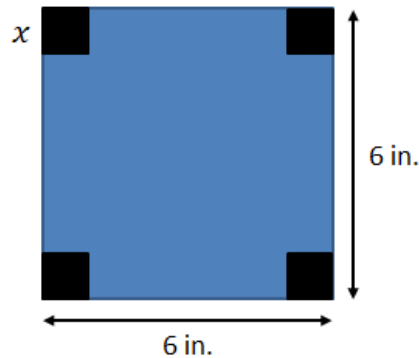
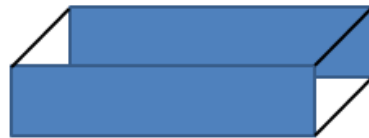


1. Erica works for a company that produces metal tins. The tins are created from a metal sheet 6 in. by 6 in. Erica’s job is to operate a machine that removes a square, with side length  $x$  in., from each corner. See picture below, not drawn to scale.



The remaining metal sheet is sent to a machine that folds up the sides to form a tray. See picture below, not drawn to scale.



- (a) Find the length and width of the tray, in terms of  $x$  (2 marks)
- (b) Show that the volume,  $V$  in<sup>3</sup>, of the tray is (2 marks)  
 $V = 4x^3 - 24x^2 + 36x$
- (c) Find  $\frac{dv}{dx}$  (3 marks)
- (d) Using your answer from part (c), find the value of  $x$  that maximizes the volume of the tin. (3 marks)
- (e) Find the maximum volume of the tin (2 marks)

Mark scheme:

- (a) Length:  $6 - 2x$  (A1)  
 Width:  $6 - 2x$  (A1)

$$(b) V = l * w * h$$

$$V = (6 - 2x)(6 - 2x)(x)$$

$$V = 4x^3 - 24x^2 + 36x$$

(M1) ft Correct substitution of their length and width into the volume of a rectangular prism formula

(A1) ft Correct multiplication of the three polynomials

$$(c) \frac{dv}{dx} = 12x^2 - 48x + 36$$

(A1)(A1)(A1)

$$(d) 0 = 12x^2 - 48x + 36$$

$$0 = x^2 - 4x + 3$$

$$0 = (x - 3)(x - 1)$$

$$x = 3, 1$$

$$x = 1$$

(1) Setting their part (c) = 0

(1) Solving for x

(A1) Knowing that 3 cannot work and that 1 is the solution

$$(e) V = 4(1)^3 - 24(1)^2 + 36(1)$$

(1) ft Correct substitution of their solution from part (d)

$$V = 16 \text{ in}^3$$

(1) ft