Math Objectives

- Students will utilize data in real-life applications of the quadratic function.
- Students will generate quadratic regression equations to model the relationship between two variables in a set of data.
- Students will interpolate and extrapolate values using their quadratic models for this real-life application.
- Students will write compound inequalities to describe the effect of various driving conditions on stopping distance.
- Students will construct viable arguments & critique the reasoning of others (CCSS Mathematical Practice).
- Students will model with mathematics (CCSS Mathematical Practice).

Vocabulary

- compound inequality
- extrapolate
- interpolate
- quadratic regression equation

About the Lesson

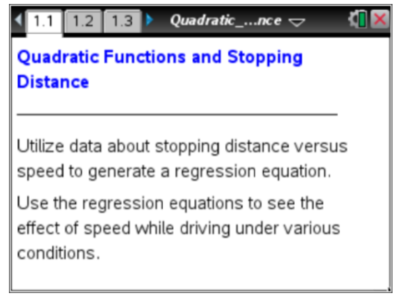
- This lesson involves analyzing data.
- As a result, students will:
  - Compare stopping distance under normal driving conditions with stopping distance while talking on a cell phone or driving in wet weather.
  - Generate quadratic equations to model the effects of these factors and compare them to the stopping distance under normal conditions.
  - Utilize their quadratic equations to make predictions.

TI-Nspire™ Navigator™

- Use Class Capture and/or Live Presenter to demonstrate using the TI-Nspire document and for monitoring student progress.
- Use Quick Poll for assessing students' understanding.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Quadratic_Functions_and_Stopping_Distance_Student.pdf

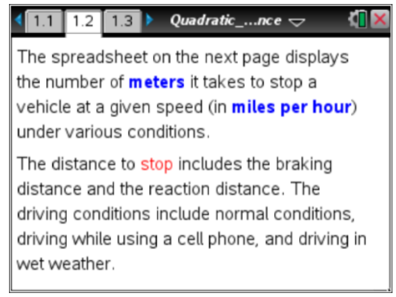
Quadratic_Functions_and_Stopping_Distance_Student.doc

TI-Nspire document

Quadratic_Functions_and_Stopping_Distance.tns

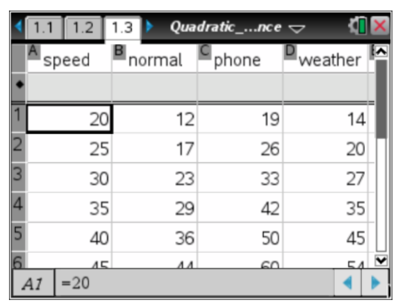
Discussion Points and Possible Answers

Move to page 1.2.



1. Read page 1.2 and move to page 1.3 to look at the spreadsheet.

> **Teacher Tip:** The information for this activity can be found at http://www.stoppingdistances.org.uk/. If possible, have students visit the Web site and utilize the simulator.
>
> Also, see the National Safety Council's Web site: http://www.nsc.org/safety_road/Distracted_Driving/Pages/distracted_driving.aspx.
>
> The *New York Times* did a series of articles on distracted driving. It also has a "game" to measure how a person's reaction time is affected by external distractions. See: http://topics.nytimes.com/top/news/technology/series/driven_to_distraction/index.html?scp=2&sq=distracted%20drivers&st=cse.



a. Describe the effect that increased speed seems to have on stopping distances under normal driving conditions. Explain why this is so.

<u>**Sample Answer:**</u> As the speed increases, the stopping distance will also increase. This makes sense since the faster you are going, the longer it takes to stop.

> **Teacher Tip:** You may want to explain to students that stopping distance is the sum of braking distance and reaction distance. When a car is traveling at a high speed, a driver will have less time to identify hazards and react to them. Vehicles will take a longer distance to stop from higher speeds. If there is a crash, the injuries will be more severe.[1]

[1] http://www.stoppingdistances.org.uk/facts/speed.htm

Braking distance is a quadratic function of speed. As speed increases, a greater distance is covered over the same time, and thus braking distance also increases.

b. Describe the effect that increased speed seems to have on stopping distances when the driver is using a cell phone. How does the increase in stopping distance compare to the increase in stopping distance under normal conditions?

Sample Answer: When the driver is using a cell phone, the stopping distance increases as the speed increases. The stopping distance for a driver using a cell phone is greater than when the driver is driving under normal conditions at the same speed. In addition to the stopping distance being greater, the increase in the stopping distance from one speed to the next is also greater for the driver using a cell phone than it is for a person driving in normal conditions.

Teacher Tip: You may want to explain to students why the stopping distance for a driver talking on a cell phone is greater than the stopping distance for the driver under normal conditions going at the same speed. Although the braking distance for the driver under both conditions is the same,[2] the reaction distance is greater since the driver is distracted by the cell phone and will take longer to react.

According to the National Safety Council, more than 1 out of every 4 motor vehicle crashes involves cell phone use at the time of the crash.[3]

c. Describe the effect that increased speed seems to have on stopping distances in wet weather. How does the increase in stopping distance compare to the increase in stopping distance under the other two driving conditions?

Sample Answer: As the speed increases, the stopping distance also increases. The stopping distance for a person driving in wet weather is greater than that of a person driving at the same speed under normal conditions, but less than that of a person driving while using a cell phone. The increase in stopping distance in wet weather from one speed to the next is greater than stopping distance under normal circumstances, but, in most cases, less than stopping distance for a driver talking on a cell phone.

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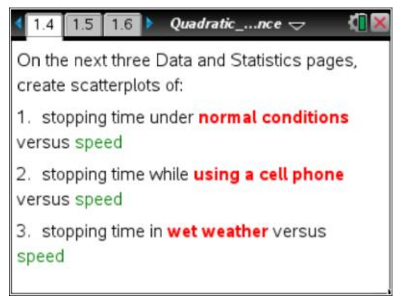
[2] See the simulator at http://www.stoppingdistances.org.uk/.
[3] See http://www.nsc.org/Pages/NSCReleasesWhitePaperonBrainDistractionDuringCellPhoneUseWhileDriving.aspx.

Teacher Tip: You may want to explain to students why the stopping distance when the roads are wet will be greater than the stopping distance under normal conditions. Although the reaction distance may be the same under both conditions,[4] the braking distance is greater because when the roads are wet, the friction between the car's tires and the road surface is reduced. The braking distance will increase dramatically when a vehicle's tires have a shallow tread depth.

Since the reaction distance for a person driving on wet roads is the same as that for a person driving under normal conditions, it is less than that for a driver using a cell phone. However, because of the wet roads, the braking distance is greater. It is important to note that even though the braking distance is greater, the stopping distance is still less than that for a person using a cell phone.

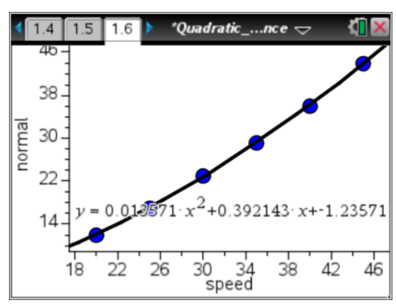
Move to page 1.4.

2. Read pages 1.4 through 1.5. On pages 1.6 through 1.8, you will see three Data & Statistics pages. On each page, click on the center of the bottom of the page to select *speed* as your independent variable. On page 1.6, click on the center of the left side of the page to select *normal* as your dependent variable. On page 1.7, select *phone* as your dependent variable, and on page 1.8, select *weather* as your dependent variable.



TI-Nspire Navigator Opportunity: *Class Capture/Live Presenter*
See Note 1 at the end of this lesson.

Scientists have determined that stopping distance is a quadratic function of speed. On each of the three pages, determine a quadratic function to model the data sets. Select **Menu > Analyze > Regression > Show Quadratic.** The graph of the quadratic function will be drawn and the equation shown. Write the equation for each of the functions.

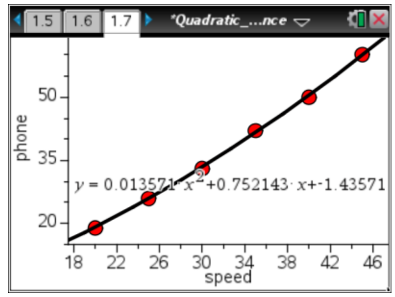


[4] See the simulator at http://www.stoppingdistances.org.uk/.

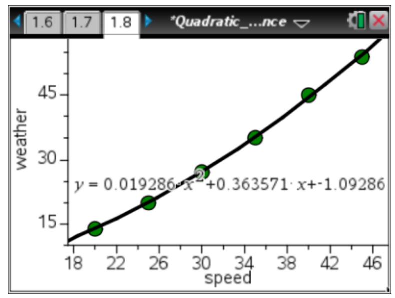
Tech Tip: To determine the quadratic functions to model the data sets, select **> Analyze > Regression > Show Quadratic**.

Normal: $y = 0.013571x^2 + 0.392143x - 1.23571$

Cell Phone: $y = 0.013571x^2 + 0.752143x - 1.43571$



Wet Weather: $y = 0.019286x^2 + 0.363571x - 1.09286$



Teacher Tip: Another idea would be to show all three data plots and graphs on one page to easily compare them. To do this, add a Data & Statistics page, click on the center of the bottom of the page, and select *speed*. Click on the center of the left side of the page, and select one of the driving conditions. Select **Menu > Plot Properties > Add Y Variable**, and choose another driving condition. Repeat for the last driving condition. To graph all regression equations, select **Menu > Analyze > Regression > Show Quadratic.** Click on a graph to display its equation.

Teacher Tip: To show all three graphs on one page, add a Data & Statistics page, tap the center of the bottom of the page, and select *speed.* Tap the center of the left side of the page, and select one of the driving conditions. Select **> Plot Properties > Add Y Variable**, and choose another driving condition. Repeat for the last driving condition. To graph all regression equations, select **> Tools**, and then select **Analyze > Regression > Show Quadratic.** Click on a graph to display its equation.

Teacher Tip: Encourage students to use different methods to find the following answers (algebraic, Scratchpad, etc.)

3. Use the regression equations to estimate, to the nearest meter, the stopping distance if a car was driving at 43 mph:
   a. Under normal conditions

      Answer: 41 meters

   b. While talking on a cell phone

      Answer: 56 meters

   c. On wet roads

      Answer: 50 meters

4. If the model continued to approximate the stopping distance for a person driving at 65 mph while talking on a cell phone, determine the stopping distance.

   Answer: 105 meters

5. A football field is 100 yards long. How do the stopping distances in questions 3 and 4 relate to the length of a football field? (Note: 1 meter $\approx$ 1.09 yards)

   Sample Answer: The stopping distances in question 3 are approximately half the length of a football field. The stopping distance for a person driving at 65 mph while talking on a cell phone, as shown in question 4, is more than the length a football field.

6. Which driving condition seems to have the greatest impact on stopping distance? Explain your reasoning.

   Sample Answer: The stopping distance for a driver talking on a cell phone seems to have the greatest impact. For the data given, the stopping distances for a driver talking on a cell phone are greater than the stopping distances for the other two driving conditions.

> **Tech Tip:** You may want to have students use the Graph Trace feature to answer the above questions. Increase the window size before using Graph Trace. Select **Menu > Analyze > Graph Trace.** Use the arrow keys on the Touchpad to drag the trace point along the function. If the trace starts to follow the data points, have the students use the ▼ to switch to the function graph. Another option is to calculate the regression equation on a Calculator page and then evaluate the function at the various speeds.

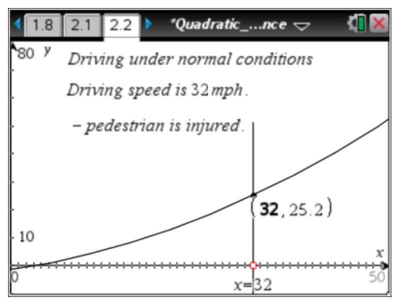
Tech Tip: To use the Graph Trace feature, select **> Analyze > Graph Trace**. The point can be traced along the graph or moved using the left and right arrows at the top of the page. To trace along a different graph, either tap it or use the up and down arrows at the top of the page.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 2 at the end of this lesson.

Tech Tip: For the next pages, if students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (👆) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text.* Then press ctrl to grab the point and close the hand (✊).

Teacher Tip: For questions 7, 8, and 9, first have students drag the open circle to see the different consequences and then determine the speed intervals for each consequence.

Move to page 2.1.

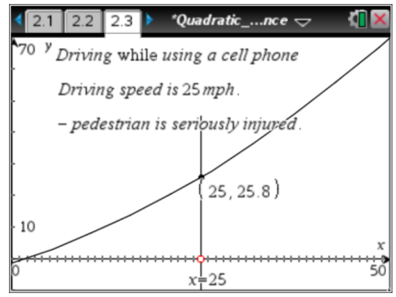


7. Read page 2.1 and then move to page 2.2. Drag the open circle to see what would happen to a pedestrian if the driver of a car tries to stop under normal conditions. Write compound inequalities to describe the speed intervals for each consequence.

<u>**Answer:**</u>
$\text{speed} \le 30$, car stops before hitting pedestrian.
$30 < \text{speed} \le 35$, pedestrian is injured.
$35 < \text{speed} \le 40$, pedestrian is seriously injured.
$\text{speed} > 40$, pedestrian is fatally injured.

TI-Nspire Navigator Opportunity: ***Live Presenter*** **and** ***Quick Poll (Open Response)***
See Note 3 at the end of this lesson.

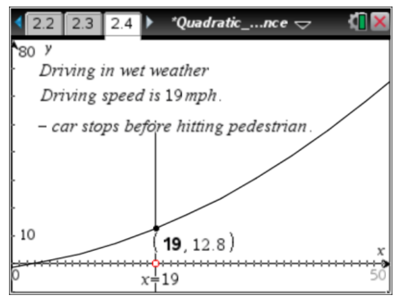
Move to page 2.3.



8. Drag the open circle to see what would happen to a pedestrian if the driver of a car tries to stop while the driver is talking on a cell phone. Write compound inequalities to describe the speed intervals for each consequence.

Answer:
$\text{speed} \le 10$, car stops before hitting pedestrian.
$10 < \text{speed} \le 20$, pedestrian is injured.
$20 < \text{speed} \le 30$, pedestrian is seriously injured.
$\text{speed} > 30$, pedestrian is fatally injured.

Move to page 2.4.



9. Drag the open circle to see what would happen to a pedestrian if the driver of a car tries to stop in wet weather. Write compound inequalities to describe the speed intervals for each consequence.

Answer:
$\text{speed} \le 25$, car stops before hitting pedestrian.
$25 < \text{speed} \le 30$, pedestrian is injured.
$30 < \text{speed} \le 35$, pedestrian is seriously injured.
$\text{speed} > 35$, pedestrian is fatally injured.

10. Examine the compound inequalities in questions 7, 8, and 9. Describe the differences of speed and conditions that would:
    a. Enable a driver to stop in time without injuring the pedestrian.

    Sample Answer: Under normal conditions, if $\text{speed} \le 30$, the car should stop before hitting the pedestrian. With a driver talking on a cell phone, if $\text{speed} \le 10$, the car should stop before hitting the pedestrian. In wet weather, if $\text{speed} \le 25$, the car should stop before hitting the pedestrian.

    b. Likely injure or seriously injure the pedestrian.

    Sample Answer: Under normal conditions, if $30 < \text{speed} \le 40$, the pedestrian will likely be injured or seriously injured. With a driver talking on a cell phone, if $10 < \text{speed} \le 30$, the pedestrian will likely be injured or seriously injured. In wet weather, if $25 < \text{speed} \le 35$, the pedestrian will likely be injured or seriously injured.

c. Likely fatally injure the pedestrian.

Sample Answer: Under normal conditions, if $speed > 40,$ the pedestrian is likely to be fatally injured. With a driver talking on a cell phone, if $speed > 30,$ the pedestrian is likely to be fatally injured. In wet weather, if $speed > 35,$ the pedestrian is likely to be fatally injured.

11. Stopping distance is determined by a combination of reaction distance and braking distance. Why do you think that the consequences for the three driving conditions vary so much?

Sample Answer: The reaction distance for a person using a cell phone is greater than the reaction distance for a person not using a cell phone. A driver using a cell phone is distracted by the cell phone and will take longer to react. This results in a significant increase in stopping distance, which will cause more serious injuries at a lesser speed.

The braking distance of a person driving in wet weather is greater than the braking distance of a person driving when the roads are dry. This occurs because, when the roads are wet, the friction between the car's tires and the road surface is reduced. This will increase stopping distance and cause more serious injuries at a lower speed.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to compare two or more relationships graphically and symbolically.
- How to utilize quadratic equations to make predictions about a given input.
- How to write compound inequalities to describe the effect of various conditions on an outcome.

Assessment

Students should be asked to produce their regression equations and the answers they obtained by extrapolating the stopping distances outside of the data set. Students should also be asked to write compound inequalities to describe the effect of various driving conditions on stopping distance. This can be done on paper, or using TI-Nspire Navigator.

TI-Nspire Navigator

Note 1

Question 2, *Class Capture/Live Presenter:* You may want to use *Live Presenter* to have a student demonstrate how to obtain the regression equation. Use *Class Capture* to monitor the students' progress throughout the activity.

Note 2

Question 3, *Quick Poll (Open Response):* Tell students that you are going to send an *Open Response Quick Poll* for them to type in one or more of their answers. This will give you an opportunity to see if students are able to extrapolate the necessary information from the regression equations. If students are having difficulty, you may want to review the procedure for obtaining the correct answer.

Note 3

Questions 7, 8, and 9, *Live Presenter* and *Quick Poll (Open Response):* You may want to use *Live Presenter* to have a student demonstrate how to drag the point. Once students obtain their inequalities, you may want to send them an *Open Response Quick Poll* to check their answers.