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| **Math Objectives*** Students will explore arithmetic sequences and series.
* Students will consider the effect of the value for the common difference and first term using Lists and Spreadsheets on the TI-Nspire CX II.
* Students will graphically analyze the arithmetic series using graphs.
* Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

**Vocabulary*** Arithmetic Sequence • Common Difference
* Arithmetic Series • Explicit Formula

**About the Lesson*** This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
* This falls under the IB Mathematics Core Content Topic 1 Algebra:

**1.2a** Arithmetic sequences and series**1.2b** Use of the formula for the nth term and the sum of the first n terms of the sequence**1.2d** Applications * As a result, students will:

 Apply this information to real world situations**HH_SW_iconsTI-Nspire™ Navigator™*** Transfer a File.
* Use Class Capture to examine patterns that emerge.
* Use Live Presenter to demonstrate.
* Use Teacher Edition computer software to review student documents.
* Use Quick Poll to assess students’ understanding

.**Activity Materials*** Compatible TI Technologies: **Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Handheld_icon.png** TI-Nspire™ CX Handhelds, Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Tablet_icon.png TI-Nspire™ Apps for iPad®, Trail Blaszer:Users:ronblasz:Documents:WIP:CL947_Platform icons:Software_icon.png TI-Nspire™ Software
 | **Tech Tips:*** This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
* Watch for additional Tech Tips throughout the activity for the specific technology you are using.
* Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

**Lesson Files:***Student Activity*Arithmetic\_Sequences\_and\_Series\_Student-Nspire.pdfArithmetic\_Sequences\_and\_Series\_Student-Nspire.docArithmetic\_Sequences\_and\_Series.tns |
| **Part 1 – Sequences and Scatter Plots** |
| Open the file *Arithmetic\_Sequences\_and\_Series.tns*. Move to **page 1.2**.* Column A, titled n, shows a finite sequence with six terms.
* Column B, titled seq1, shows the term numbers.
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| Find the differences between consecutive terms of the sequence in Column Band record them in Column C, title it diff. * For the first difference, in the first row of Column C, subtract the second term of Column 2 from the first by typing **=b2 – b1**. Do this for the next four rows: **=b3 – b2, b4 – b3,** etc.
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| Now enter the following data into the first 6 rows of Column D: 5, 8, 13, 21, 34, 55. Title it seq2.* Column 4 shows a finite sequence with six terms.
* L1 shows the term numbers for this sequence.

Find the consecutive differences for the Column D sequence and record them in Column E using the same method from before finding the differences in Column B. |  |
| Move to **page 1.3** and graph the sequences in Columns B and D.Press the **var** button and select **n**. Move down to the y input. Press the **var** button a second time and select **seq1**, press **enter**.  |  |
| Press tab and repeat for **seq2** $\left(x \leftarrow n and y \leftarrow seq2\right)$, then press **enter**. Once both are graphed, press **menu**, **4 Window/Zoom**, **9 Zoom – Data**. |  |
| **1.** For each sequence, write the differences between the consecutive terms and give a description of the scatter plot. a. Column B: seq1 **Answer:** 1.25, 1.25, 1.25, 1.25, 1.25. Possible answer: The points of the scatter plot form a  straight line that slants up to the right. b. Column D: seq2**Answer:** 3, 5, 8, 13, 21. Possible answer: The points of the scatter plot form a curve. c. Study the graphs and the differences you found in Column C and Column E. Make a conjecture. **Answer:** Students should make conjectures about the scatter plot of a sequence and the  differences between the consecutive terms. They should conjecture that for sequences with a common difference, the points form a straight line.  |
| Move back to **page 1.2** and clear the data from Columns B, C, D, and E. Leave the natural numbers in Column A. To clear, select the equals box below each heading, press **menu**, **3 Data**, **4 Clear Data**.* Enter the following sequences into Column B and Column D.

Column B: 3, -2, -7, -12, -17, -22Column D: 1, 2, 4, 7, 11, 16* Recalculate the differences between consecutive terms and record them in Columns C and E.

Move to **page 1.3** and press **menu**, **4 Window/Zoom**, **9 Zoom – Data**. |  |
| **2.** For each sequence, write the differences between the consecutive terms and give a description of the scatter plot.a. New Column B Sequence  **Answer**: –5, –5, –5, –5, –5. Possible answer: The points of the scatter plot form a straight  line that slants down to the right.b. New Column D Sequence **Answer**: 1, 2, 3, 4, 5. Possible answer: The points of the scatter plot form a curve. c. With a classmate, discuss how your observations affect your conjecture about the scatter plot of a sequence and the differences between the consecutive terms. Share your discussion with the class. **Answer**: Students should find that the new data reinforces their conjecture that for sequences  with a common difference, the points form a straight line.  |
| **Part 2 – Explicit Formulas and Sums** |
| An **arithmetic sequence** is formed by adding a fixed number, called a **common difference** $(d)$, to each previous term (this number can be positive or negative).The explicit formula for the *n*th term in an arithmetic sequence is $u\_{n}= u\_{1}+ \left(n-1\right)∙d,$ * $u\_{n}$ is any term of a sequence
* $n$ is the term number
* $u\_{1}$ is the first term
* $d$ is the common difference

You can use this formula to calculate any term in an arithmetic sequence.  |
| Move to **page 2.1** and generate a sequence in Column B to display the first 30 terms of $u\_{n}=7.5+ \left(n-1\right)∙1.25$.* Title Column B as seq3.
* Move down to the equal row below the heading seq3. Press **menu**, **3 Data**, **1 Generate Sequence**.
* Enter **7.5 + (n–1)\*1.25**, n0 is **1**, nMax is **30**, and the step is **1**. Press enter or select OK.

Note: There are two rows that are optional to fill. You can enter your initial term as 7.5 or leave it blank and you do not have to enter a ceiling value since we are only looking at the first 30 terms. |  |
| **3.** Simplify the formula $u\_{n}=7.5+ \left(n-1\right)∙1.25$ by distributing and combining like terms. Use this formula in the sequence command (equal row below the heading Column C) to generate 30 terms of this sequence in Column C.  Note: If the handheld asks if this is a column reference or a variable reference due to a **Conflict Detected**, select **Variable Reference**.  Explain what you notice about the terms in Columns B and C.  **Answer:** The terms that appear in Columns B and C should be the same. |
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| **Part 3 – Practice Finding the Sum of a Series** |
| The expression consisting of summing the terms in a sequence is called a ***series***. To find the ***finite*** sum of the first $n$ terms of an arithmetic sequence algebraically, you will use the formula: $S\_{n}= \frac{n}{2}\left(2u\_{1}+ \left(n-1\right)∙d\right) or S\_{n}= \frac{n}{2}\left(u\_{1}+ u\_{n}\right)$Move to page 2.2. You can use the handheld to check the sum of the 30 terms by hand on the home screen enter **sum(seq3)**. The Sum command can be found by pressing **menu**, **6 Statistics**, **3 List Math**, **5 Sum of Elements**. Seq3 can be typed in manually or found by pressing the **var** button. |  |
| **4.** Find the sum of the first 30 terms of this sequence in Column B algebraically. Check your result using the **sum** command. **Answer**: 768.75**5.** Now, let’s look at another sequence. Find the sum of the first 80 terms of the sequence below, using the **Data and Statistics** page to generate the following sequence as was done in part 2 and the **sum()** command on **page 3.1**.62, 67, 72, 77, 82…a. Find the explicit formula for this sequence in simplified form. **Answer**: $u\_{n}=5n+57$ b. Find the sum of the first 80 terms. **Answer**: 20,760

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| **TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)***(to review before the Further IB Extension)Find the pattern of the following sequence: 5, -3, -11, -19, …Tell why the sequence is arithmetic or not. |

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| **Further IB Extension**The Clemson Tigers football team play in the multilevel Memorial stadium. The closer you are to the field, the higher the ticket prices. The ticket prices for the first 4 rows of a Tigers football game are as follows:Row 1: $120 per ticket; Row 2: $117 per ticket; Row 3: $114 per ticketThese ticket prices continue in an arithmetic pattern.(a) Find the common difference between each consecutive row price. [1 mark] **Answer**: $d=114-117=117-120= -3$ A1 The ticket prices go down by $3 each row. |

 (b) Calculate the price of a ticket in row 20. [2 marks]

 **Answer**: $u\_{20} =120+ \left(20-1\right)\left(-3\right)$ (M1)

 $= 120+ \left(19\right)\left(-3\right)$

 $=120-57$

 $=63$ A1

 (c) Find the total cost of buying 2 tickets in each of the first 20 rows. [3 marks]

 **Answer**: $S\_{20}= \frac{20}{2}\left(2\left(120\right)+\left(20-1\right)∙\left(-3\right)\right) or S\_{20}= \frac{20}{2}\left(120+63\right)$ (M1)

 $=10\left(240-57\right) =10\left(183\right)$

 $=10\left(183\right) =1830$

 $=1830$ A1

 Multiplying their sum by 2 (tickets)

 $2(1830)$

 $=\$3660$ A1

*\*\*Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*