

1. Water is being poured into a cone of height  $h$  cm and base radius  $r$  cm. The height of the cone is twice the radius of the base. The height of the water is increasing at a rate of  $\frac{1}{3}$  cm/min. Find the rate at which the water is being poured, in  $\text{cm}^3/\text{min}$ , when the height is 6 cm. (5 marks)

Mark scheme:

$$V = \frac{1}{3}\pi r^2 h$$

Given:  $h = 2r$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12}\pi h^3 \quad (\text{M1})$$

$$\frac{dv}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} \quad (\text{A1})$$

$$\frac{dv}{dt} = \frac{1}{4}\pi(6)^2 \left(\frac{1}{3}\right) \quad (\text{M1})(\text{A1})$$

$$\frac{dv}{dt} = \frac{1}{4}\pi(36) \left(\frac{1}{3}\right)$$

$$\frac{dv}{dt} = 3\pi \text{ cm}^3/\text{min} \quad (\text{A1})$$