

It's To Be Expected**ID: 11857****Time required***20 minutes***Activity Overview**

In this activity, students will use a tree diagram to find theoretical probabilities and use this information in lists to find the expected value.

Topic: Expected Value

- *Theoretical probability*
- *Expected value*
- *Binomial Distributions*

Teacher Preparation and Notes

- *This activity introduces tree diagrams and expected value. A homework assignment is included.*
- *Students should have knowledge of the solve command and combination and permutation operations on the handheld.*
- *Students should know the difference between independent and dependent events. They should know the multiplication rule for independent events.*
- **To download the student worksheet, go to education.ti.com/exchange and enter "11857" in the keyword search box.**

Associated Materials

- *ItsToBeExpected_Student.doc*

Suggested Related Activities

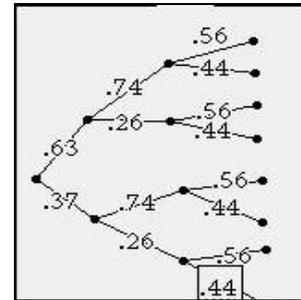
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Makin' It Through The Winter (TI-84 Plus family) — 11936*
- *Binomial Probabilities (TI-Nspire technology) — 9836*

Creating a Tree Diagram

Students are given a scenario involving three basketball players with different shooting percentages. First discuss with them whether the events are independent or dependent and why. Then have them change the labels of the diagram on their worksheet to show the percentages associated with each player. Aisha has a 63% chance of making a basket and a 37% chance of missing, Bria has a 74% chance of making her shot and a 26% chance of missing, and Carmen has a 56% change of making her shot and a 44% chance of missing.

The diagram given represents one way to calculate the probabilities. Discuss with students other ways to set up the diagram, such as with Bria or Carmen shooting first, and whether that would change the probabilities.



With the help of the diagram, students can calculate the eight probabilities. From these probabilities, they will determine the probabilities of none, one, two, or all three of the girls making their shots. The students should follow a path through the diagram to guide their calculations.

Discuss with students the multiplication rule that allows them to calculate the probabilities. Ask students what other probabilities they can calculate. For example:

$0.63 \cdot 0.74 \cdot 0.56$	0.261072
$0.63 \cdot 0.74 \cdot 0.44$	0.205128
$0.63 \cdot 0.26 \cdot 0.56$	0.091728
$0.63 \cdot 0.26 \cdot 0.44$	0.072072
$0.37 \cdot 0.74 \cdot 0.56$	0.153328
$0.37 \cdot 0.74 \cdot 0.44$	0.120472
$0.37 \cdot 0.26 \cdot 0.56$	0.053872
$0.37 \cdot 0.26 \cdot 0.44$	0.042328

What is the probability that Aisha will make her shot and Bria will miss her shot?
 $((0.63)(0.26) = 0.1638$ or 16.38%)

- (8 outcomes, ^m means miss)
 $ABC, AB^mC, ABC^m, AB^mC^m, A^mBC, A^mB^mC, A^mBC^m, A^mB^mC^m$
- 0.63; 0.37
- 0.74; 0.26
- 0.56; 0.44
- 0.042328
- $0.072072 + 0.120472 + 0.053872 = 0.246416$
- $0.205128 + 0.091728 + 0.153328 = 0.450184$
- 0.261072
- \$12,080.80

$$\begin{array}{r}
 0.37 \cdot 0.26 \cdot 0.44 \\
 0.042328 \\
 0.072072 + 0.120472 \\
 2 + 0.053872 \\
 .246416
 \end{array}$$

Introducing Expected Value

Next students are introduced to expected value. The **expected value** is the sum of the products of probabilities of the outcomes and their payoffs. In this contest, different numbers of baskets win different dollar amounts. Students will use the List Editor to determine what the girls can expect to earn for their charity.

Before entering the probabilities, discuss the expected value formula with students. It uses the probabilities as weights on the payoffs. The payoff of the outcome with the highest probability should be the most likely choice.

Step 1: Students will enter the outcomes (0, 1, 2, or 3 baskets) in list **L1**. They will then enter the probability of each outcome in list **L2**.

L1	L2	L3	3
0	.04233	0	
1	.24642	5000	
2	.45018	12000	
3	.26107	20000	
-----	-----	-----	
L3(5) =			

Step 2: They then enter the associated payoff in list **L3**.

Step 3: To calculate probability*payoff (list L4), students should arrow up to the formula bar of **L4**, type **L2** \times **L3**, and press **ENTER**. This will compute the product for all rows in **L2** and **L3** and store the results.

L2	L3	L4	4
.04233	0	-----	
.24642	5000		
.45018	12000		
.26107	20000		
-----	-----		
L4 = L2 * L3			

Step 4: The expected value is the sum of the values in list L4. Students can use the sum command or add up the cells by hand. To use the sum command, return to the Home screen and type: **2nd** **[LIST]** and choose **MATH > sum(**. Then enter the name of the list for which you want the sum (**L4**), and press **ENTER**.

sum(L4)	12080.82
---------	----------

Students should find the expected value to be \$12,080.80. Actual results may differ based on the rounding that occurs.

Be sure to discuss what this result means. Ask the students, *Should the charity expect to earn \$12,080.80 from the contest?* Students may be tempted to say yes based on the term “expected value.” Use the tree diagram to show these students that it is very possible that the charity will earn *no* money from the contest. The expected value is simply what they would earn on average if the contest were repeated many times.

To drive this concept home, calculate the expected value of purchasing a lottery ticket for a 3-digit lottery where the three digits chosen must exactly match the winning digits (order is important). The payoff for matching all three digits in the correct order is \$5,000. Otherwise the payoff is \$0. There are 720 ($10 \text{ nPr } 3$) different permutations of the three digits, so the probability of winning is $\frac{1}{720} = 0.001389$. The expected value is $(0.001389 * 5,000) \approx \7 . But should you expect to win \$7 each time you buy this type of lottery ticket? No! The probability of winning nothing (and losing the \$1 you paid for the ticket) is greater than 99%.

Note: In the lottery described in the Extension problem, the order in which the numbers are chosen does not matter. That problem involves a combination.

The following table encompasses the calculations of the entire activity. The first three columns display the chance of each girl making or missing the shot.

Aisha	Bria	Carmen	Probability	Success		
0.63	0.74	0.56	0.261072	3	0.261072	20000
0.63	0.74	0.44	0.261072	2	0.450184	12500
0.63	0.26	0.56	0.091728	2		
0.63	0.26	0.44	0.072072	1	0.246416	5000
0.37	0.74	0.56	0.153328	2		
0.37	0.74	0.44	0.120472	1		
0.37	0.26	0.56	0.053872	1		
0.37	0.26	0.44	0.042328	0	0.042328	0

Extension – Putting it All Together

1. See table to the right.
2. 42¢ (may differ based on rounding)
3. 58¢
4. \$1 (The break-even point is when the expected value is equal to what someone pays to play.)
5. Approx. \$144,000,000 (To determine the break-even point for mega millions lottery students may use the **solve** command to find the value for **x** in the equation):

$(0.181885 + \frac{1}{175771536} x) - 1 = 0$, or they can solve the equation by hand. Let the jackpot be **x**.)

Prob	Payoff	Prob*Payoff
5.069115E-9	4.2 E7	0.239028
2.56102E-7	250000	0.064025
0.0000001	10000	0.014512
0.000065	150	0.009796
0.000073	150	0.010885
0.003268	7	0.022876
0.001185	10	0.011848
0.007092	3	0.021277
0.013333	2	0.026667

To use the **solve** command, press $\boxed{2\text{nd}}$ [CATALOG] and choose **solve(**. In parentheses enter the side of the equation set equal to zero, the variable, and the guess.

```
solve(.181885+(1
/175771536)X-1,X
,1)
143801330.2
```

NOTE: To use the command, the equation must be set equal to 0.

If using MathPrint™ OS:

Students can use the fraction template when using the solve command. To do this, press $\boxed{\text{ALPHA}}$ [F1] and select **n/d**. Then enter the numerator, press $\boxed{\downarrow}$, and then enter the denominator. Press $\boxed{\rightarrow}$ to move out of the fraction template.

```
solve(.181885+
1/n/d
2: Un/d
3: n/d4Un/d
4: n/d
FRAC|FUNC|MTRX|YVAR
```

The students can also calculate the number of different sets of five numbers out of 56 numbers by using the **nCr** command, found in the **PRB** menu ($\boxed{\text{MATH}}$). They should multiply the calculated value by 46 to determine the number of different sets of lottery numbers for this type of lottery.

```
56 nCr 5
3819816
Ans*46
175711536
```

Note: The calculator uses E to display some of the numbers in scientific notation.