**Monday Night Calculus**

**Polar Equations**

**Exercises**

**1. Spiraling Under Control**

Consider the curve $C$ given by the polar equation $r = \dfrac{2\theta}{\pi}$ for $0 \le \theta \le 2\pi$.

**(a)** Sketch the graph of the curve $C$ and find an equation of the tangent line to the curve at the point where $\theta = \dfrac{3\pi}{4}$.

**(b)** Find the first value in the interval $0 \le \theta \le 2\pi$ for which the tangent line to the curve $C$ is vertical.

**(c)** The region $R$ is bounded by the curve $C$ and the line segment that connects the origin to the point $(x, y) = (4, 0)$. Find the area of the region $R$.

**(d)** Find the length of the curve $C$.

**2. Rabbit Ears (Bifolium)**

Consider the curve $C$ defined by the polar equation $r(\theta) = 12 \sin\theta \cos^2\theta$ for $0 \le \theta \le \pi$.

**(a)** Sketch the graph of the curve $C$. Find the polar coordinates $(r, \theta)$ of the point on the curve in the first quadrant that is farthest from the origin.

**(b)** Find an equation of the line tangent to the curve $C$ at the point found in part (a).

**(c)** Find the total area enclosed by the curve $C$.

**3. An Infinity Curve**

Consider the curve $C$ defined by the polar equation $r = 5\sqrt{\cos 2\theta}$.

**(a)** Sketch the graph of the curve $C$.

**(b)** There are two horizontal lines tangent to the curve. Find these lines and the values for $\theta$, $0 \le \theta \le 2\pi$, at which they occur.

**(c)** Find $\displaystyle\lim_{\theta \to (\pi/4)^-} \frac{dr}{d\theta}$ or explain why it does not exist.

**(d)** Find $\displaystyle\lim_{\theta \to (\pi/4)^-} \frac{dy}{dx}$ or explain why it does not exist.

**(e)** Find the total area enclosed by the curve $C$.
Hint: Carefully consider the domain of $r$.