Topic 5: Calculus

Graphical behaviors of functions and the relationship between $f, f', \text{ and } f''$

1. Given: 
\[ f(x) = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 1 \]

(a) Find $f'(x)$ 

(b) Find the intervals of $x$ for which $f(x)$ is increasing and decreasing

(c) Find $f''(x)$

(d) Find the intervals of $x$ for which $f(x)$ is concave up and concave down

Mark scheme:

(a) \[ f'(x) = 2x^2 - 7x - 4 \] (A1)(A1)(A1)

(b) \[ 0 = 2x^2 - 7x - 4 \] Setting their first derivative = 0

Solving for $x$
\[ 0 = (2x + 1)(x - 4) \]
\[ x = -\frac{1}{2}, 4 \]

Increasing: \( (-\infty, -\frac{1}{2}) \cup (4, \infty) \) (A1) ft

Decreasing: \( (-\frac{1}{2}, 4) \) (A1) ft

(c) \[ f''(x) = 4x - 7 \] (A1) ft (M1) ft

(d) \[ 0 - 4x - 7 \] Setting their second derivative = 0
\[ x = -\frac{7}{4} \] (A1) ft

Concave up: \( (-\infty, -\frac{7}{4}) \) (A1) ft

Concave down: \( (-\frac{7}{4}, \infty) \) (A1) ft