Monday Night Calculus

Integration by Parts

Exercises

1. Use integration by parts to evaluate the integral $\int_0^1 x^2 \tan^{-1} x \, dx$

2. **(a)** Evaluate the integral $\int \sin^2 x \, dx$ by using the trigonometric identity $\sin^2 x = \dfrac{1 - \cos 2x}{2}$ and the substitution $u = 2x$.

(b) Evaluate the integral $\int \sin^2 x \, dx$ by using integration by parts with $u = \sin x$ and $dv = \sin x \, dx$.

3. **(a)** Use integration by parts to show that

$$\int x^n \ln x \, dx = \frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) \qquad \text{for } n \neq -1$$

(b) Fill in the gap for this formula. That is, evaluate the integral when $n = -1$.

4. Uncle Stanley found a differentiable function f with the property $f'(x) = -f(x)$ for all x.

(a) Show that for Uncle Stanley's function

$$\int f(x) \sin x \, dx = \frac{-\sin x - \cos x}{2} \cdot f(x) + C$$

(b) Evaluate $\int f(x) \cos x \, dx$ where f is Uncle Stanley's function.

(c) Explain the consequences of using integration by parts to evaluate $\int f(x) e^x \, dx$.

(d) Find a candidate for Uncle Stanley's function.