1. A study was done to determine whether the amount of sleep a student gets affects their GPA. A random sample of high school seniors was taken in the Unites States. The results are in the following table.

|  | GPA $<\mathbf{2 . 0}$ | $\mathbf{2 . 0} \leq \mathrm{GPA}<\mathbf{3 . 0}$ | $\mathbf{3 . 0} \leq$ GPA $\leq 4.0$ |
| :--- | :---: | :---: | :---: |
| Hours of Sleep $<\mathbf{7}$ | 10 | 12 | 6 |
| $\mathbf{7} \leq$ Hours of Sleep $<\mathbf{8}$ | 7 | 14 | 18 |
| $\mathbf{8} \leq$ Hours of Sleep | 9 | 12 | 25 |

A $\chi^{2}$ test was performed at the $1 \%$ significance level. The critical value for this test is 13.77 .
(a) State the null hypothesis.
(b) Write down the degrees of freedom.
(c) Write down:
i. The $\chi^{2}$ statistic.
ii. The associated $p$-value.
(d) State, giving a reason, whether the null hypothesis should (2 marks) be accepted.

Mark scheme:
(a) $\mathrm{H}_{0}$ : The GPA of a high school senior is independent of the number of hours of sleep they receive.
(b) Degrees of Freedom: $(3-1)(3-1)=4$
(c)
i. $\quad \chi^{2}=8.63912$
ii. $\quad p$-value $=0.070781$
(d) Method 1: $\quad \chi^{2}<$ critical value $8.64<13.277$
Fail to reject the null hypothesis
Method 2: $p$-value $>$ significance level $0.070781>0.01$
Enough evidence to support null hypothesis

