1. Erica works for a company that produces metal tins. The tins are created from a metal sheet 6 in. by 6 in. Erica’s job is to operate a machine that removes a square, with side length $x$ in., from each corner. See picture below, not drawn to scale.

The remaining metal sheet is sent to a machine that folds up the sides to form a tray. See picture below, not drawn to scale.

(a) Find the length and width of the tray, in terms of $x$  

(b) Show that the volume, $V$ in³, of the tray is  

$$V = 4x^3 - 24x^2 + 36x$$

(c) Find $\frac{dv}{dx}$

(d) Using your answer from part (c), find the value of $x$ that maximizes the volume of the tin.

(e) Find the maximum volume of the tin

Mark scheme:

(a) Length: $6 - 2x$  
Width: $6 - 2x$  

(A1)

(A1)
(b) \( V = l \times w \times h \)
\[
V = (6 - 2x)(6 - 2x)(x)
\]
\[
V = 4x^3 - 24x^2 + 36x
\]

(c) \( \frac{dv}{dx} = 12x^2 - 48x + 36 \)

(d) \( 0 = 12x^2 - 48x + 36 \)
\[
0 = x^2 - 4x + 3
\]
\[
0 = (x - 3)(x - 1)
\]
x = 3, 1

   x = 1

(e) \( V = 4(1)^3 - 24(1)^2 + 36(1) \)
\[
V = 16 \text{ in}^3
\]