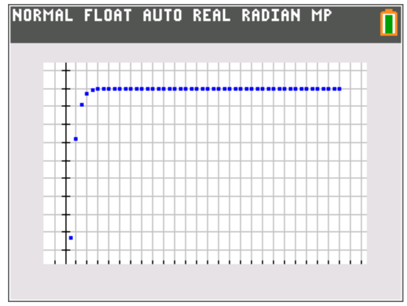
Summing Up Geometric Series
Student Activity

Name _______________

Class _______________

In this activity, you will explore the concept of finding the sum of an infinite geometric series. Reviewing the concepts of when you can find the sum of an infinite geometric series will be the first task, discussing with your classmates not only what the characteristics of a geometric sequence are, but also the key characteristic that allows you to add every term of the infinite sequence and still get a non-infinite sum.



Let us review the characteristics of a geometric sequence. A **geometric sequence** is a sequence of terms where the ratio of every two consecutive terms is constant. The constant ratio is referred to as the **common ratio** or ***r***. To find subsequent terms of a geometric sequence, multiply a term by ***r***. The nth term, u_n, formula for a geometric sequence is $\boldsymbol{u_n = u_1 \cdot r^{n-1}}$, where u_1 is the first term, and *r* is the common ratio.

Problem 1 – Geometric Sequence Practice

1. Find the next three terms of each infinite geometric series.

   (a) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ (b) $2+\frac{3}{2}+\frac{9}{8}+\cdots$

2. Discuss with a classmate how you would find the next three terms of each series. Explain your results.

3. Sigma notation is used at times to express a series. The symbol for sigma, Σ, actually means the sum of. Using the nth term formula from above and the sigma notation, write an expression in terms of *n* that describes each of the series from number 1.

4. Discuss with a classmate how you would find the sum of each series in number 1. Explain your results.

Problem 2 – Finding the Sum of a Geometric Series

There are two types of geometric series. There is the partial sum or finite series and then there is the infinite series.

1. Discuss with a classmate the formula to find a partial sum of a geometric series. Explain what you would need to use the formula and if there are any restrictions.

2. Explain how you could use sigma notation to find the partial sum of a geometric series as well. Explain what you would need to use sigma.

3. Discuss with a classmate the formula to find an infinite sum of a geometric series. Explain what you would need to use the formula and if there are any restrictions.

4. Explain how you could use sigma notation to find the infinite sum of a geometric series as well. Explain what you would need to use sigma.

5. Given the geometric sequence 1, -4, 16, -64, …, find the partial sum of the first 9 terms.

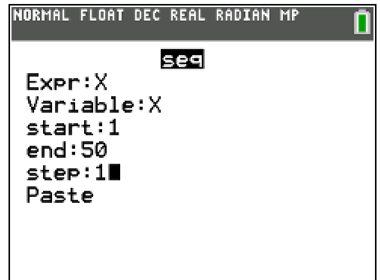
6. Given the geometric sequence 9, 3, 1, $\frac{1}{3}$, …, find the infinite sum.

7. Write the sequences in questions 5 and 6 in sigma notation. Explain if you can use your handheld to verify your answers using sigma notation.

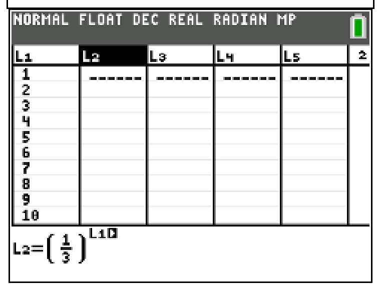
Problem 3 – Visualizing an Infinite Geometric Series

Use Lists to display the terms of each series.

Press **stat**, **enter** to access the table of data screen. In **L1**, enter **seq(x,x,1,50)** in the top most cell. The **seq(** command can be found by pressing **2nd**, **stat [list]** and arrowing over to **OPS** and selecting **5:seq(**. Enter the information in the **seq** exactly as shown in the screen to the right.

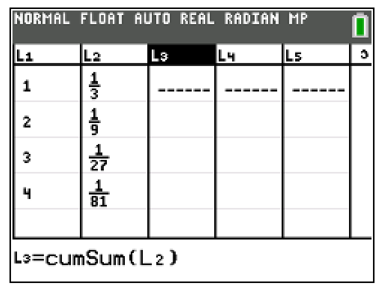


In the top most cell of **L2**, type $\left(\frac{1}{3}\right)^{L_1}$ and **enter**.



Next, we will graph the series.

First, we will need to generate a list with the cumulative sums of the terms of the sequence. To do this, move to the top most cell of **L3**, press **enter**, then press **2nd**, **stat [list]** and arrow over to **OPS** and select **6:cumSum(**. Then type **2nd**, **2 [L2]** and press **enter**.



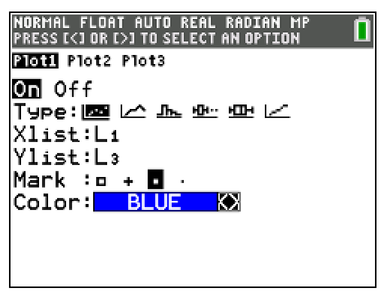
This will list the first 50 partial sums of the series in **L3**.

You can view a graph for each series by creating a scatter plot of the values of the partial sums of the series.

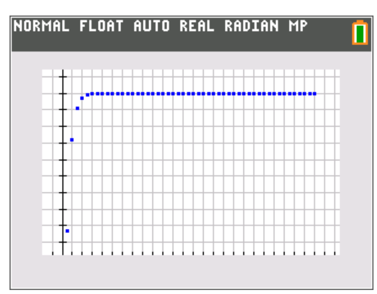
To create a scatter plot, select **2nd**, **y= [stat plot]**, **1**.

Set up as shown in the figure to the right.

To view the graph, press **zoom**, **9:ZoomStat**.



To get an even better view of the behavior of the partial sums, you can change the scaling of the *x* and *y*-axes. Press window and change each of the following: **Xscl:** 2 **Yscl:** 0.2**.** The graph should look like the screen shown to the right.



1. Discuss with a classmate the characteristics you notice about the graph of the partial sums. Write down what you notice.

2. Discuss with a classmate what you expect the total sum of all of the terms of the geometric series would be. Explain how you arrived at your conclusion.

3. Write an expression for the sum of the infinite series. Find the value of this sum. Explain how you found this sum.

4. Express your answer from Question 6 in sigma notation.

5. Suppose you change the base of $\left(\frac{1}{3}\right)$ to a 3.
   a. Express the sum of the terms as an infinite sum.

   b. Describe what happens to this sum as each term increases. Explain your answer.

6. Give an example of an infinite geometric series that you think would have a finite sum and an example of one that you think would not have a finite sum. Explain your reasoning.

7. Based on the information above, describe what conjecture that must be true about the ratio of the consecutive terms of an infinite geometric series for the series to have a finite sum.

8. Find the values of the ratio *r* where an infinite geometric series appears to have a finite sum.

Further IB Application

A local coffee shop had an amazing first year after it opened, earning $40,000 of profit. Unfortunately, the profits have been decreasing by 10% each year after the first. Assuming that this trend continues, find the total profits the shop hopes to earn over the course of its lifetime.