Monday Night Calculus

Local Linearity and L'Hospital's Rule

10/12 Question

1. Find the limit, if it exists.

(a) $\lim_{x\to(\pi/2)} \dfrac{\cos x}{x - \dfrac{\pi}{2}}$

(b) $\lim_{x\to(\pi/2)} \dfrac{\sin x - 1}{x - \dfrac{\pi}{2}}$

(c) Sketch the graphs of $f(x) = x - \dfrac{\pi}{2}$, $g(x) = \cos x$, and $h(x) = \sin x - 1$ in the same viewing window. Zoom in on the graphs at the point $\left(\dfrac{\pi}{2}, 0\right)$ and use local linearity to explain how the graphs relate to the limits found in parts (a) and (b).

2. Consider the limit $\lim_{x\to(\pi/2)} (\sec^2 x - \tan^2 x)$

(a) Find the value of the limit by writing $\sec x$ and $\tan x$ in terms of $\sin x$ and $\cos x$, and then using L'Hospital's Rule.

(b) Find the value of the limit by using a trigonometric identity.

3. Suppose f has a continuous derivative, and the line tangent to the graph of $y = f(x)$ at the point where $x = 5$ has the equation $y = 3x - 8$. Consider the limit

$$\lim_{x\to 5} \frac{f(x)^2 - 49}{x^2 - 25}$$

(a) Find the limit using L'Hospital's Rule.

(b) Find the limit by factoring the difference of squares in the numerator and denominator, and without using L'Hospital's Rule.

4. (Bonus Problems) Find the limit.

(a) $\lim_{x\to\infty} \left(1 + \dfrac{1}{x}\right)^x$

(b) $\lim_{x\to 0^+} (1 - 3x)^{2/x}$