Monday Night Calculus

Limits at Infinity and Infinite Limits

9/14 Question

The graphs of the functions \( f \) and \( g \) are given in the figures. The dashed lines in the figures represent horizontal or vertical asymptotes. The \( x \)-axis is a horizontal asymptote for both graphs.

![Graph of f(x)](image1)

1. Use the graph of \( f \) to evaluate each limit. If a limit does not exist, explain why.

   (a) \( \lim_{x \to -2^-} f(x) = \infty \)
   
   (b) \( \lim_{x \to -2^+} f(x) = -\infty \)
   
   (c) \( \lim_{x \to -2^-} f(x) \) DNE

   (d) \( \lim_{x \to 2^-} f(x) = \infty \)
   
   (e) \( \lim_{x \to 2^+} f(x) = \infty \)
   
   (f) \( \lim_{x \to 2^-} f(x) = \infty \)

   (g) \( \lim_{x \to 0} f(x) = 0 \)

   (h) \( \lim_{x \to \infty} f(x) = 0 \)

2. Use the graph of \( g \) to evaluate each limit. If a limit does not exist, explain why.

   (a) \( \lim_{x \to \infty} g(x) = -3 \)

   (b) \( \lim_{x \to \infty} g(x) = 0 \)

   (c) \( \lim_{x \to 0} g(x) = 0 \)

3. Use the graphs of \( f \) and \( g \) to evaluate each limit, if it exists. If the limit does not exist, explain why. Or, explain why neither conclusion is possible.

   (a) \( \lim_{x \to 2^+} \frac{f(x)}{g(x)} = \infty \)

   (b) \( \lim_{x \to 2^-} \frac{g(x)}{f(x)} = 0 \)

   (c) \( \lim_{x \to -2} \frac{g(x)}{f(x)} = 0 \)

   (d) \( \lim_{x \to \infty} [f(x) + g(x)] = 0 \)

   (e) \( \lim_{x \to 0} \frac{f(x)}{g(x)} \) CBD*

   (f) \( \lim_{x \to \infty} [f(x) \cdot g(x)] = 0 \)

CBD: Cannot Be Determined; not enough information is given in order to evaluate this limit.
**Bonus Problem**

Evaluate \( \lim_{x \to 0} \frac{1}{x} \sin \left( \frac{1}{x} \right) \)

Consider one-sided limits.

\[
\lim_{x \to 0^+} \frac{1}{x} \sin \left( \frac{1}{x} \right) \text{ DNE}
\]

This limit is not in an indeterminate form.

As \( x \to 0^+ \), \( \frac{1}{x} \to \infty \) (increases without bound), and \( \sin \left( \frac{1}{x} \right) \) oscillates between \(-1\) and \(1\) infinitely often.

So, the product is oscillating between positive and negative values, getting larger in absolute value as \( x \to 0^+ \).

Therefore, the limit does not exit.

One can make a similar argument for the limit at \( x \to 0^- \).

So, \( \lim_{x \to 0^-} \frac{1}{x} \sin \left( \frac{1}{x} \right) \text{ DNE} \)

There is no vertical asymptote at \( x = 0 \).