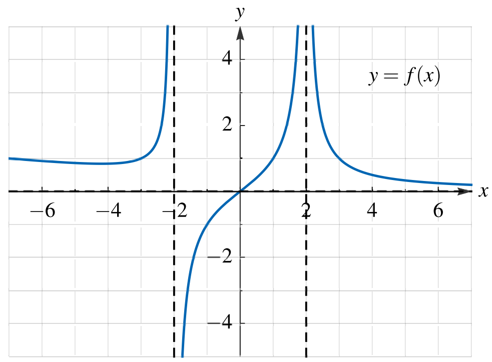
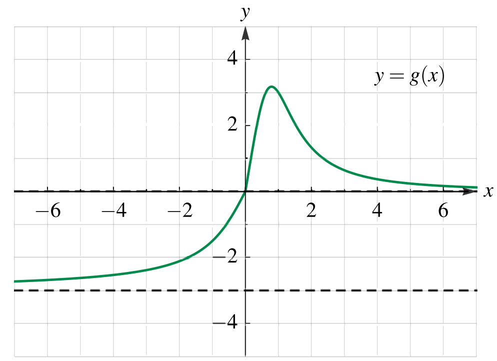
Monday Night Calculus

Limits at Infinity and Infinite Limits

9/14 Question

The graphs of the functions f and g are given in the figures. The dashed lines in the figures represent horizontal or vertical asymptotes. The x-axis is a horizontal asymptote for both graphs.





1. Use the graph of f to evaluate each limit. If a limit does not exist, explain why.

(a) $\lim_{x\to -2^-} f(x) = \infty$ **(b)** $\lim_{x\to -2^+} f(x) = -\infty$ **(c)** $\lim_{x\to -2} f(x)$ DNE

(d) $\lim_{x\to 2^-} f(x) = \infty$ **(e)** $\lim_{x\to 2^+} f(x) = \infty$ **(f)** $\lim_{x\to 2} f(x) = \infty$

(g) $\lim_{x\to 0} f(x) = 0$ **(h)** $\lim_{x\to \infty} f(x) = 0$

2. Use the graph of g to evaluate each limit. If a limit does not exist, explain why.

(a) $\lim_{x\to -\infty} g(x) = -3$ **(b)** $\lim_{x\to \infty} g(x) = 0$ **(c)** $\lim_{x\to 0} g(x) = 0$

3. Use the graphs of f and g to evaluate each limit, if it exists. If the limit does not exist, explain why. Or, explain why neither conclusion is possible.

(a) $\lim_{x\to 2^+} \dfrac{f(x)}{g(x)} = \infty$ **(b)** $\lim_{x\to 2^-} \dfrac{g(x)}{f(x)} = 0$ **(c)** $\lim_{x\to -2} \dfrac{g(x)}{f(x)} = 0$

(d) $\lim_{x\to \infty} [f(x) + g(x)] = 0$ **(e)** $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$ CBD* **(f)** $\lim_{x\to \infty} [f(x) \cdot g(x)] = 0$

CBD: Cannot Be Determined; not enough information is given in order to evaluate this limit.

Bonus Problem

Evaluate $\lim_{x \to 0} \frac{1}{x} \sin\left(\frac{1}{x}\right)$

Consider one-sided limits.

$\lim_{x \to 0^+} \frac{1}{x} \sin\left(\frac{1}{x}\right)$ DNE

This limit is not in an indeterminate form.
As $x \to 0^+$, $\frac{1}{x} \to \infty$ (increases without bound), and $\sin\left(\frac{1}{x}\right)$ oscillates between -1 and 1 infinitely often.

So, the product is oscillating between positive and negative values, getting larger in absolute value as $x \to 0^+$.

Therefore, the limit does not exit.

One can make a similar argument for the limit at $x \to 0^-$.

So, $\lim_{x \to 0} \frac{1}{x} \sin\left(\frac{1}{x}\right)$ DNE

There is no vertical asymptote at $x = 0$.