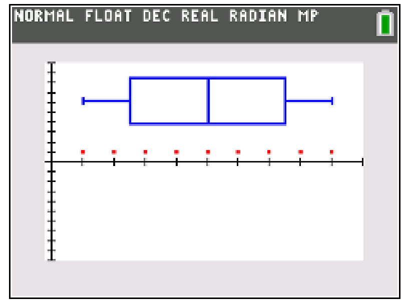



## Math Objectives

- Students will organize data and find the Five Number Summary.
- Students will use their handhelds to verify the data analysis that have produced by hand.
- Students will interpret their data analysis using the visual of a Box and Whisker diagram.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

## Vocabulary

- Outlier
- Quartile
- Inter-Quartile Range
- Five Number Summary
- Box and Whisker Diagram

## About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 4 Statistics and Probability:

  **4.1:** **(a)** Interpretation of Outliers

  **4.2:** **(a)** Presentation of Data (discrete and continuous)

  **(d)** Production and understanding of box and whisker diagrams

  **4.3**: **(a)** Measure of central tendency (mean, median, and mode)

  **(c)** Measures of dispersion (Range and Inter-Quartile range)

  As a result, students will:

  - Apply this information to real world situations.

## Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

## Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

**Tech Tips:**

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**
*Student Activity*
84CE-GiveMeFive-Student.pdf
84CE-GiveMeFive-Student.doc

In this activity, students will discuss and describe the center and spread of a univariate data set by way of a five number summary and visually by a box & whisker diagram. Students will then apply this knowledge to real life applications to enhance their ability to understand this math in statistical data analysis.

**Teacher Tip:** This is a great time to lead your students through the process of entering the data on their handheld and showing them how to analyze and discuss both the Five Number Summary and a Box and Whisker Plot.

**Introduction**

A univariate set of data is a list of numbers that describes the different value of a variable characteristic across a range of different units. For example, if a study involved finding out the height of a range of people, each person whose height is measured is statistically considered to be a 'unit'. Height is the characteristic that varies (variable) and the list of height measurements is called the data.

When describing a group of data, there are generally two main types of things to consider:

a) Measure of center – this is a single value that could be used as a representative of the entire data set (e.g. mean, median, mode)
b) Measure of spread – this is a number that indicates how spread out the data are (e.g. standard deviation, range, inter-quartile range)

**Problem 1 – The Fantastic Five**

A five number summary is a convenient way of describing a set of data as it provides us with information about both center and spread. Consider the set of data: {1, 2, 3, 4, 5, 6, 7, 8, 9}. We can see that the numbers are already ordered from lowest to highest.

1. Find the *minimum* value in the data set. We call this value **MinX**.

   **Solution: 1**

2. Find the *maximum* value in the data set. We call this value **MaxX**.

   **Solution: 9**

3. Find the *middle* value in the data set. We call this value **MedianX**.

   **Solution: 5**

4. Look at the numbers that are less than the Median, find the median of *this* set of numbers. Discuss with a classmate what the median would be if this data set was an even number of data and if this data set was an odd number of data. Find the name of this piece of data.

**Solution: 2.5**

As there are only four numbers in this group, you have to work out what number is exactly half way between two digits, for this data set it will be a fraction, we call this value $\mathbf{Q_1}$ or the **Lower Quartile**.

5. Look at the numbers that are less than the Median, find the median of *this* set of numbers. Discuss with a classmate what the median would be if this data set was an even number of data and if this data set was an odd number of data. Find the name of this piece of data.

**Solution: 7.5**

As there are only four numbers in this group, you have to work out what number is exactly half way between two digits, for this data set it will be a fraction, we call this value $\mathbf{Q_3}$ or the **Upper Quartile**.

6. You have found $Q_1$ and $Q_3$. Discuss with a classmate what $Q_2$ is.

**Solution:** $Q_2$ is the median.

The five numbers that are your answers to questions 1 to 5 are called the **five number summary**. Usually the five number summary is written in the order MinX, $Q_1$, Median, $Q_3$, MaxX. The median (Answer to question 3) is the measure of center. The other numbers provide indications of spread.

- MaxX minus MinX is the **Range**.
- $Q_3$ minus $Q_1$ is the **Inter-Quartile Range (IQR)**.

7. Find the *range* for the data set {1, 2, 3, 4, 5, 6, 7, 8, 9}.

**Solution: 8**

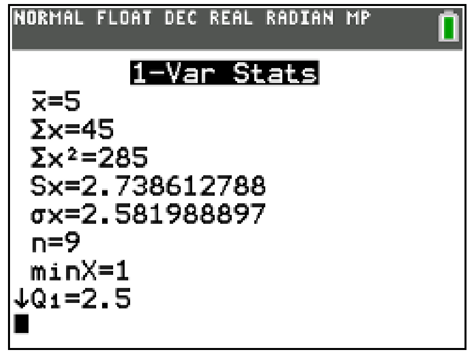
8. Find the *Inter-Quartile Range* for the data set {1, 2, 3, 4, 5, 6, 7, 8, 9}.

**Solution: 5**

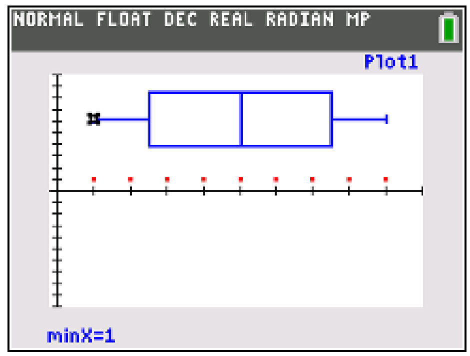
**Problem 2 – Automatic Calculation of the Five Number Summary**

On the handheld, press **stat**, **1: edit** and enter the data set {1, 2, 3, 4, 5, 6, 7, 8, 9} into $L_1$ and the data set {1,1,1,1,1,1,1,1,1,1} into $L_2$.

The data values are now entered into a List under the column title of $L_1$. Calculate the summary statistics for the data by pressing **stat**, **CALC**, **1: 1-VAR STATS** (list is $L_1$ and press calculate) to display the statistics we will be referencing in this activity.




Next, create a graphical representation of the data set, called a Box Plot or a Box & Whisker Diagram. Press **2$^{nd}$ y =** (**statplot**). Under Plot1, turn it on, select the first box plot, and make sure your xlist is $L_1$. Under Plot2, turn it on, select the scatter plot and make sure your xlist is again $L_1$ and your ylist is $L_2$ to create a dot plot beneath the Box Plot. If you press trace and the left and right arrows, the five numbers of the five number summary will be revealed. Note that they are in line with corresponding numbers on the scale below it.




Go back to your data list (stat, edit) and change the final value from a nine to a ten and return to your Box Plot.

1. Explain why $Q_1$, the median and $Q_3$ do not change when the data point (9) is increased.

   **Solution:** The 9 is in a different quartile and remains in the same quartile even when it is increased.

2. Keep changing this final value. Explain what happens to the whisker when this data point is moved further and further away from the rest of the data. Find at what value, approximately, this significant change occurs.

   **Solution:** The length of the whisker increases until it 'snaps' and the point becomes an outlier … just past 15.

3. Return to your lists page (**stat**, **edit**) and enter the data set {9, 3, 8, 5, 7, 4, 1, 6, 2} into $L_3$. Discuss with and state the affects this may have on the Box Plot and statistics calculations. Explain why you think this is so.

**Solution:** It doesn't make any difference as the values haven't changed, they're just re-ordered. The five number summary is based on the ordered values therefore they will be returned to their corresponding order before calculation.

4. Compute the five number summary of the data set, as you did at the start of problem 2, by placing this data on your lists page under $L_4$: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Validate the five number summary by hand.

**Solution:** MinX = 1, $Q_1$ = 3, MedX ($Q_2$) = 5.5, $Q_3$ = 8, MaxX ($Q_4$) = 10
(by inspection) {1, 2, **3**, 4, 5} $\frac{5+6}{2} = 5.5$ {6, 7, **8**, 9, 10} (by inspection)

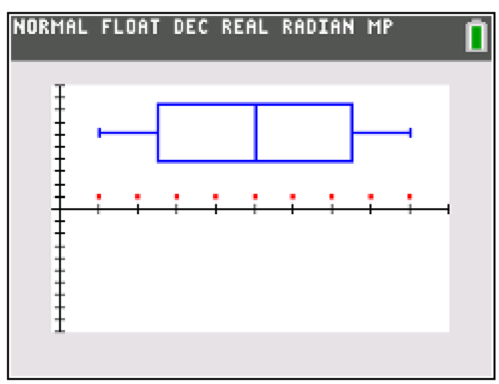
5. Use your answer to the previous question to find a data set that has a five number summary made up entirely of integers.

**Solution:** Answers may vary.
Examples: {1, 2, 3, 4, 5, 5, 7, 8, 9} or {1, 2, 3, 4, 6, 6, 7, 8, 9}

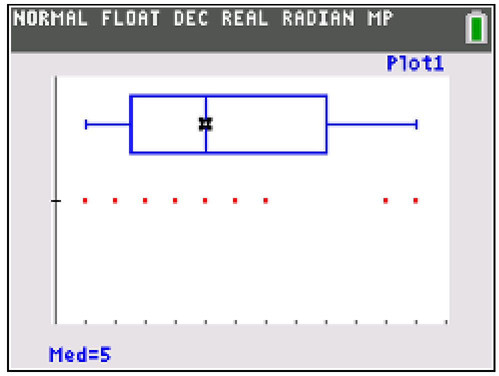
**Problem 3 – Consideration of shape and skew**

So far the data sets we have considered have been **symmetrical**. That is, the Box Plot is geometrically symmetrical and has a vertical line of symmetry at the median. This means that $Q_1$ is as far below the median as $Q_3$ is above it and MinX is as far below the median as MaxX is above it. You may also have noticed that the **mean** value (the first value shown when finding the 1-VAR STATS) is always the same as the median.

Using the original data set: {1, 2, 3, 4, 5, 6, 7, 8, 9}, note that the median value is 5, as also is the mean. Looking at the Box Plot for these data, notice that it is perfectly symmetrical.

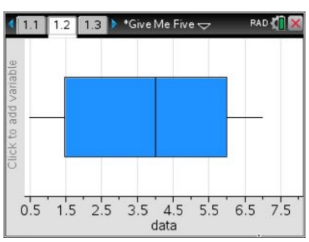
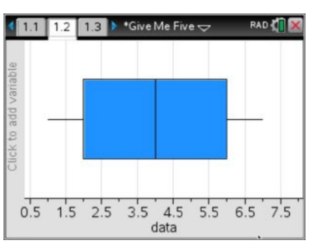


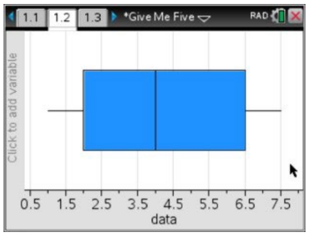
Going back to your lists page, replace the final two values of $L_1$ (8, 9) with 11 and 12. Return to the graph. $Q_3$ will move up to about 9. Notice that the distribution is now no longer symmetrical. The part of the box that is between the median and $Q_3$ is bigger than the part between the median and $Q_1$. The distribution is now said to be **positively skewed** or **skewed right**.




Notice also that, although the median is still 5, the mean value has moved up to about 5.6. Restore the original data set and repeat this process to show a **negative skew** or **skew left**.

1. Match each of the following Box Plots with its matching description of symmetry and comment about measure of center.

| **Box Plot** | **Description of Shape** | **Comment on measure of center** |
|---|---|---|
|   | Symmetrical | Mean > Median |
|   | Positively skewed | Mean < Median |




Negatively skewed

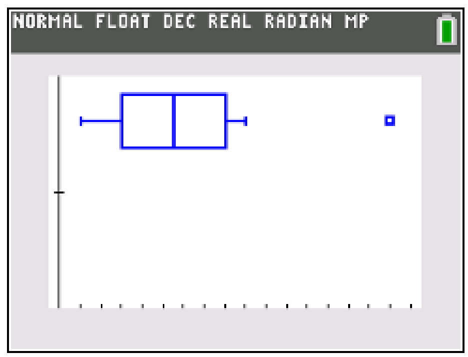
Mean = Median

**Teacher Tip:** This would be a good point to go a little further with this problem. You can have wonderful discussion about each type of skew and ask the students to give real life examples for each.

**Outliers and Fences**

Restore your data list to the set: {1, 2, 3, 4, 5, 6, 7, 8, 9}.

Now add in a 10th value to the list. Make this value 16. Observe the corresponding Box Plot. Notice that the value 16 doesn't appear within the main box or whisker, but is shown as a dot on its own. This is because the score 16 is so far away from the other data points that it is considered to be an **outlier**.




Experiment by replacing the 16 with values that are closer to the original data set. Try replacing it with 14, 13, 12, 11, 10.

A numerical value that determines the threshold for outliers can be computed and is referred to as the **upper fence value** where the outlier is above the median and **lower fence value** where the outlier is below the median. The upper and lower fences are defined by using the Inter-Quartile Range (IQR).

Upper Fence Value = Q3 + 1.5 x IQR

Lower Fence Value = Q1 - 1.5 x IQR

**Example**

If you have a set of 8 scores {1, 2, 3, 4, 5, 6, 7, 8}, such that $Q_1 = 2.5$, $Q_3 = 6.5$ and the IQR = 4.

Upper fence = 6.5 + 1.5 x 4
= 12.5

Lower Fence = 2.5 – 1.5 x 4

= -3.5

2. Discuss with a classmate and explain how it is possible to calculate the IQR whilst a single outlier is changed.

**Solution:** As $Q_1$ and $Q_3$ remain unchanged when the outlier is changed, the IQR will also remain unchanged.

**Further IB Application**

The scores of a mathematics test given to period 1 are shown below.

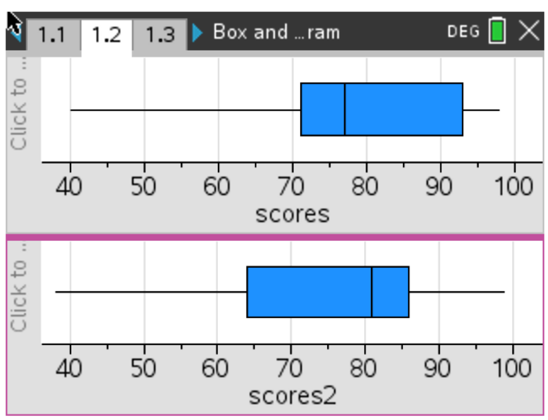
40, 62, 65, 71, 73, 74, 75, 77, 80, 90, 92, 93, 96, 97, 98

For the data, the lower quartile is 71 and the upper quartile is 93.

(a) Show that the test score of 40 would not be considered an outlier.

**Solution:** (93 – 71) x 1.5 or 22 x 1.5 seen anywhere or 33 seen anywhere
71 – 33
38
40 > 23
So is not an outlier

The same mathematics test was given to period 2 and the box and whisker diagram showing their scores (**scores2**) and comparing them to the scores of period 1 (**scores**) are below.




A fellow mathematics teacher looks at the box and whisker diagrams and believes that period 2 performed better than period 1.

(b) Using the diagrams above, state one reason that may support the mathematics teacher's opinion and one reason that may counter it.

**Solution:** The median score for the second period class is higher than the median score for the first period class.

Then:

But the scores are more spread out in the second period class.

Or

But the scores are more inconsistent in the second period class.

Or

But the lowest scores are in the second period class.

Or

But the lower quartile is lower in the second period class.

**Teacher Tip:** This is a good place to have students discuss this situation and see if they can add more questions, scenarios and discussions to the problem.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*