Monday Night Calculus

Volume

Exercises

1. Let R be the region in the first quadrant bounded by the graph of $y = \dfrac{4}{\sqrt{1+x^2}}$, the coordinate axes, and the vertical line $x = 1$. Find the volume of the solid obtained when R is rotated about the x-axis.

2. **(a)** Let R be the region in the first quadrant bounded by the graph of $y = 36 - x^2$ and the coordinate axes. A container has the shape of the solid formed by rotating the region R about the x-axis. If the units on the axes are centimeters, how many liters of water does the container hold?

(b) Suppose a second container has the shape of the solid formed by rotating the region R (described in part (a)) about the y-axis. Find the resulting volume of the container.

3. Let R be the region bounded by the graphs of $y = 2 - x^2$ and $y = e^x$. Find the volume of the solid generated when R is rotated about the x-axis.

4. Let R be the region bounded by the graph of $y = \sqrt{x}$, the x-axis, and the vertical line $x = 4$. Let S_1 be the solid obtained by rotating the region R about the x-axis. Let S_2 be the solid obtained by rotating the region R about the line $y = 2$.

(a) Which solid, S_1 or S_2, has the greater volume? Show the calculations that support your conclusion.

(b) There is a constant $c \neq 2$ such that the volume of the solid of revolution obtained by rotating the region R about the horizontal line $y = c$ is the same as the volume of S_2. Set up an equation involving integrals that could be used to solve for c, and use it to find c.