

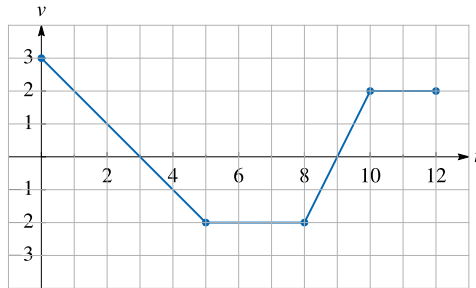
Monday Night Calculus

Rectilinear Motion

Exercises

1. Two objects oscillate along a vertical axis, starting at the same initial position $y = 5$ at time $t = 0$. The position of Object A at time t , $t \geq 0$, is given by $y_1(t) = 5e^{-t} \cos t$, and the position of Object B at time t , $t > 0$, is given by $y_2(t) = \frac{5 \sin t}{t}$.
- (a) Find the first time $t_a > 0$ at which Object A has position 0. What is Object A's velocity, speed, and acceleration at that time?
 - (b) Find the first time $t_b > 0$ at which Object B has position 0. What is Object B's velocity, speed, and acceleration at that time?
 - (c) Find the position of Object B at time t_a (the time found in part (a)). Are Objects A and B getting closer or are they getting farther apart at this time? Justify your answer.
 - (d) Find the position of Object A at time t_b (the time found in part (b)). Are Objects A and B getting closer or are they getting farther apart at this time? Justify your answer.
 - (e) Over the time interval $0 \leq t \leq \pi$, find the average velocity of Object A and Object B.
 - (f) Which object traveled the greater total distance over the time interval $0 \leq t \leq 2\pi$? Show the computations that lead to your answer.
 - (g) Find $\lim_{t \rightarrow \infty} (y_1(t) - y_2(t))$ or explain why the limit does not exist.
 - (h) On the interval $0 \leq t \leq 2\pi$, at what time t are the two objects farthest apart? How far apart are they at this time?

2. The graph in the figure below shows the vertical velocity for an elevator as a function of time, where the velocity is measured in units of feet per second and time is measured in units of seconds, with $0 \leq t \leq 12$ seconds. The initial height, or position, of the elevator is $y(0) = 6$ feet above the ground.



- (a) Find the acceleration of the elevator at time $t = 2$ seconds. Indicate units of measure.
- (b) Is the elevator speeding up or slowing down at time $t = 4$ seconds? Explain your reasoning.
- (c) Find the average velocity of the elevator over the time interval $0 \leq t \leq 12$ seconds.
- (d) Find the time at which the elevator reaches its greatest height above the ground. What is that height?
- (e) Does the elevator ever go below ground level ($y = 0$)? Justify your answer.
- (f) Find the acceleration of the elevator when it is at its lowest level.
- (g) Find the height of the elevator at time $t = 12$ seconds.