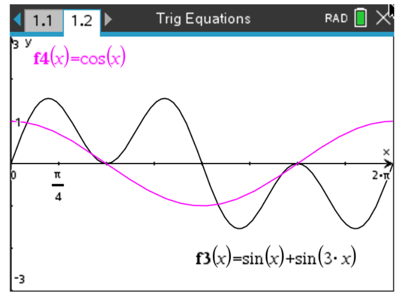
Can You Solve A Trig Equation?

MATH NSPIRED



Math Objectives

- Students will solve basic multi-step trigonometric equations within a finite interval.
- Students will review and use trigonometric identities to help in the solving of trigonometric equations.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Finite Interval
- Identity
- Verify
- Radian

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 3 Geometry and Trigonometry:

AI HL 3.8: **(a)** The definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.
(b) Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$
(c) $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$
(d) Graphical methods of solving trig equations in a finite interval.

AA 3.5: **(a)** The definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle.
(b) $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$
(c) Exact values of trig ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.

3.6: **(a)** Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$
(b) Double angle identities for $\cos\theta$ and $\sin\theta$
(c) The relationship between trig ratios.

3.8: **(a)** Solving trig equations in a finite interval both graphically and analytically.
(b) Equations leading to quadratic equations in $\sin x$, $\cos x$, or $\tan x$.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
CanYouSolveATrigEquation-Student-Nspire.pdf
CanYouSolveATrigEquation-Student-Nspire.doc

As a result, students will:

- Apply this information to real world situations.

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

In this activity, students will practice how to solve equations involving trigonometric functions within a finite interval. Starting with basic two-step equations and then progressing to equations involving the use of trigonometric identities, students will review how to solve these equations both analytically and graphically. By the end of the activity, students will then apply this trig equation solving knowledge to real world situations.

Teacher Tip: This activity is meant to support your teaching of both simplifying and verifying trig identities, along with the solving of trig functions. The answers given in the activity will be in radian measure, but if you prefer to work with degrees, feel free to adjust the directions in the student document.

Before we start solving the trig equations, please know that students will be finding answers for this activity in radian measure and depending on the problem, students need to know how to find both exact answers or answers rounded to three significant figures.

Problem 1 – Solving Basic Trig Equations

Solve the following equations for $0 \leq x < 2\pi$, algebraically, leaving the exact answers in radian measure.

1. $2 \cdot \sin x + 1 = 0$

   Solution: $\sin x = -\frac{1}{2}$

   $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

2. $4 \cdot \cos^2 x - 3 = 0$

   Solution: $cos^2 x = \frac{3}{4}$

   $\cos x = \pm\frac{\sqrt{3}}{2}$

   $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3. $\sin x \cdot \cos x = \sin x$

   Solution: $\sin x \cdot \cos x - \sin x = 0$

   $\sin x(\cos x - 1) = 0$

   $\sin x = 0 \;\; and \;\; \cos x = 1$

   $x = 0, \pi$

4. $\cos^2 x - 2 = \cos x$

   Solution: $cos^2 x - \cos x - 2 = 0$

   $(\cos x - 2)(\cos x + 1) = 0$

   $\cos x = 2 \;\; and \;\; \cos x = -1$

   Not possible! $x = \pi$

5. With a classmate, discuss how you might graphically solve the equations above. Verify your discussed method(s) by checking one or two of the problems above graphically.

   Possible Discussion: Graph the left side of the equation in ***f1(x)*** and the right side of the equation in ***f2(x)*** and find their point of intersection.

Teacher Tip: When graphing, please discuss with the students or remind them of the difference between radian and degree measure. This is also a great time to discuss how important an accurate window is on the handheld graph.

Problem 2 – Using Identities to Verify Trig Equations

Here are some Trig Identities that might be helpful to use in Problem 2:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad cos^2\theta + sin^2\theta = 1$ $\quad \sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Show that:

6. $\cos x \cdot \tan x = \sin x$

   Possible Solution: Answers will vary.

   $\cos x \cdot \frac{\sin x}{\cos x} = \sin x$

   $\sin x = \sin x$

7. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

   Possible Solution: Answers will vary.

   $(1 - \sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta$

   $1 - 2\sin^2\theta = 1 - 2\sin^2\theta$

8. $\frac{\cos^2 x - 1}{\cos^2 x} = -\tan^2 x$

Possible Solution: Answers will vary

$\frac{-\sin^2 x}{\cos^2 x} = -\tan^2 x$

$-\tan^2 x = -\tan^2 x$

9. With a classmate, discuss how you can verify these trig expressions are equivalent on your handheld. Verify your discussed method(s) by checking one or two of the problems above.

Possible Discussion: On a calculator page, type in (for example) $\cos x \cdot \tan x = \sin x$, the handheld will display true or false if the left side of the equations is equivalent to the right side of the equation or not, respectively.

Teacher Tip: Reiterate to the students that there are always multiple ways to verify trig equations and the solutions above are a sample of **ONE** method of verification.

Problem 3 – Using Identities to Solve Trig Equations

Using Trig Identities, solve the following equations for $0 \le x < 2\pi$, leaving the exact answers in radian measure. Use the methods discussed with a classmate above to check your answers.

10. $\sin^2 x + 1 = \cos x$

Possible Solution:

$\sin^2 x - \cos x + 1 = 0$

$1 - \cos^2 x - \cos x + 1 = 0$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x + 2)(\cos x - 1) = 0$

$\cos x = -2 \quad and \quad \cos x = 1$

Not Possible $\quad x = 0$

11. $\cos x = \sin 2x$

Possible Solution:

$\cos x = 2\sin x \cdot \cos x$

$0 = 2\sin x \cdot \cos x - \cos x$

$0 = \cos x\,(2\sin x - 1)$

$\cos x = 0 \quad and \quad 2\sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

12. $4 - \cos 2x = 3$

Possible Method for Solution:

$4 - (2\cos^2 x - 1) = 3$

$-(2\cos^2 x - 1) = -1$

$2\cos^2 x - 1 = 1$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi$

Extension – Real World Scenario

Texas Instruments has designed a way to control a drone with your handheld. They wanted to test its flight path and height traveled. The function below represents the drone's altitude, $h(t)$ in feet, t seconds after the beginning of its flight.

$$h(t) = 20 + 10\sin^2\left(\frac{1}{3}t\right)$$

If the flight starts at *t = 0* seconds, find at what times the drone will achieve a height of 25 feet over the first 15 seconds. Round your answer(s) to three significant figures.

Solution: Graphically, students can graph the function in ***f1(x)*** and 25 in ***f2(x)*** and find the point of intersections. Those would occur at the times: $t = 2.36, 7.07, and\ 11.8\ seconds$.

Analytically:

$25 = 20 + 10sin^2\left(\frac{1}{3}t\right)$

$\frac{1}{2} = sin^2\left(\frac{1}{3}t\right)$ $\qquad let\ x = \frac{1}{3}t$

$\frac{1}{2} = sin^2(x)$

$\sin x = \pm\frac{\sqrt{2}}{2}$ $\qquad x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Since $x = \frac{1}{3}t$ and $t = 3x$, $t = 2.36, 7.07, 11.8, and\ 16.5\ seconds$, but since 16.5 is outside of the timeframe given, we eliminate that answer.

Further IB Application

(a) Show that $\cos 2x + \sin^2 x = \frac{\sin 2x}{2\tan x}$.

Possible Solution: $1 - 2\sin^2 x + \sin^2 x = \frac{2\sin x\cos x}{2\left(\frac{\sin x}{\cos x}\right)}$

$$1 - \sin^2 x = \sin x\cos x\left(\frac{\cos x}{\sin x}\right)$$

$$\cos^2 x = \cos^2 x$$

For part (b), exact answer(s) should be in radian measure.

(b) Hence or otherwise, solve $\cos 2x + \sin^2 x = \frac{1}{2}$.

Possible Method for Solution: $\cos^2 x = \frac{1}{2}$

$$\cos x = \pm\frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
Any part to any Problem in the activity would be a great way to quickly assess your student's understanding of solving trig equations with and without the use of identities.

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review and apply the idea of solving trig equations, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*