Monday Night Calculus

Strategy for Testing Series

Exercises

1. Determine whether each series converges or diverges. Explain you reasoning.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{2+n}{2n}$ **(c)** $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ **(e)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^(2n)}{\sqrt{n}}$ **(f)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

2. For $n = 1, 2, 3, \ldots$, let $a_{2n} = \dfrac{1}{n}$ and $a_{2n-1} = -\dfrac{1}{2^n}$, so that

$$\sum_{i=1}^{\infty} a_i = -\frac{1}{2} + 1 - \frac{1}{4} + \frac{1}{2} - \frac{1}{8} + \frac{1}{3} - \frac{1}{16} + \frac{1}{4} + \cdots$$

Does the alternating series test apply? Does this series converge or diverge?

3. Let f be the function defined by $f(x) = 2x^3 - 5x^2 + 7x - 6$.

(a) Find the second degree Taylor polynomial, q_0, or f about $x = 0$ and the second degree Taylor polynomial, q_2 for f about $x = 2$. Graph f, q_0, and q_2 on the same coordinate axes.

(b) Find the third degree Taylor polynomial, p, for f about $x = 2$. Graph p and f on the same coordinate axes.

(c) Use the LaGrange error bound to explain why an nth degree Taylor polynomial approximation about any value $x = a$ to an nth degree polynomial function will be always be an exact match.

4. The Maclaurin series for $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

(a) Use the first 5 nonzero terms of the series to approximate $\sin(1.0)$.

(b) What is the alternating series error bound for the approximation found in part (a)?

(c) What is the Lagrange error bound for the approximation found in part (a)?

5. Use the Integral Test error bound to estimate the error in using $\displaystyle\sum_{n=1}^{10} \frac{1}{n^3}$ to approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$