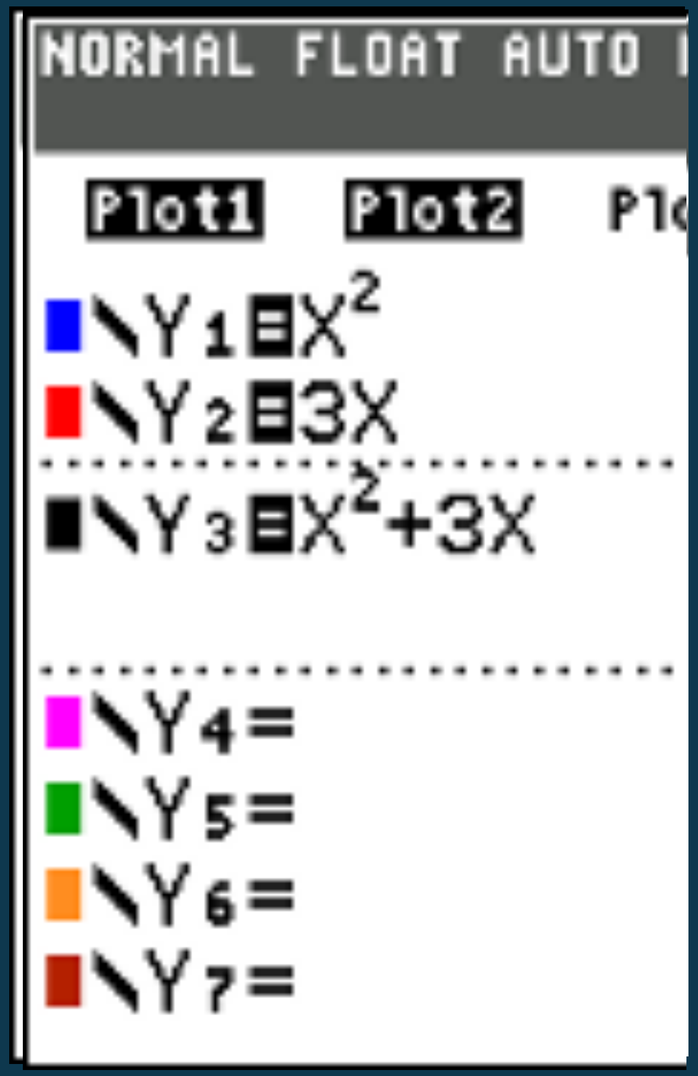


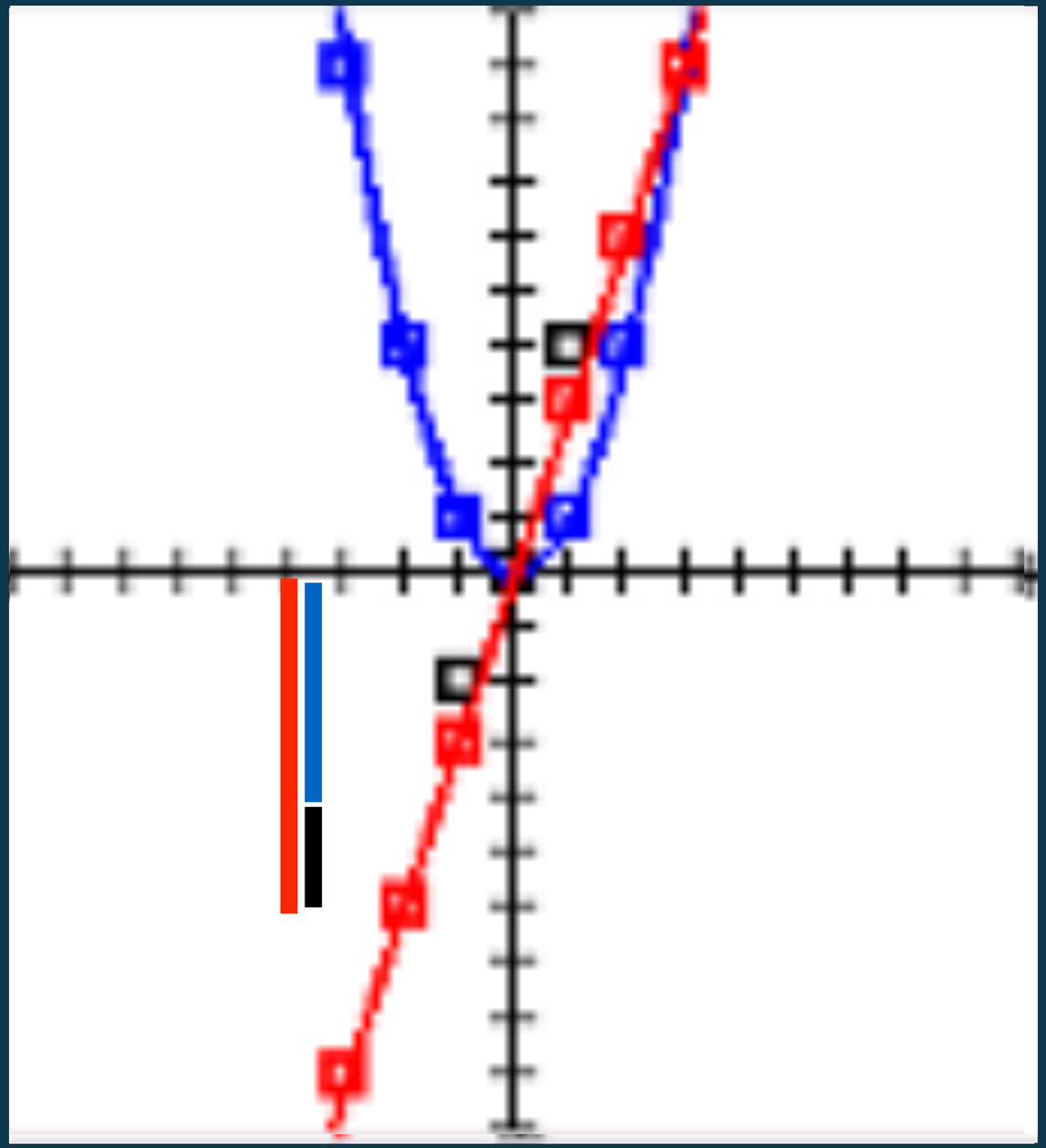

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

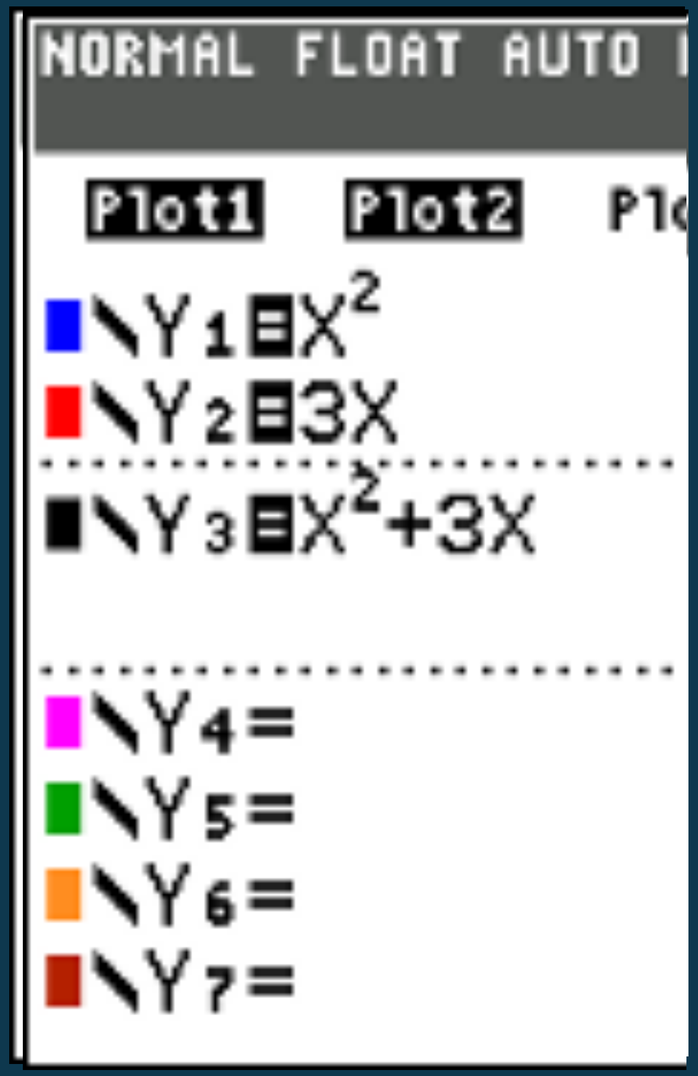


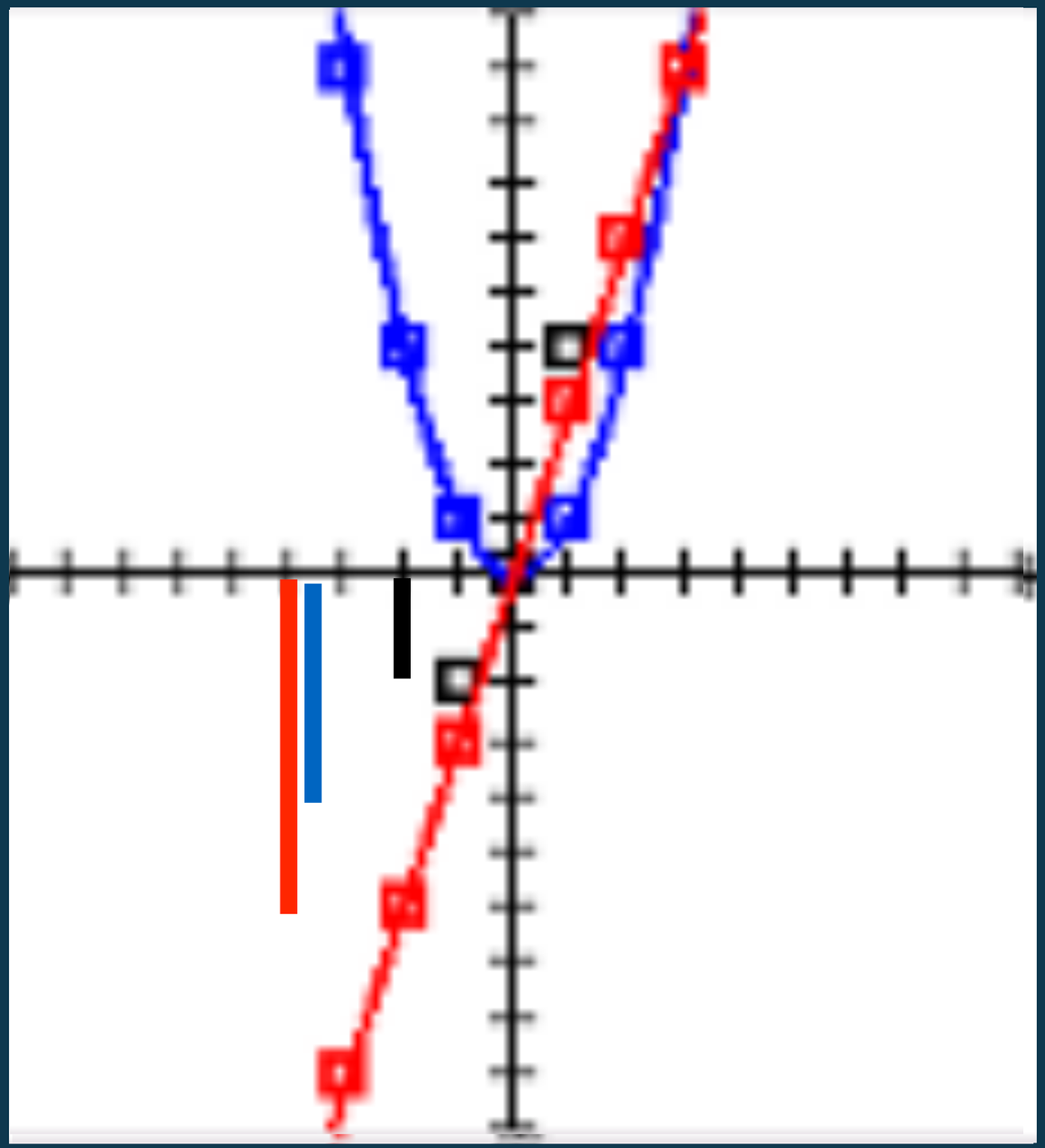

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

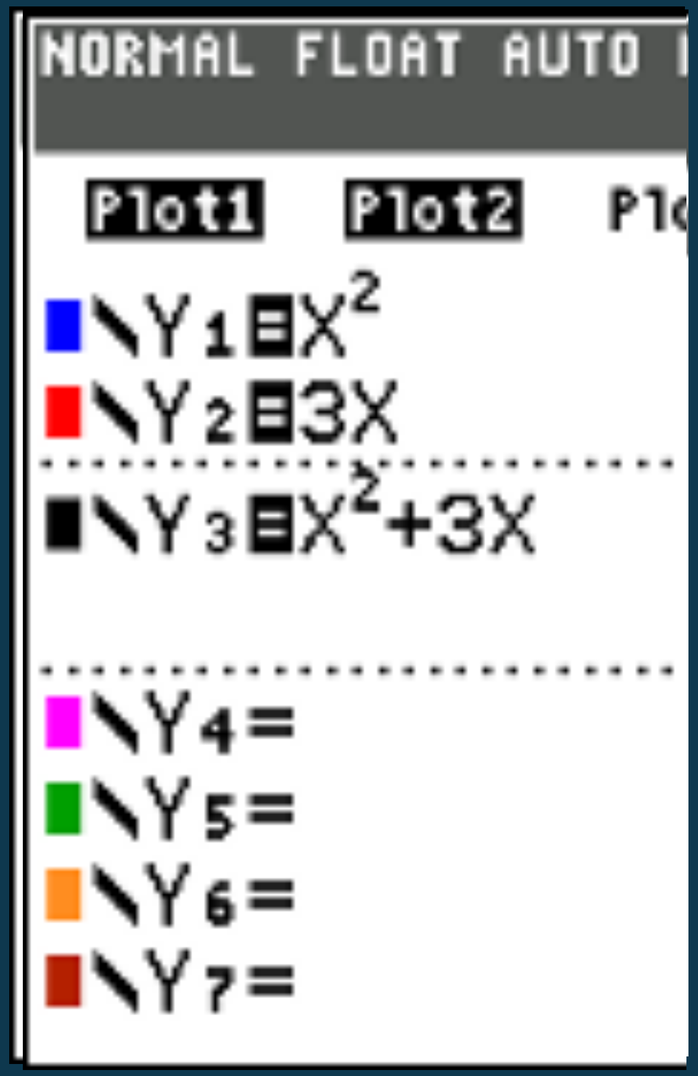


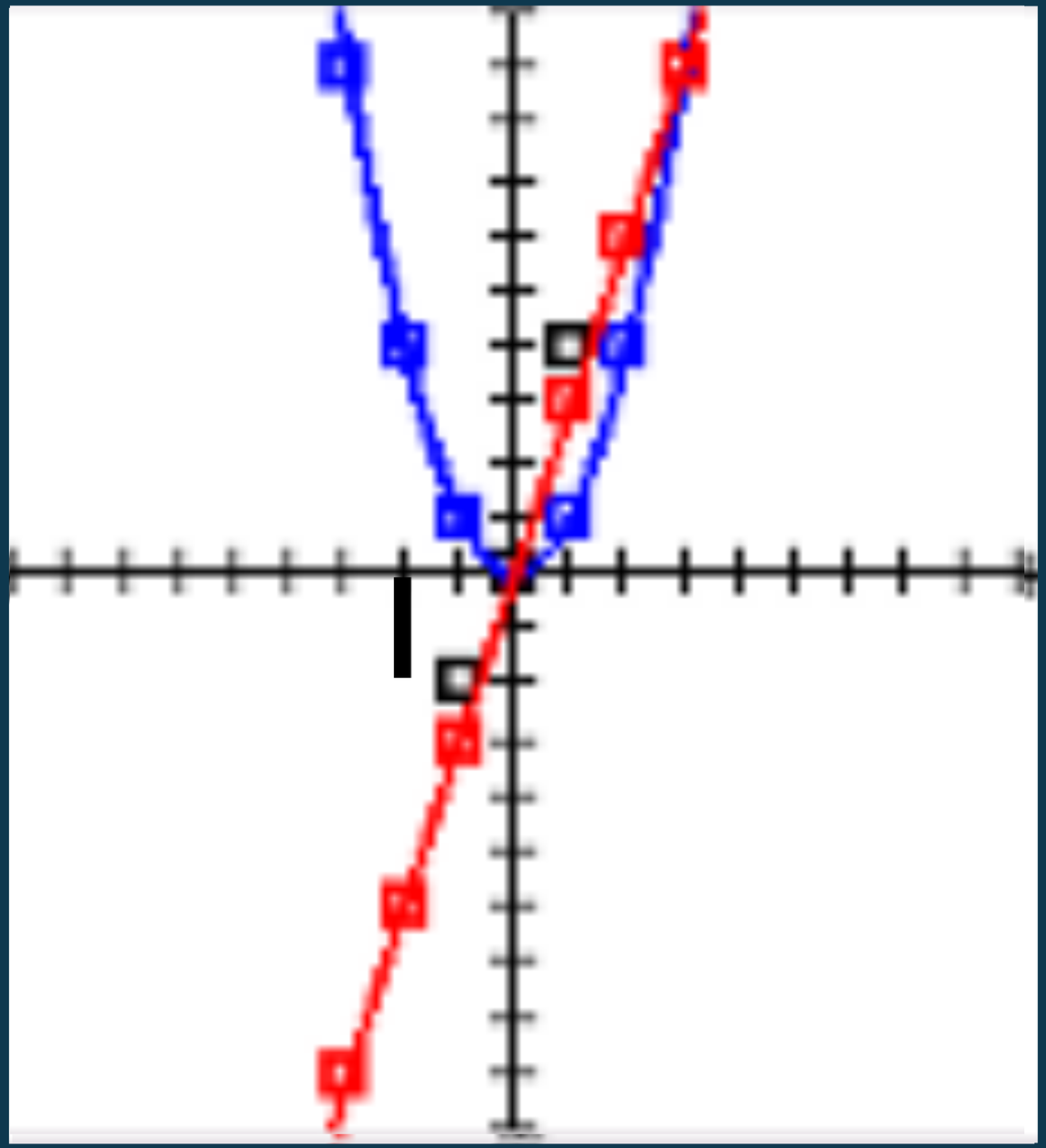

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=

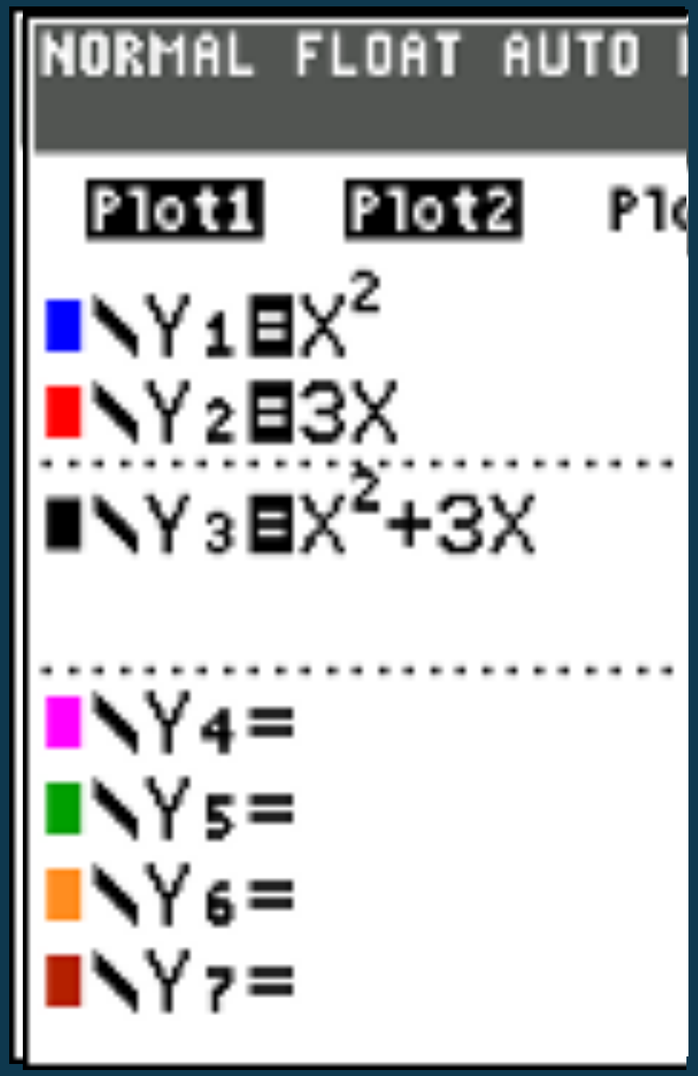


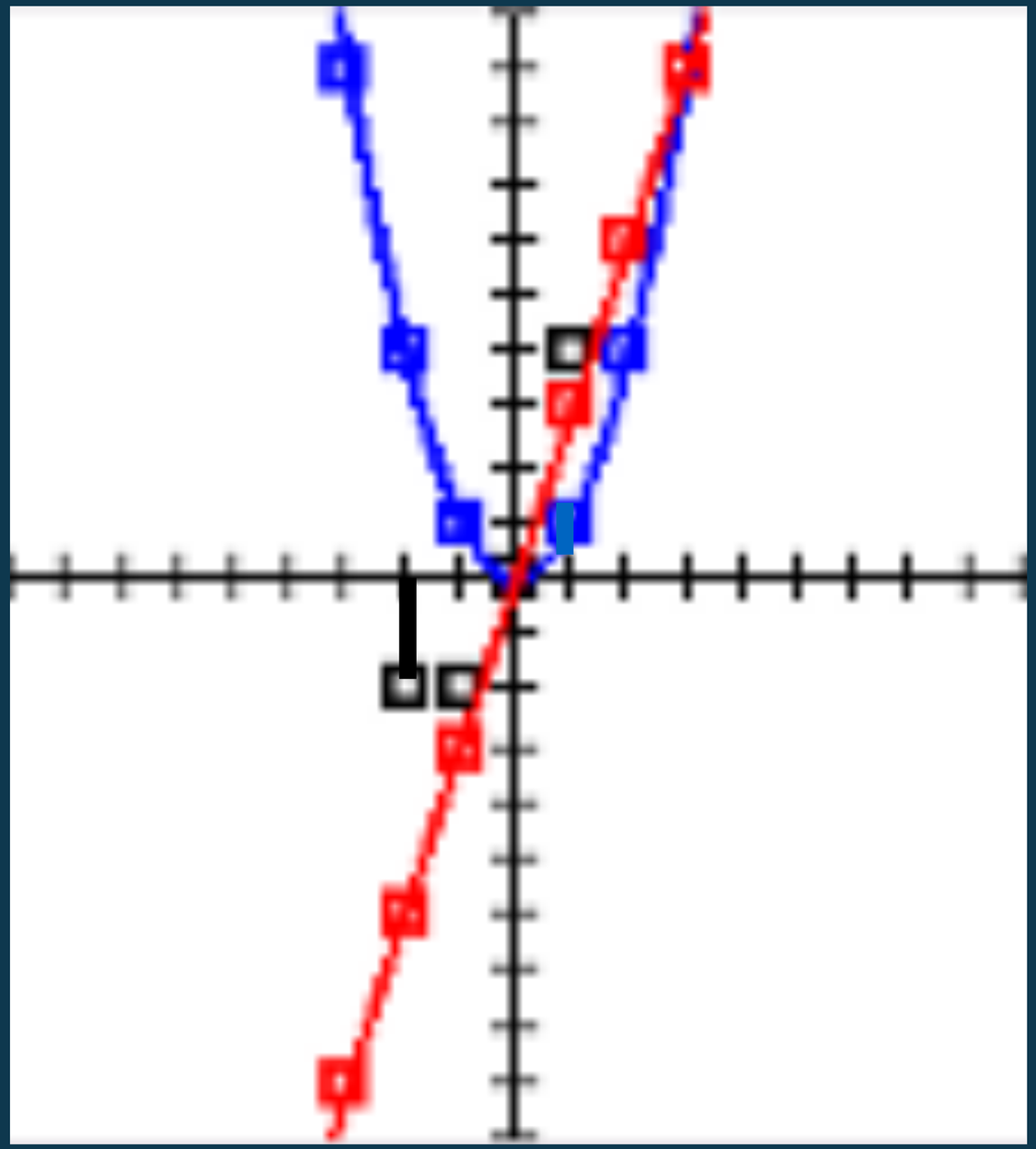

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

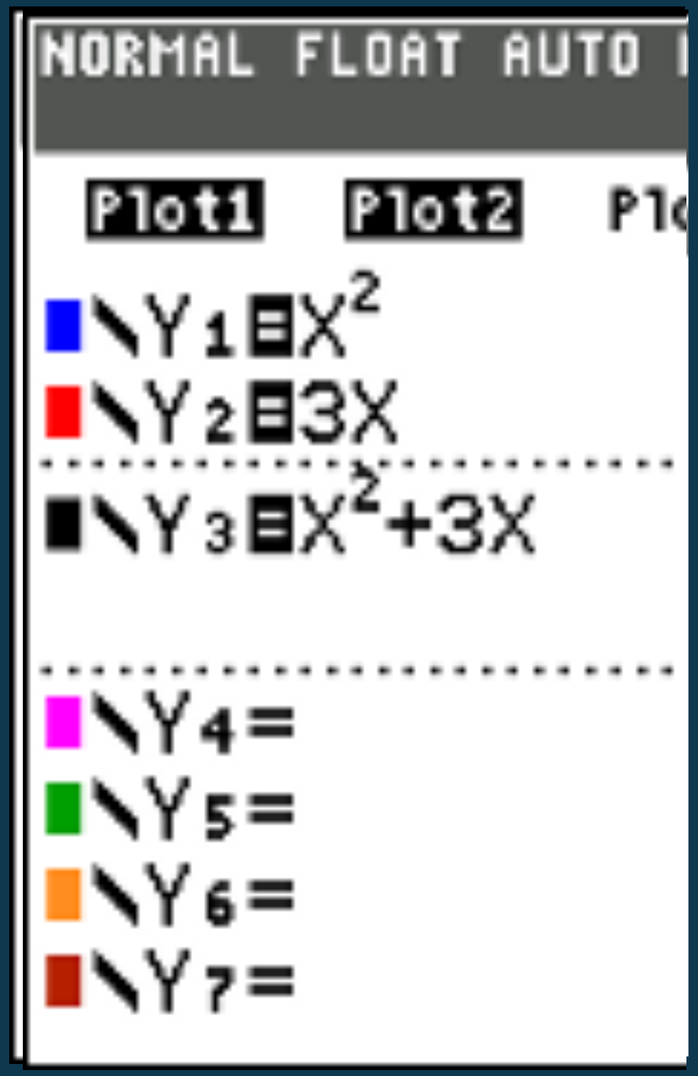


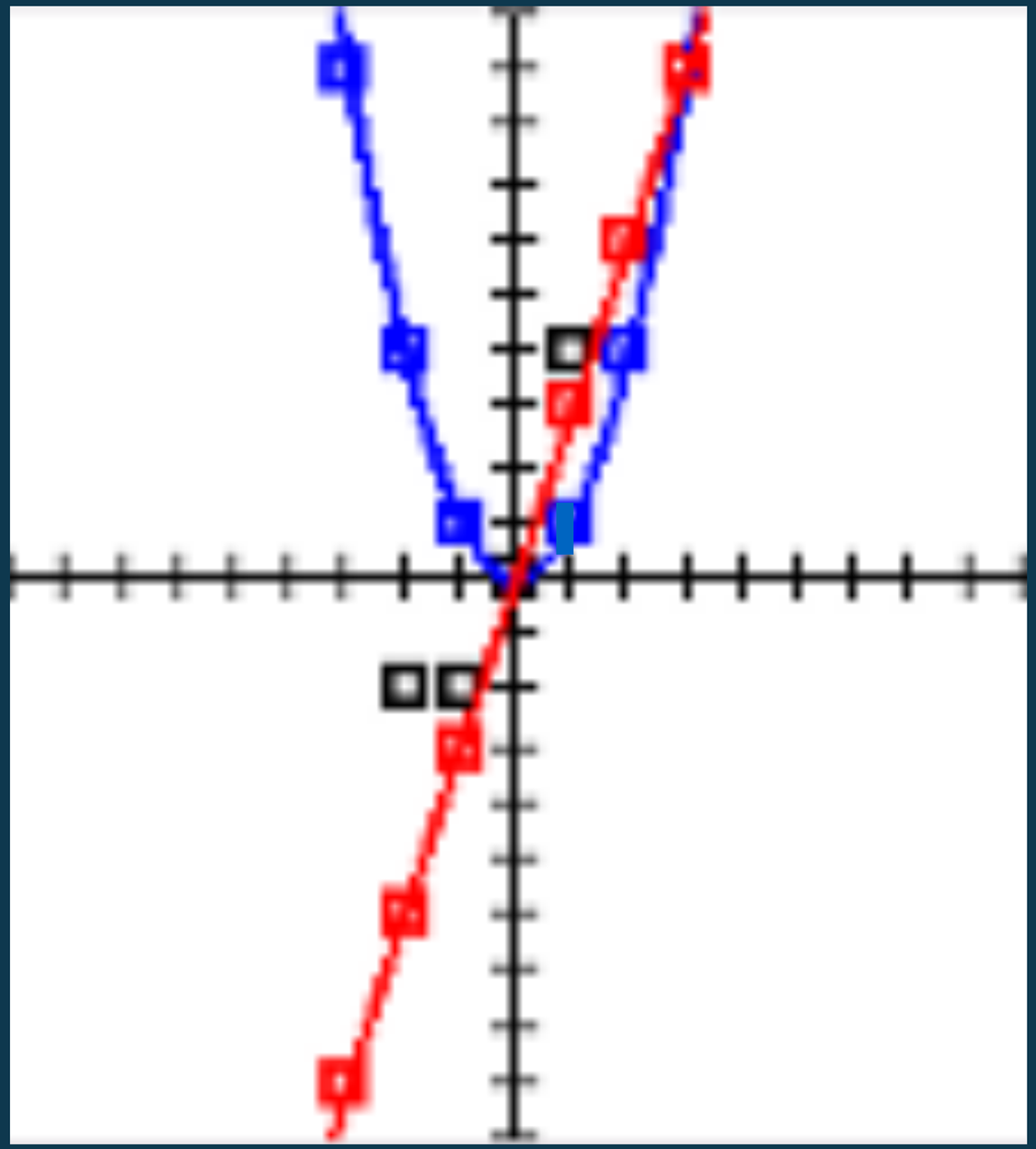

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

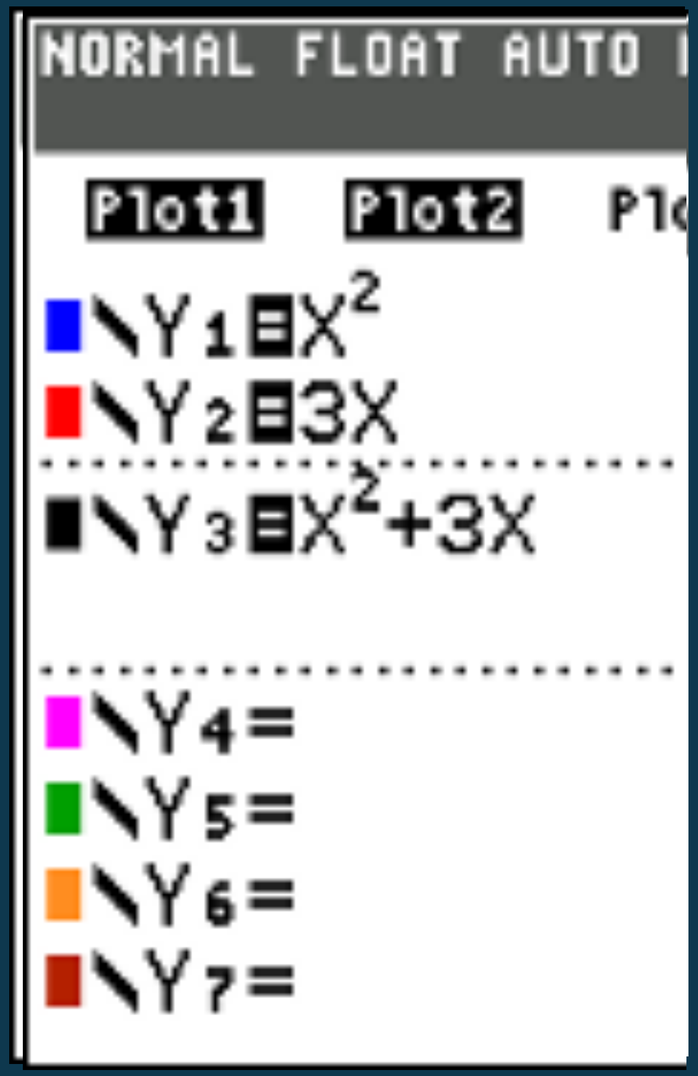


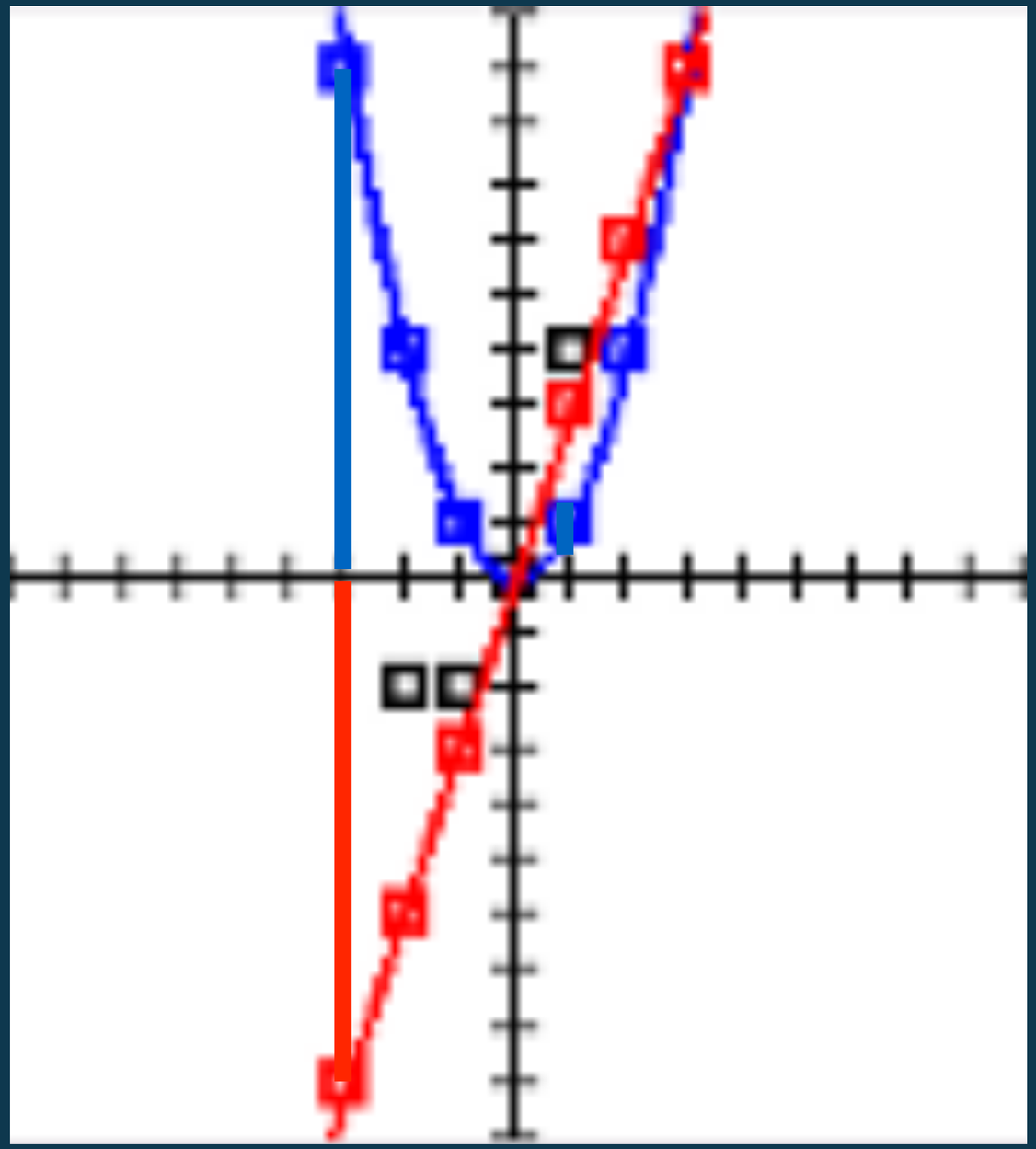

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=

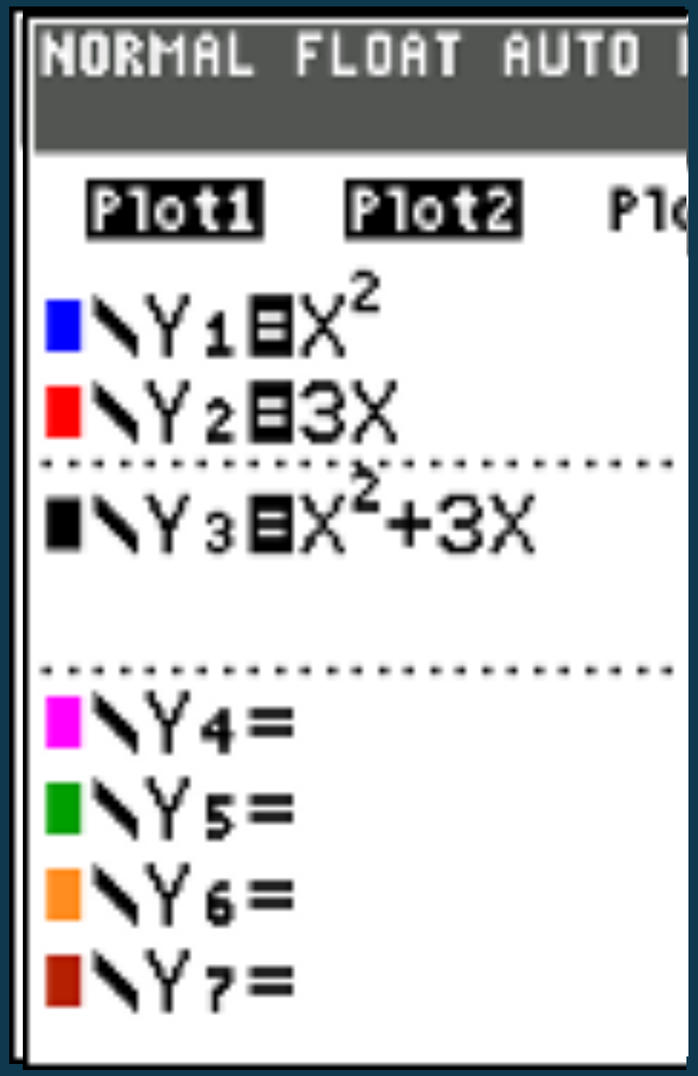


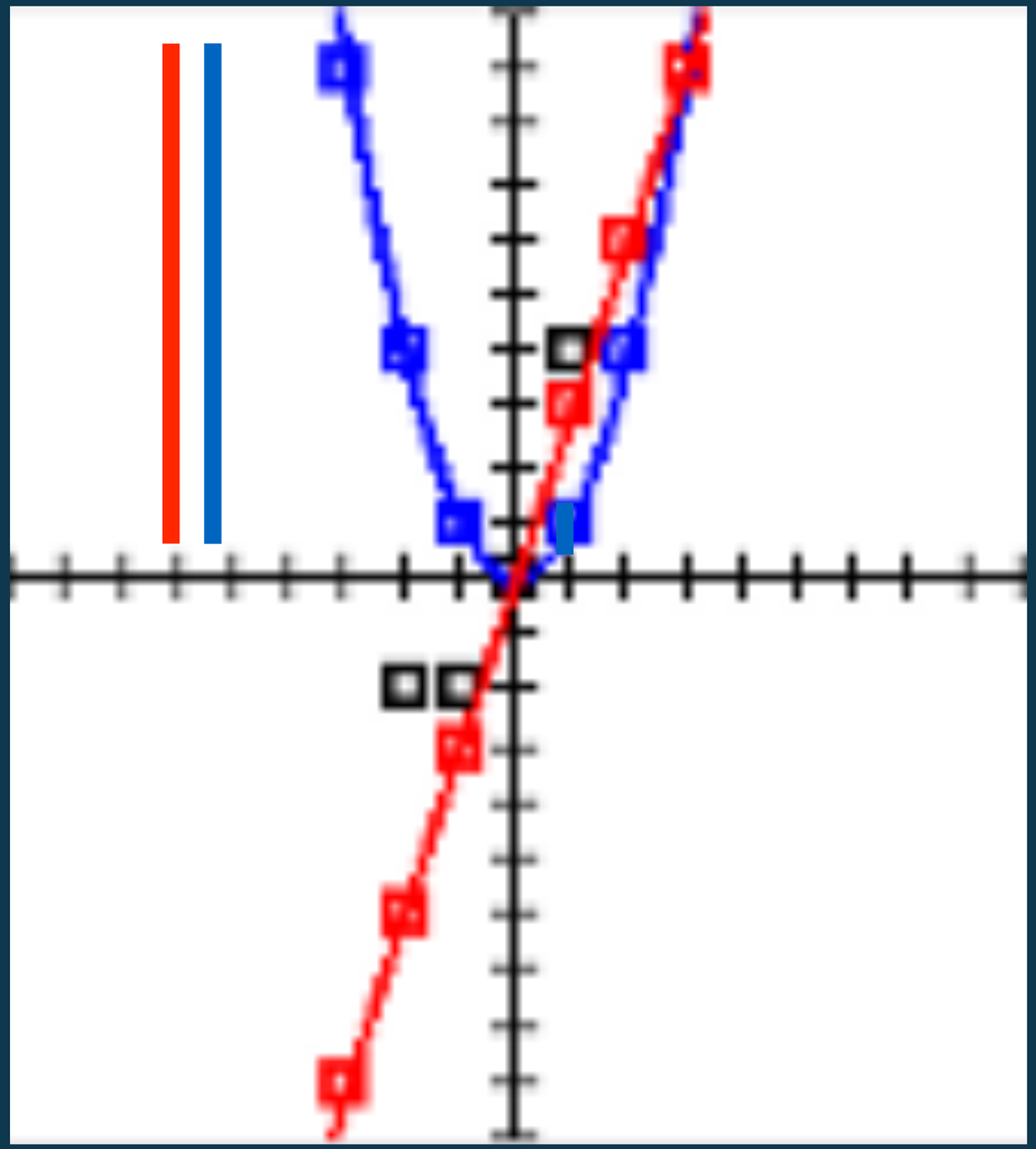

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

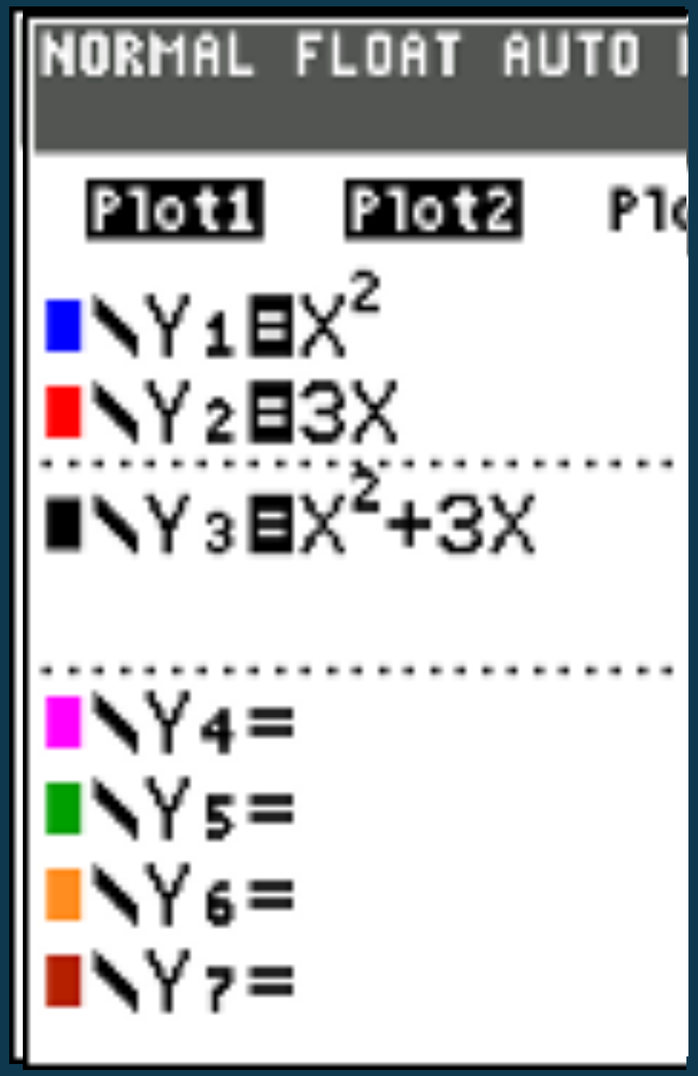


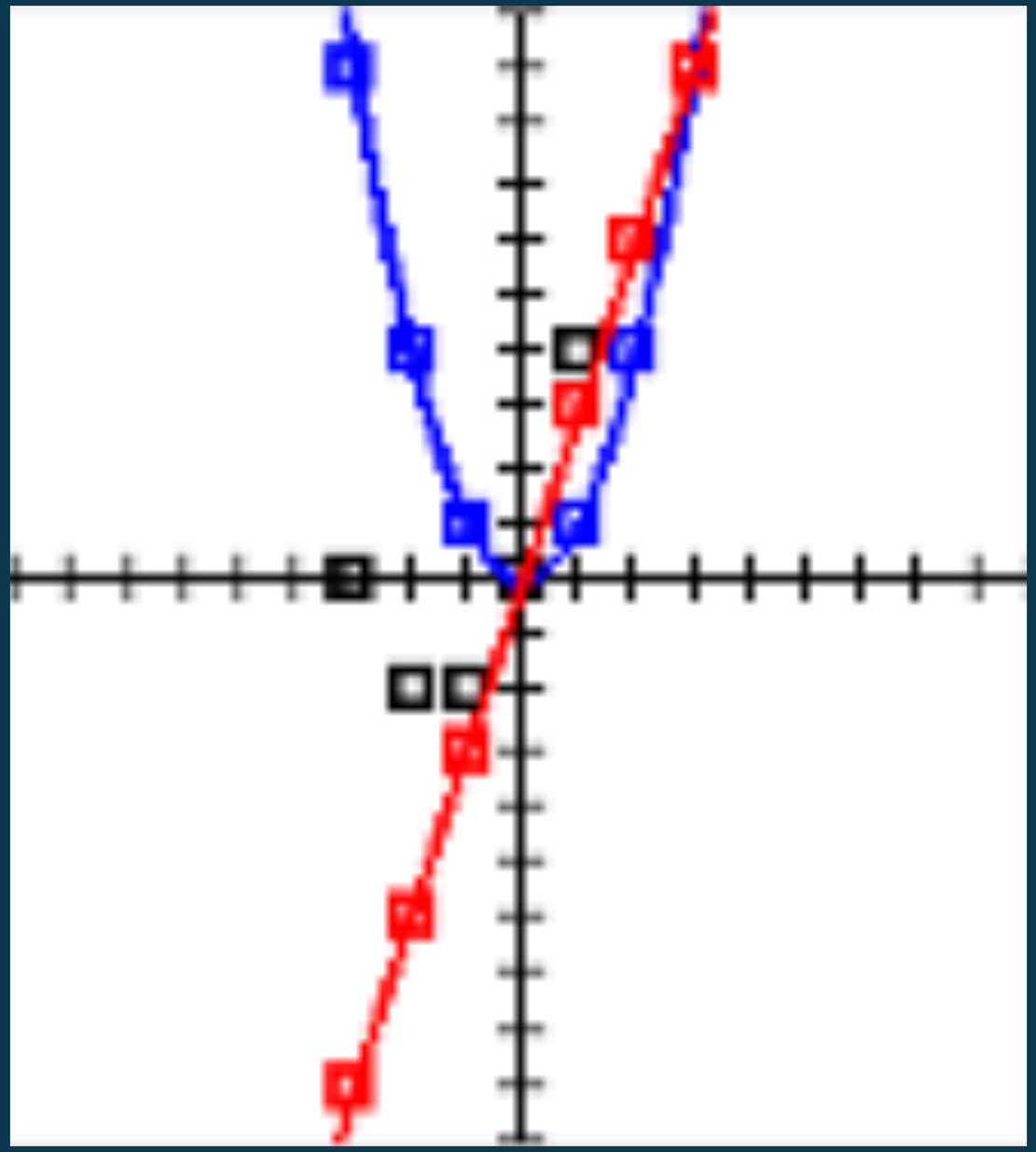

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=

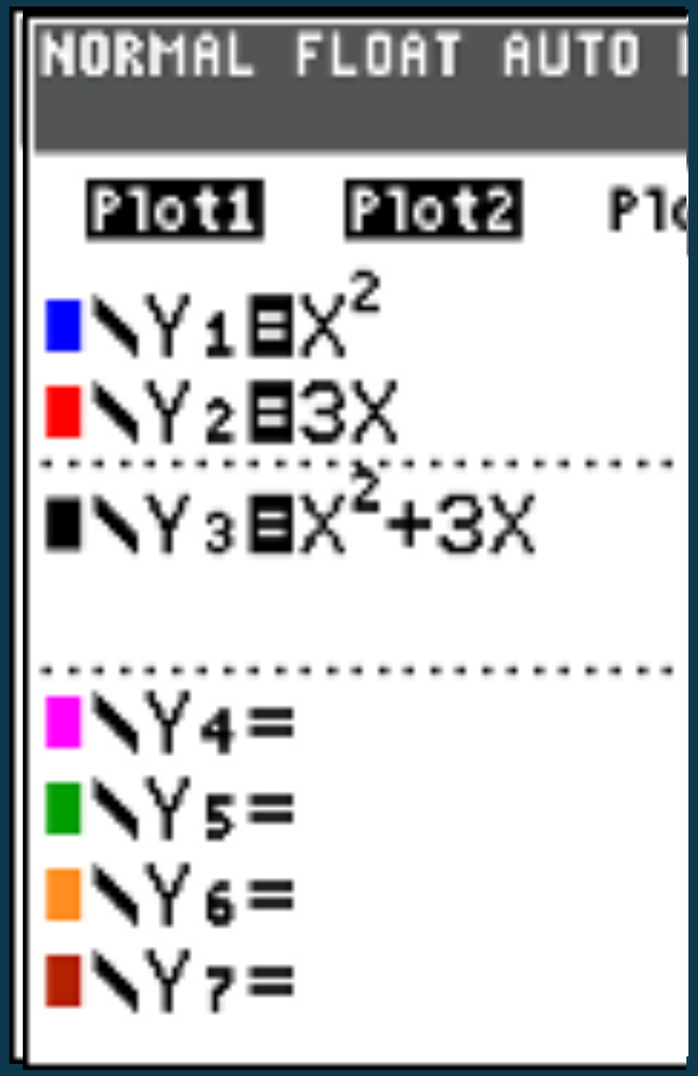


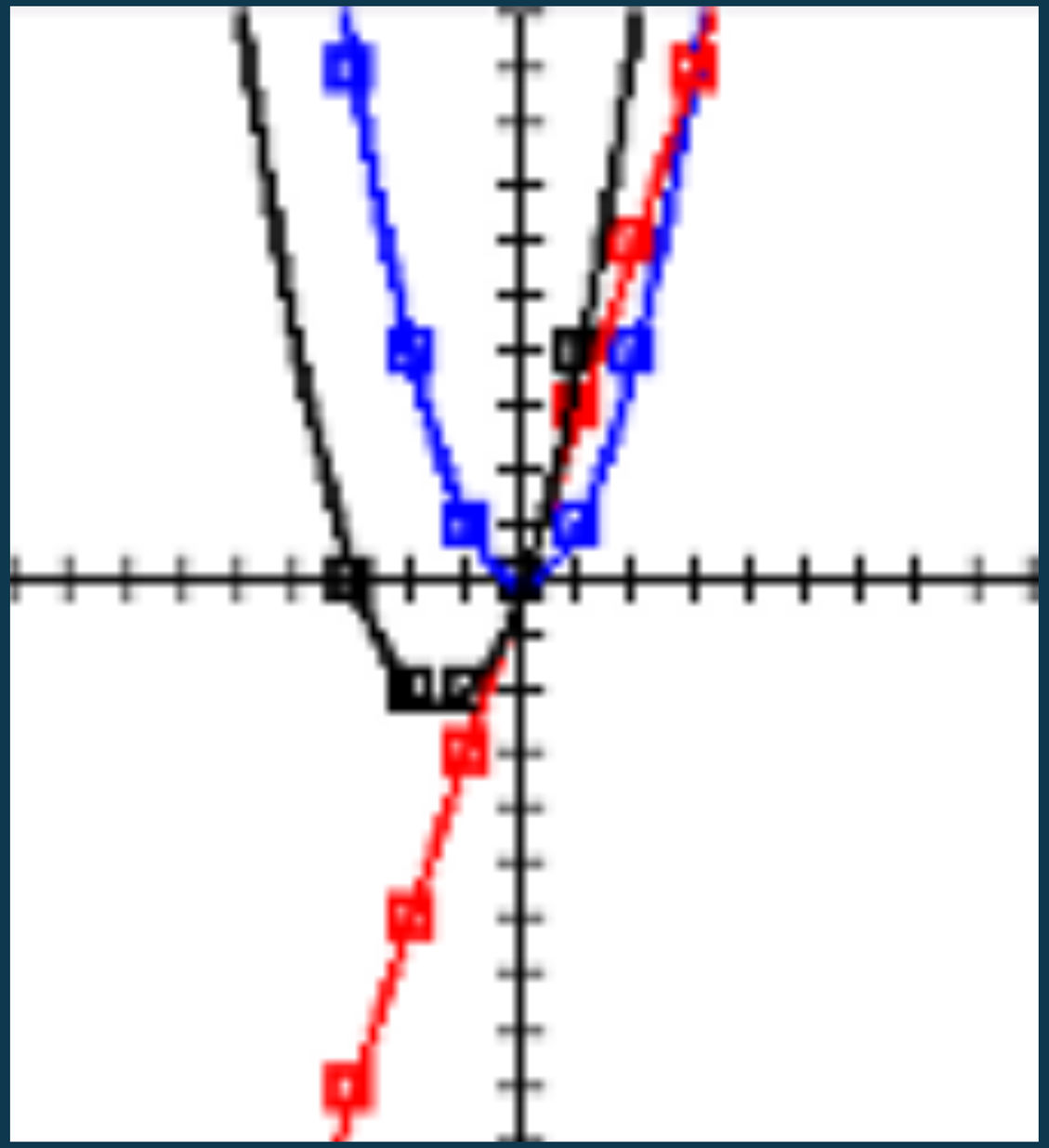

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=

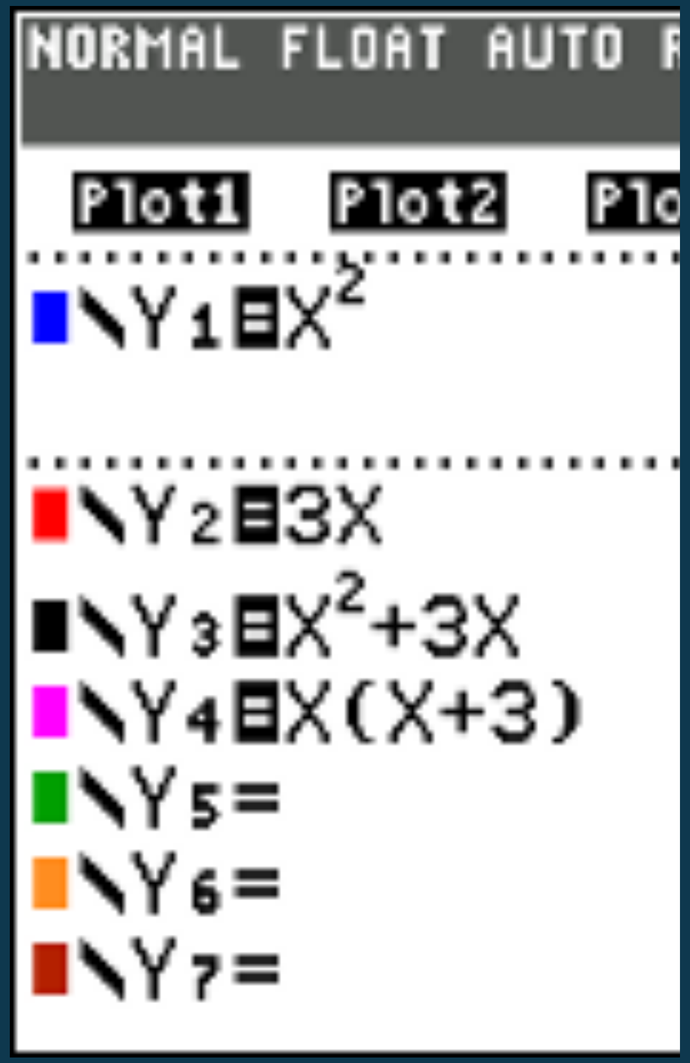


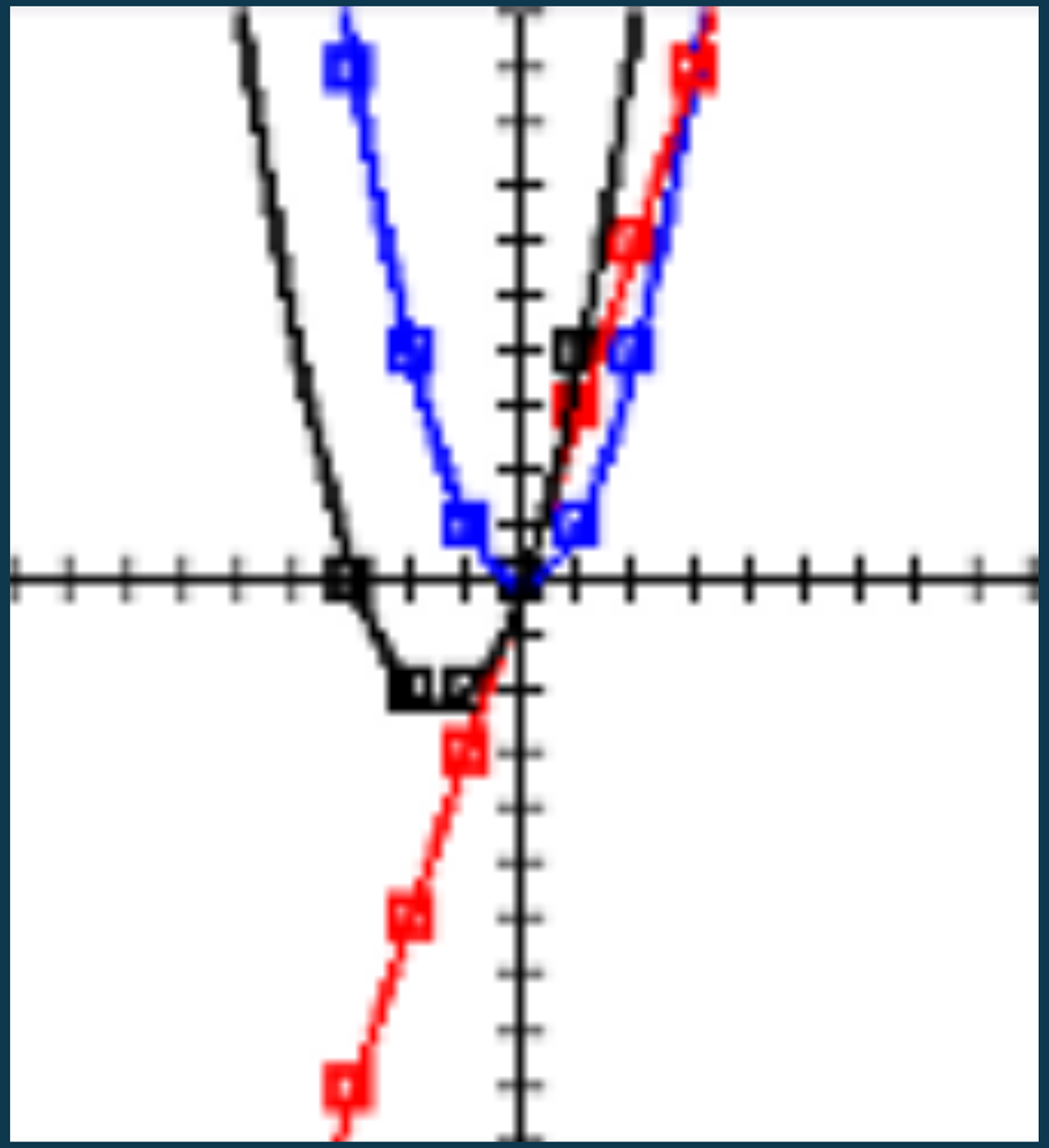

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=
Y5=
Y6=
Y7=




NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X2
Y2=3X
Y3=X2+3X
Y4=
Y5=
Y6=
Y7=

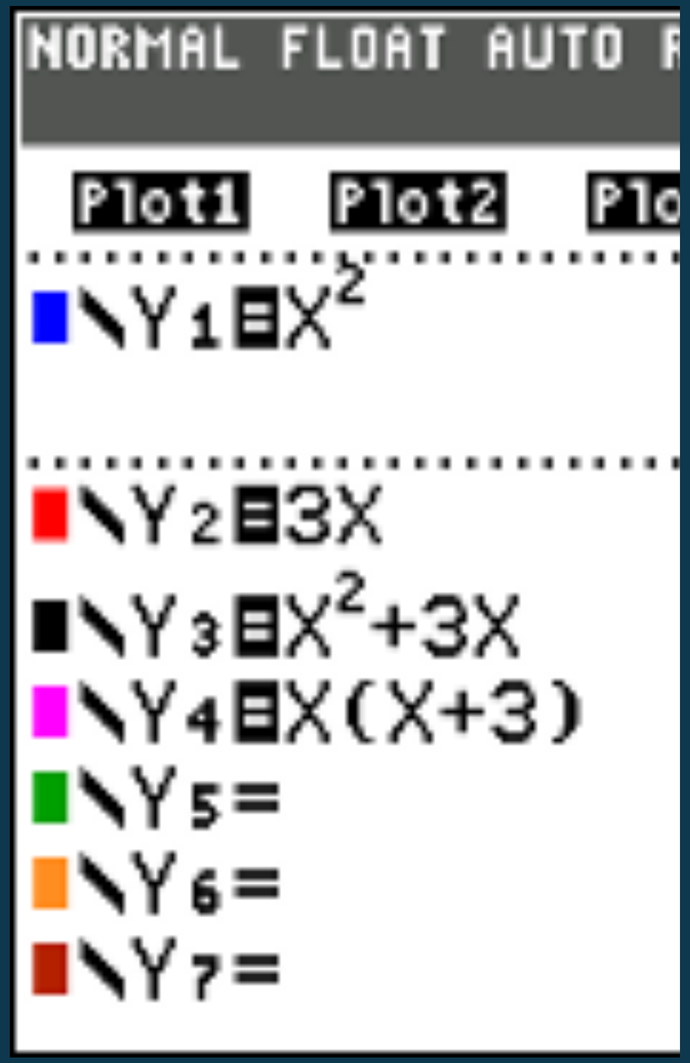


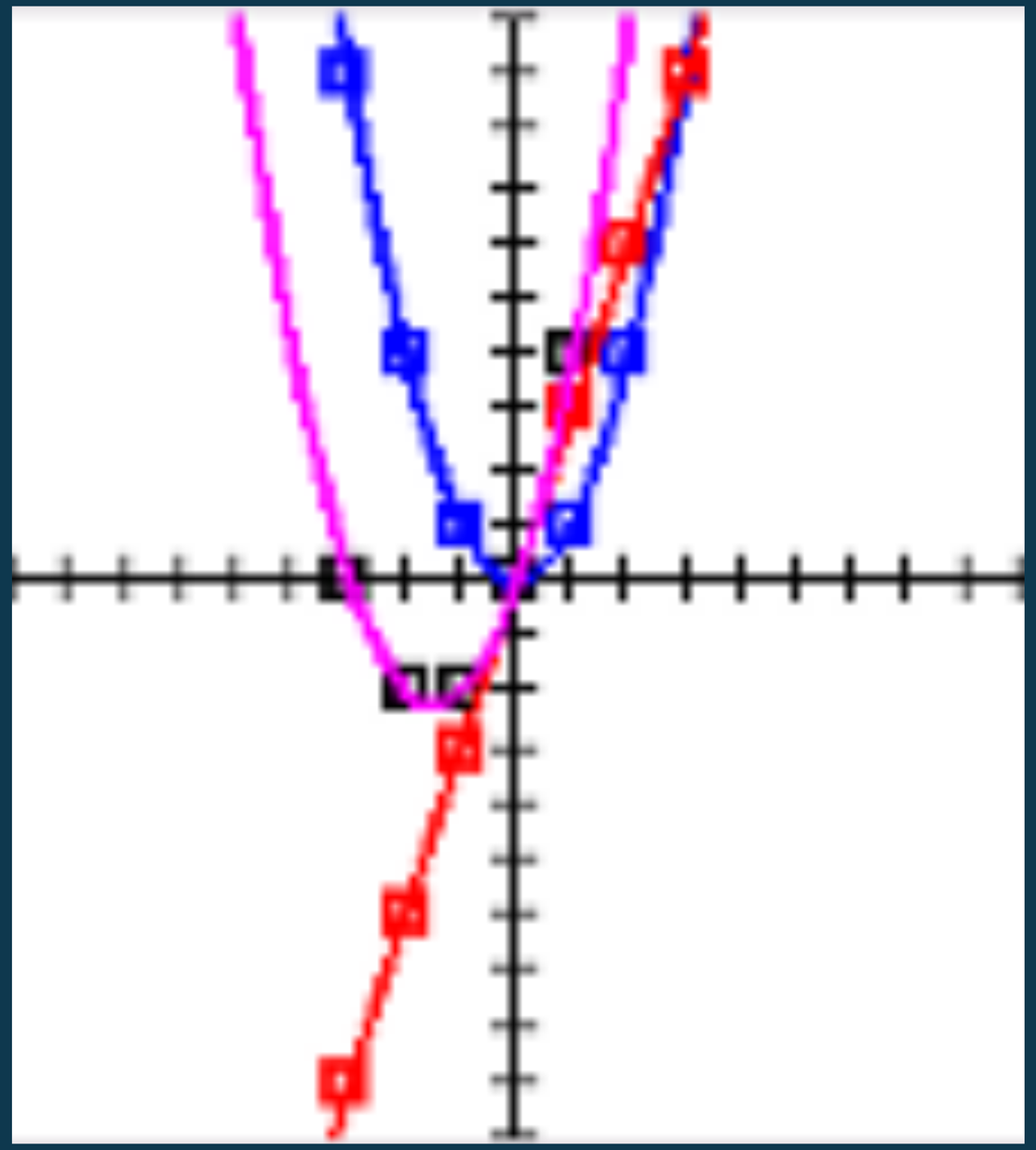


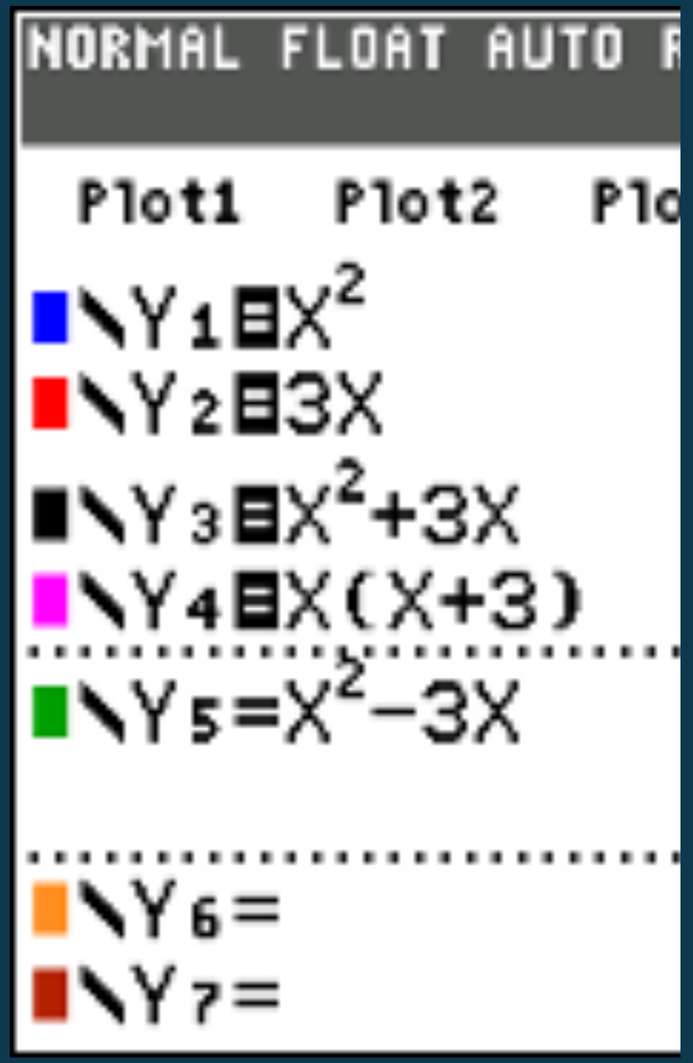


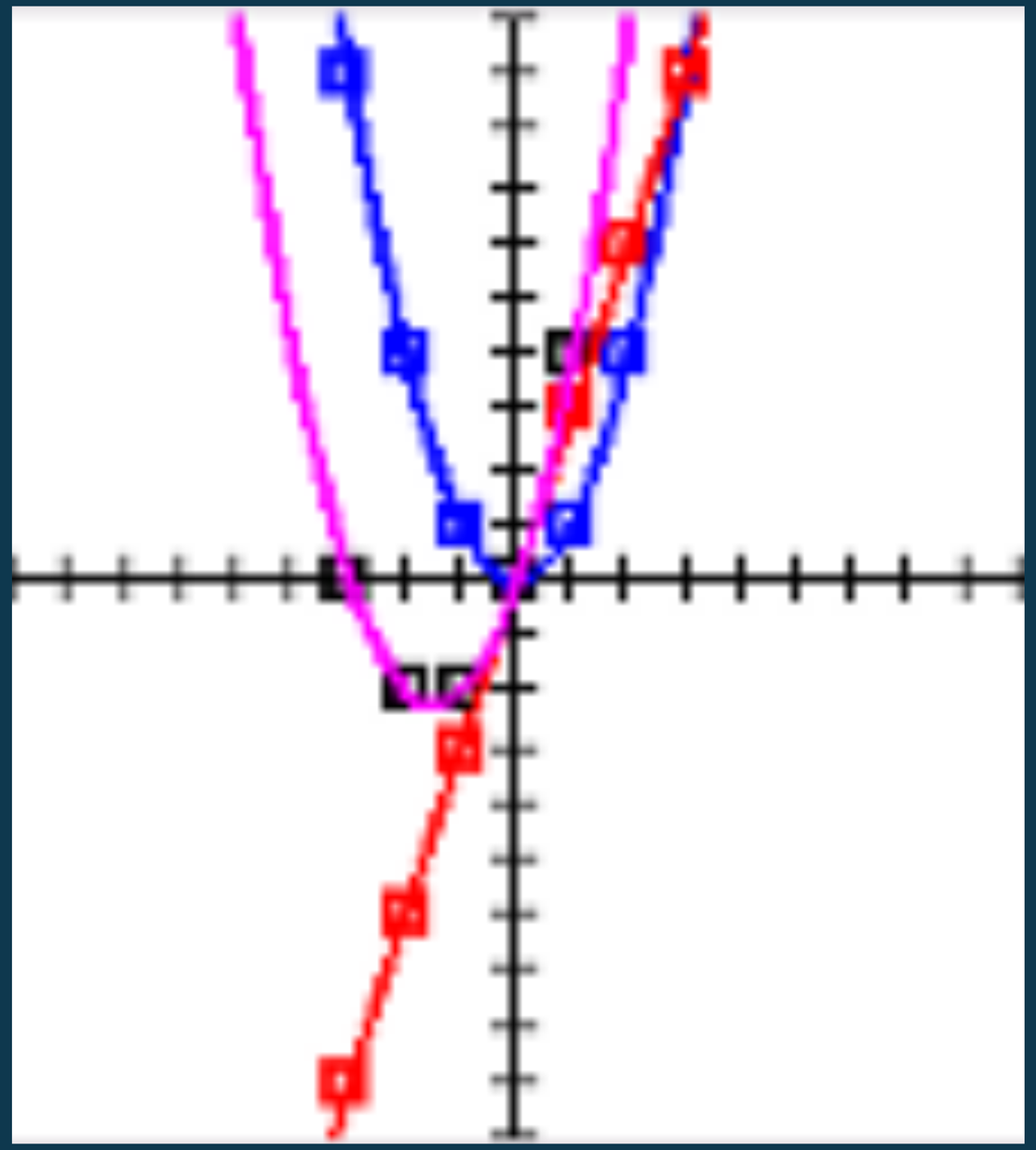
@pwharris

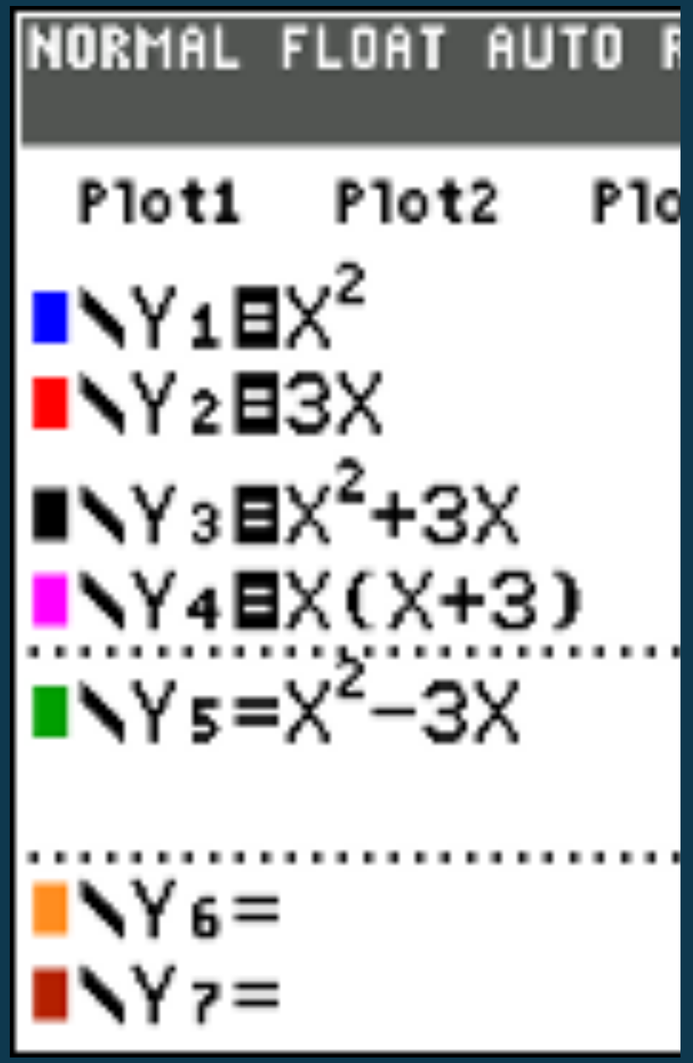
PAM HARRIS

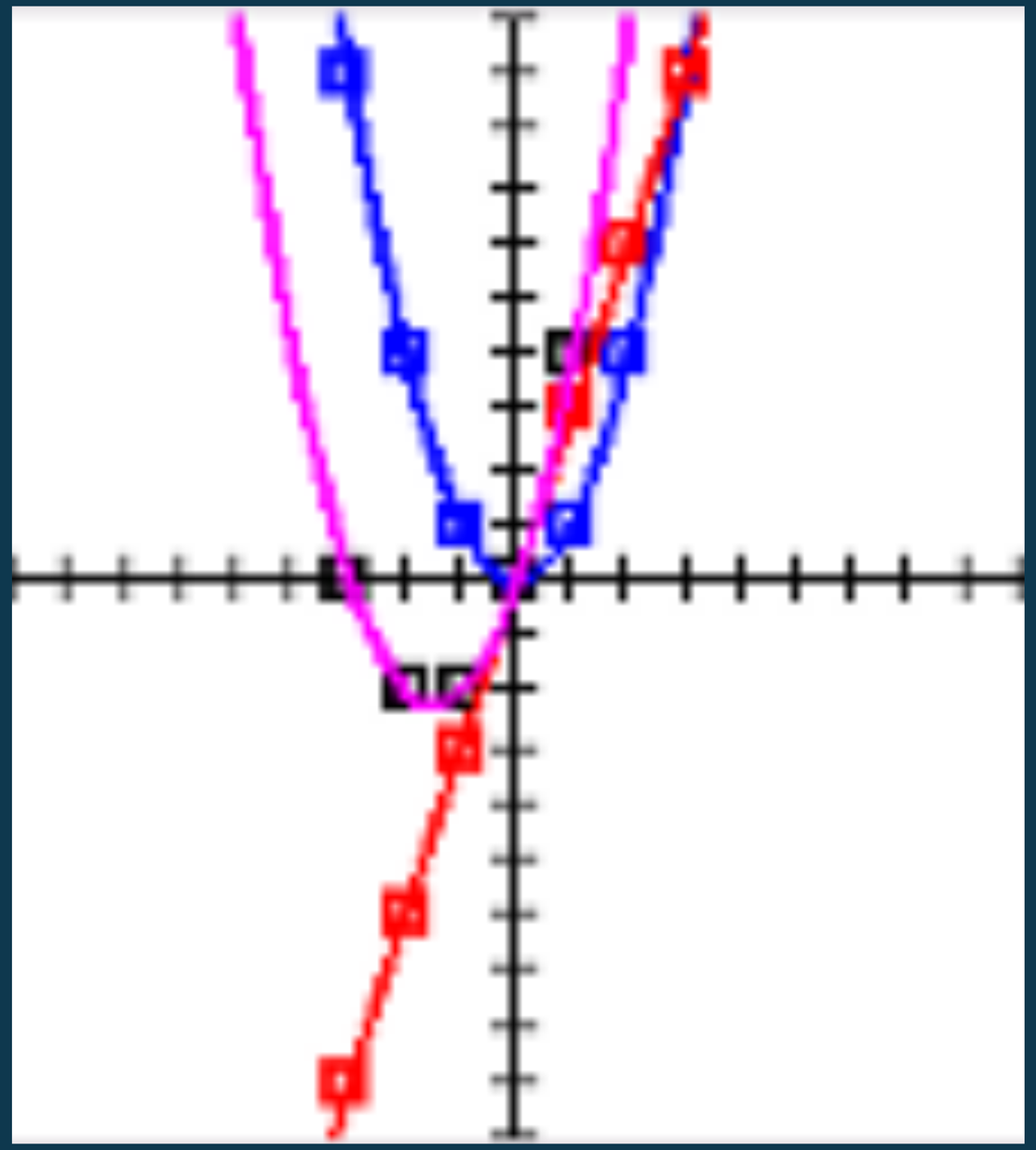

NORMAL FLOAT AUTO
Plot1
Plot2
Y1=X²
Y2=3X
Y3=X²+3X
Y4=X(X+3)
Y5=
Y6=
Y7=

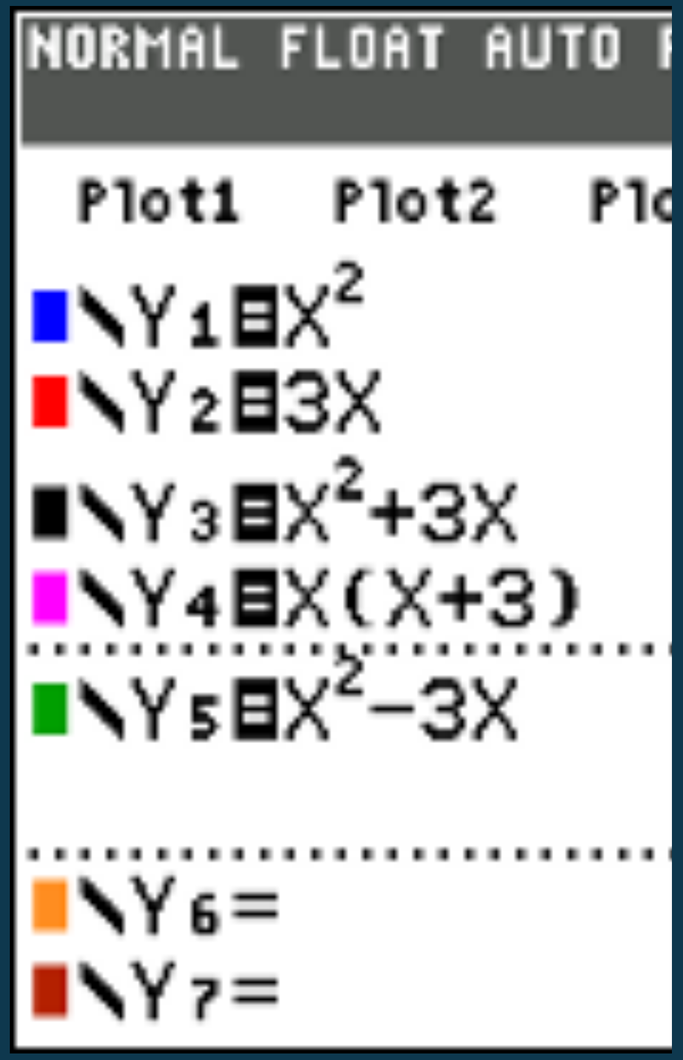


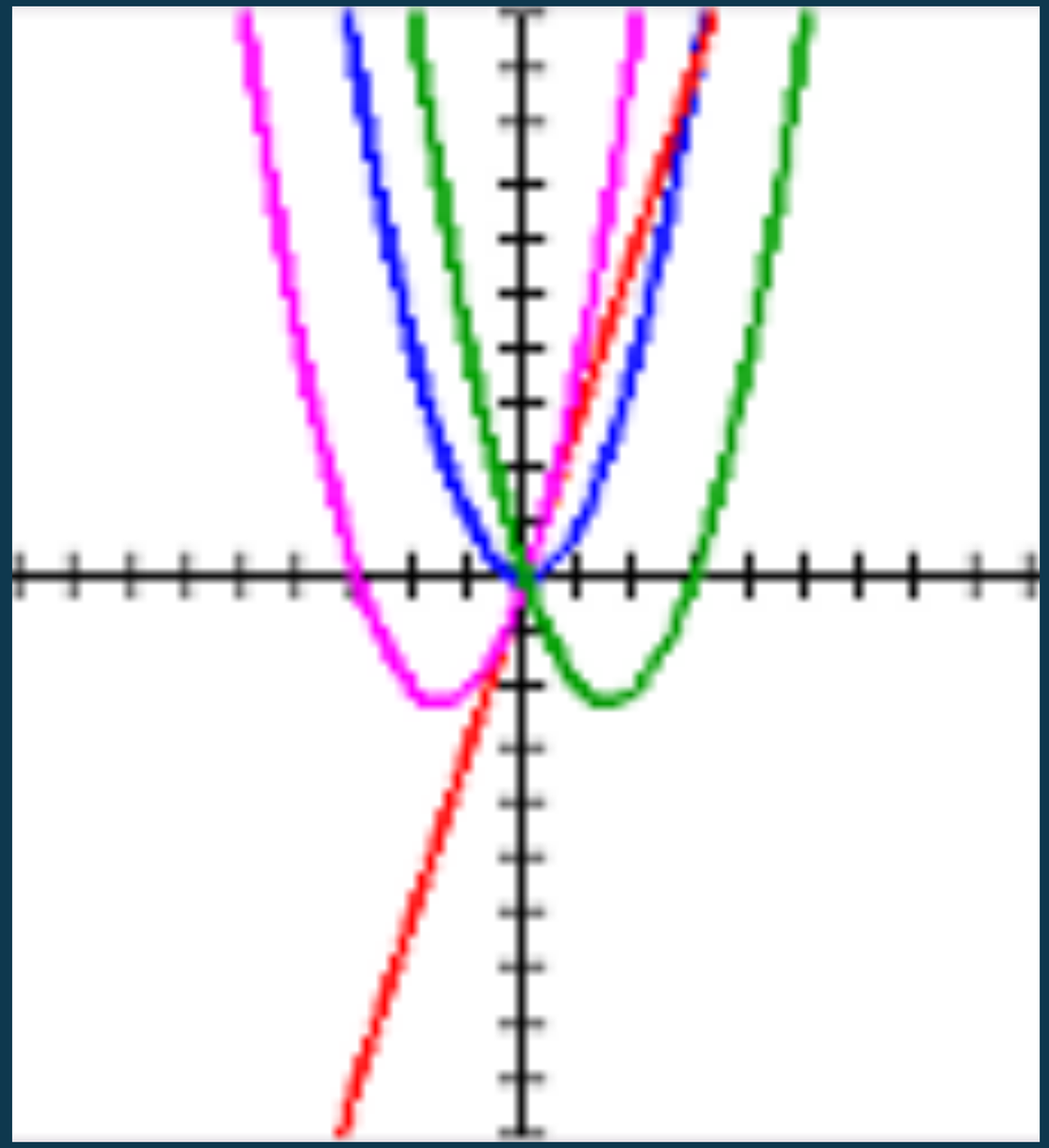

NORMAL FLOAT AUTO
Plot1 Plot2 Plo
Y1=X2
Y2=3X
Y3=X2+3X
Y4=X(X+3)
Y5=X2−3X
Y6=
Y7=

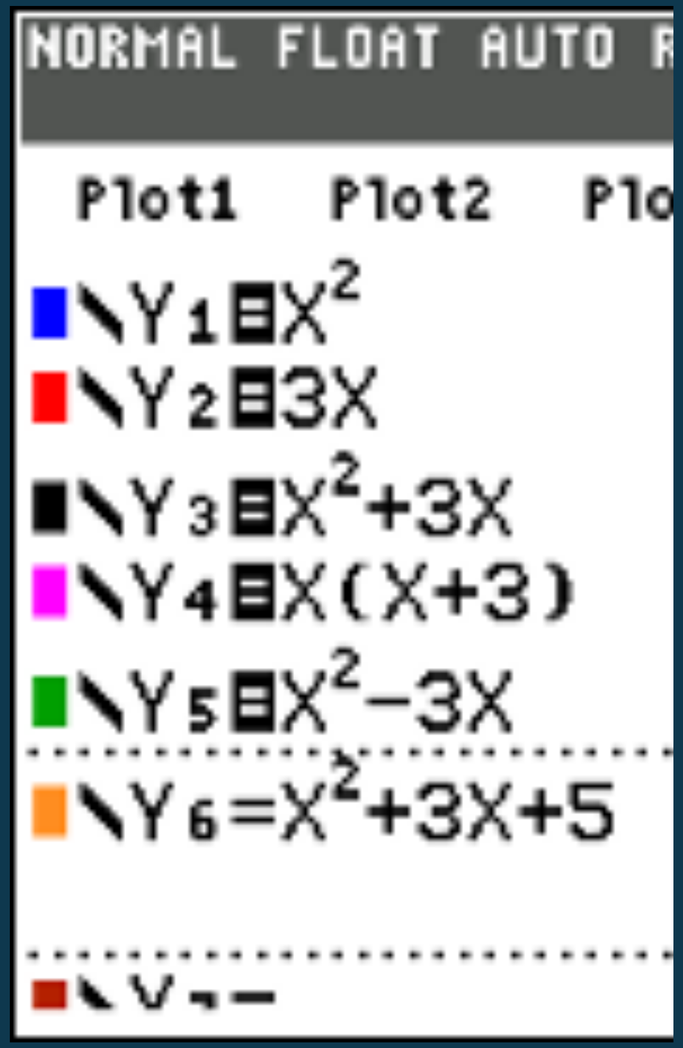


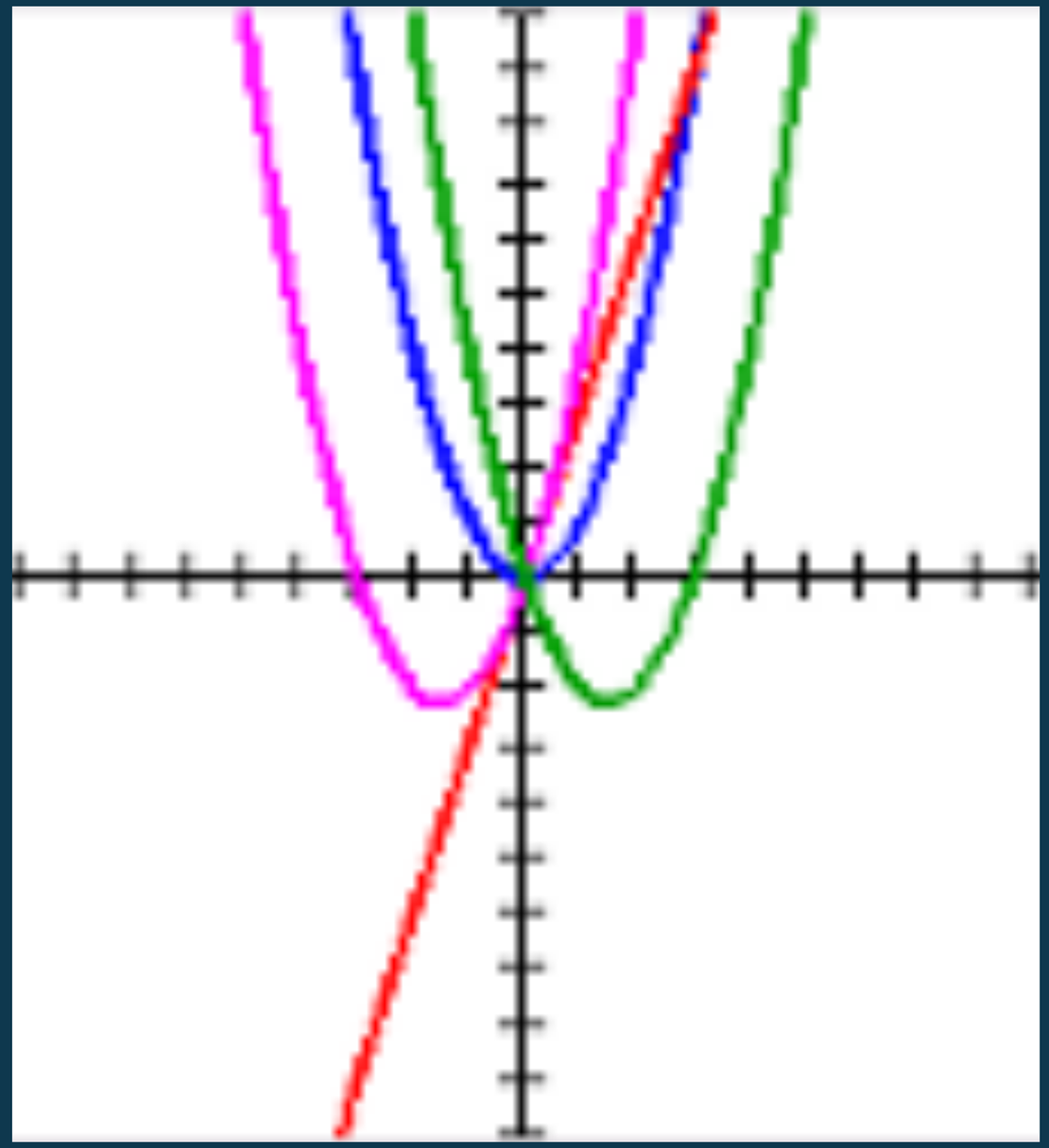

NORMAL FLOAT AUTO
Plot1 Plot2 Plo
Y1=X2
Y2=3X
Y3=X2+3X
Y4=X(X+3)
Y5=X2−3X
Y6=
Y7=



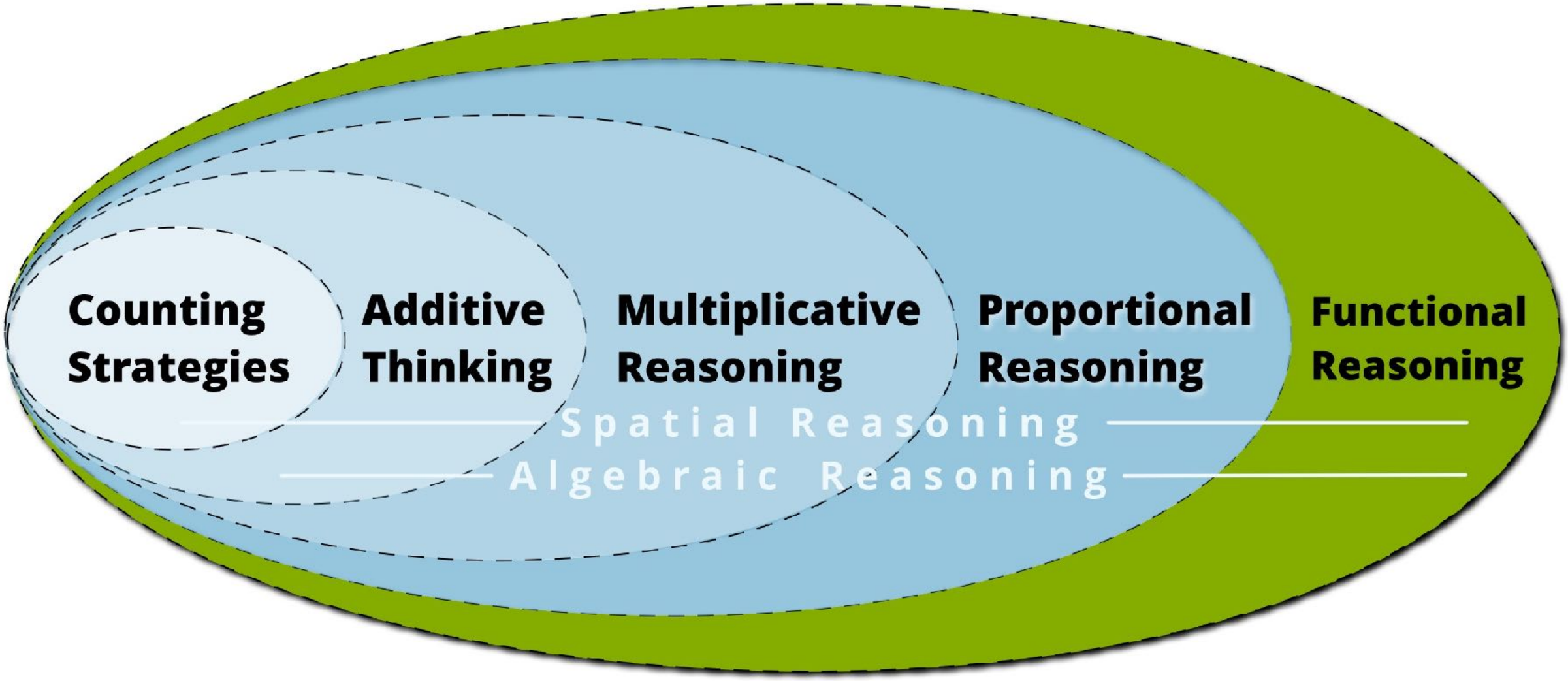

NORMAL FLOAT AUTO
Plot1 Plot2 Plo
Y1=X2
Y2=3X
Y3=X2+3X
Y4=X(X+3)
Y5=X2−3X
Y6=
Y7=




NORMAL FLOAT AUTO R
Plot1 Plot2 Plo
Y1=X2
Y2=3X
Y3=X2+3X
Y4=X(X+3)
Y5=X2−3X
Y6=X2+3X+5



The Development of Mathematical Reasoning



@pwharris

PAM HARRIS

IF NOT ALGORITHMS, THEN WHAT?

Phil Daro

AGAINST ANSWER GETTING

PAM HARRIS

equipping math teachers with content & pedagogy for student success

Home | See Pam at a Conference | Learn Now | Attend a Workshop | Booking | About

Empowering math teachers everywhere.

We help teachers teach more students more math more effectively. You are in the right place if you strive to help your students be sense makers, co-creaters of their mathematical knowledge, thinking like a mathematician. If you deemphasize algorithms and rote-memorization. If you believe that math is figureoutable!

Receive weekly emails to help you become the math teacher you want to be.

First name | Email | SIGN UP

MathStratChat

Be clever • Be original • Be inspired

Receive weekly emails
to help you become the math
teacher you want to be.

First name

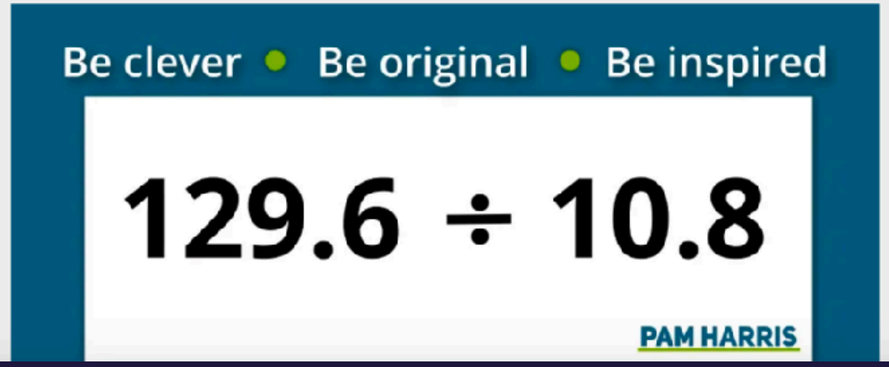
Email

Subscribe now

Join us every Wednesday evening on Twitter (#MathStratChat), **Facebook**, or **Instagram** for a new MathStratChat problem. Share your favorite strategy and see how others solve the same problem.

Every week we add a new Twitter Moment summary of the **#MathStratChat**. Click on the graphics below to read some of our favorite replies.

Most recent #MathStratChat:



Multiplication:

Be clever • Be original • Be inspired

2.7×8.1

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

49×22

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{8}{11} \times \frac{3}{4}$

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{3}{4} \times \frac{2}{5}$

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{4}{5} \times \frac{10}{3}$

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

85×47

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

89×42

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{9}{10} \times \frac{2}{3}$

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

36×76

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

3.2×4.8

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

I turned 52 last week!

Assuming 52 weeks in a year, how old am I in weeks?

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

18×51

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

8.1×24.3

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

How much does a 24 pound turkey cost at $1.25 per pound?

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

26×89

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{5}{6} \times \frac{3}{10}$

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

24×33

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

95×44

PAM HARRIS

MathStratChat

Be clever • Be original • Be inspired

$\frac{5}{12} \times \frac{3}{5}$

PAM HARRIS

MathStratChat

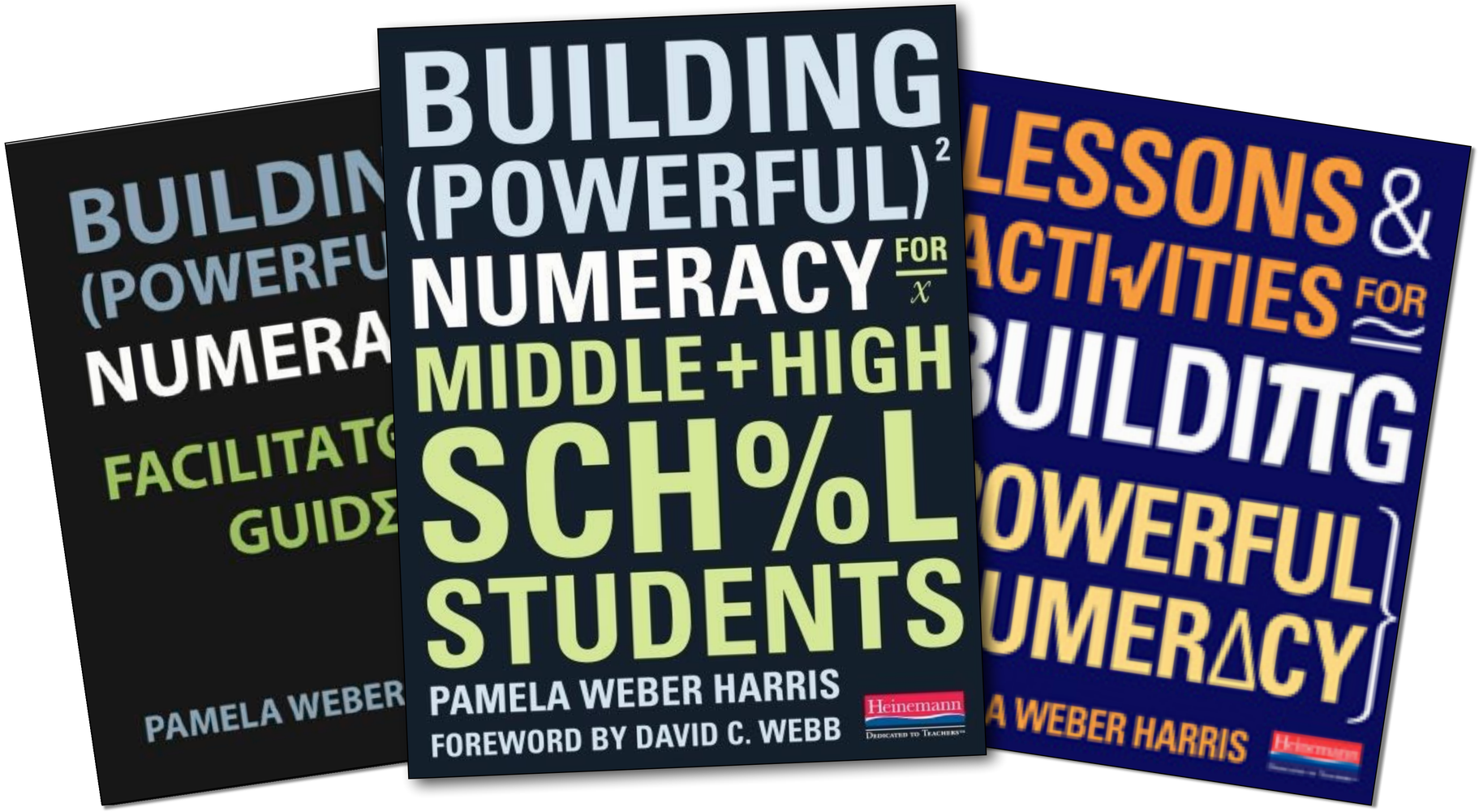
Be clever • Be original • Be inspired

25×18

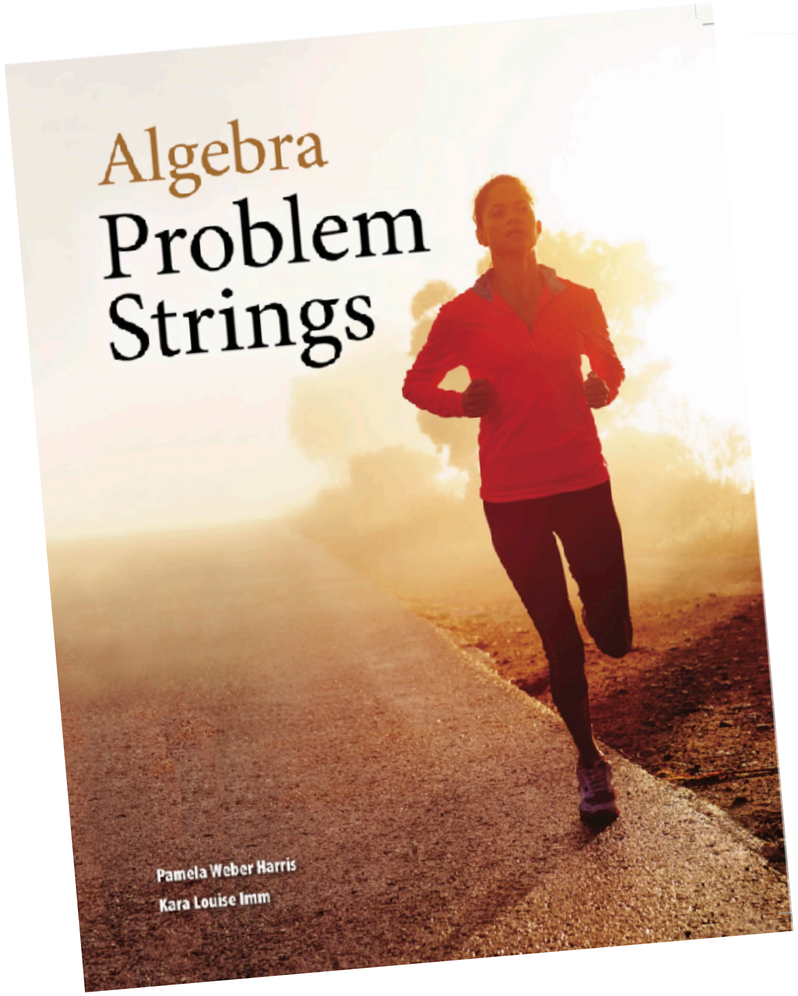
PAM HARRIS

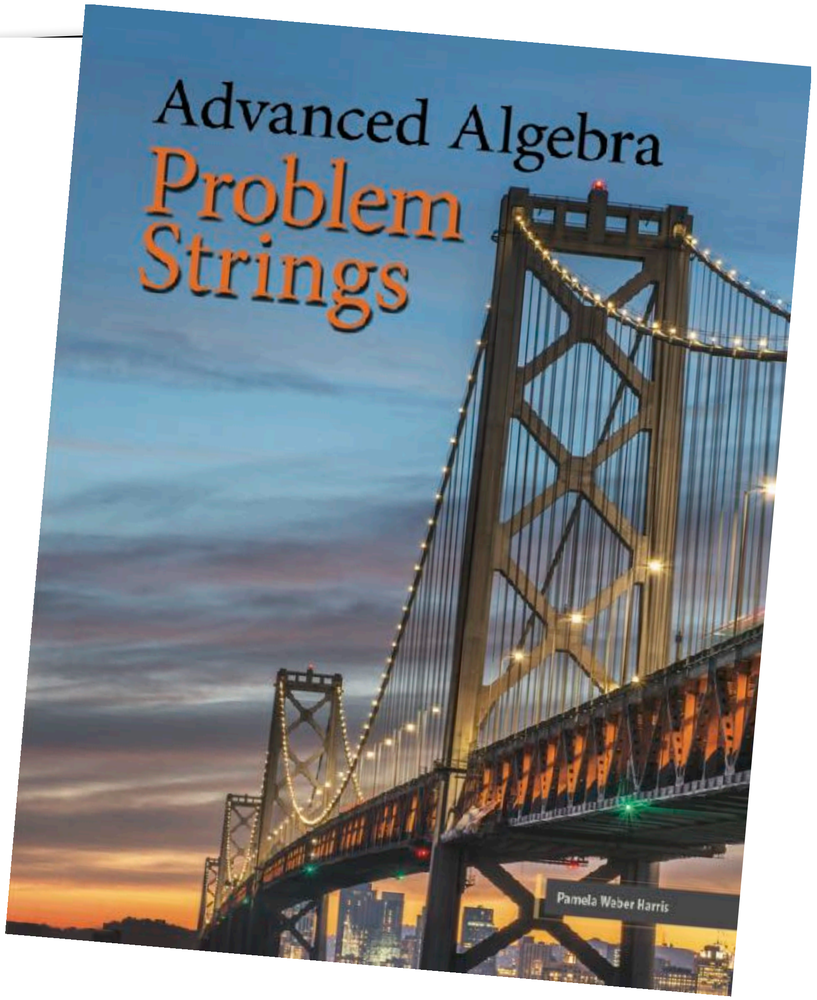
MathStratChat

Numeracy Books



Problem Strings





ONLINE COURSES

VIDEOS
Problem Strings

REAL MATH

~~FAKE~~ MATH

EXPERIENCE MATH

MATHEMATIZE

MENTOR MATHEMATICIANS

IT'S ABOUT RELATIONSHIPS

- Among the numbers and structures to solve problems
- Between teachers and students to mentor and nurture young mathematicians

PAM HARRIS

equipping math teachers with content & pedagogy for student success

http://bit.ly/mathreason

@pwharris
www.MathIsFigureOutAble.com
facebook.com/MathIsFigureOutAble
pharris@byu.net