



# Building Concepts: Probability, Diagrams, and Tables

TEACHER NOTES

## Lesson Overview

In this TI-Nspire lesson, students describe simple and compound events using tree diagrams and tables and use those displays to determine the probabilities of events.



The probability of a compound event can be found by listing the outcomes for one event followed by the other, creating a probability tree, or by using the product rule for independent events.

## Learning Goals

1. Distinguish between an experiment in which the outcomes are equally likely and one in which the outcomes are not equally likely;
2. organize a list of all possible outcomes of a chance experiment involving compound events;
3. find the probability of an outcome involving compound events.

## Prerequisite Knowledge

*Probability, Diagrams, and Tables* is the thirteenth lesson in a series of lessons that explore statistics and probability. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *What is a Probability?* Students should understand:

- the concept of theoretical probability;
- how to identify a sample space;
- the characteristics of probability.

## Vocabulary

- **probability:** the chance that something will happen; how likely it is that an event will happen
- **compound events:** probabilities of two or more events happening at once
- **equally likely:** events that have the same probability of occurring
- **tree diagram:** a graphic that shows all the possible outcomes of an event

## Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

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## Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Probability Diagrams and Tables\_Student.pdf
- Probability Diagrams and Tables\_Student.doc
- Probability Diagrams and Tables.tns
- Probability Diagrams and Tables\_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.



**Deeper Dive:** These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

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## Mathematical Background

In Lesson 12, *What is Probability?*, students learned about theoretical probability and long-run relative frequency (empirical) probability and considered cases where the theoretical probability of an outcome is known and where it is not. They developed the notion of a probability model, identified a sample space as the set of possible individual outcomes in a probability model with a probability for each possible non-overlapping outcome of the chance process, and noted that the total probability of all these possible outcomes in a sample space is 1. They also investigated characteristics of probability as a number (between and including 0 to 1). This lesson moves to a next step, one involving compound events (such as a head and a six when tossing a coin and a number cube), where the probability of a compound event can be found by listing the outcomes for one event followed by the other, creating a probability tree, or by using the product rule for independent events.

Students need experience in creating organized counting of possible outcomes from chance processes, which can be done using diagrams, particularly probability trees and tables. At this level, they do not work with more formal rules for permutations and combinations.

The product rule is developed using an area model with the approach to multiplying fractions that involves identifying the fraction of the whole area that is shaded. This development also uses the formula for finding the area of a rectangle, which suggests that the product of two fractions has to equal the product of the numerators and the product of the denominators. The probabilities in this lesson are all equally likely, but the TNS activity could be used with probabilities that are not equally likely (the focus of Lesson 14, *Unequally Likely Events*).

Events in which all of the outcomes have the same chance of occurring are called *equally likely events*. This lesson will work with events where the outcomes are equally likely and where one event is followed by another, independent, event; together the two independent events are called a *compound event*.



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## Part 1, Page 1.3

Focus: Tables and tree diagrams can be used to organize all of the outcomes of a chance process that consists of an event followed by a repetition of the event.

Page 1.3 shows two compound events where each event has two outcomes.

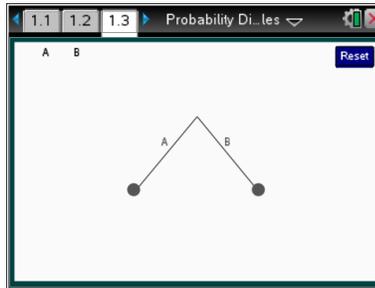
**Outcomes - Stage 1** changes the number of outcomes in Stage 1.

**Stage 2** displays a tree with two stages. Note: this is currently limited to match the outcomes in Stage 1.

**Probability** displays different values; select one and drag it to a segment in Stage 1 to assign it to an outcome. Select a white space to deselect.

Selecting a dot or selecting a corresponding area of the table highlights the associated branch. Selecting a second time removes the highlight.

**Reset** clears the page completely.



### TI-Nspire Technology Tips

**menu** accesses page options.

**A** or **B** assigns the selected probability to the corresponding outcome.

**esc** lets go of a selected probability.

**ctrl del** clears the page completely.



## Class Discussion

***This question is designed to make clear that if an event has two outcomes, they are not necessarily equally likely, an important consideration in making probability statements. Not every event has two equal chances of happening (it did or it did not). All of the events in this activity have equally likely outcomes.***

Have students...

- ***Think of at least three events that have exactly two outcomes and give the outcomes.***

Look for/Listen for...

Answers will vary. Students may suggest tossing a coin with outcomes heads and tails, rolling a number cube with outcomes either even or odd, having either a boy or a girl baby, making or missing a basket, passing or not passing a test.



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## Class Discussion (continued)

- *In which of your examples are the two outcomes “equally likely”—that is both outcomes have the same chance to occur?*

Answers will vary. The coin toss, the even/odd and the boy/girl have even outcomes (assuming that ‘a baby is a girl’ and ‘a baby is a boy’ have the same probabilities). The probability for most people of making a basket is not the same as missing a basket. Some people are more likely to miss—making maybe 1 basket out of 10 tries—while a basketball player is more likely to make a basket and if they are really good might make the basket 9 out of 10 tries. The same is true of passing or failing a test; much depends on how much someone studied, whether they know the material and so on.

*The following questions deal with two stage events (two separate independent happenings) where the first and second stages have the same number of possible outcomes.*

*The diagram on page 1.3 is called a ‘tree diagram’. It could represent an event with two outcomes. Suppose the event represented on page 1.3 was having a baby and that the branch A to the left was having a girl and the branch B to the right was having a boy.*

- *What is the probability of having a girl? A boy?*

Answer:  $\frac{1}{2}$

Select  > Probability and choose the probability from the list. Move the probability of having a boy and the probability of having a girl to the corresponding letter on each branch and select when a pencil appears to assign a probability to the branch. (Choose a white space to deselect the probability.)

- *Suppose the family has two children. Select  > Stage 2 > Stage 2 (select stage 2 twice). Remember that A represents having a girl and B represents having a boy. Explain why the probabilities showed up where they did on the tree.*

Answer: The second stage was like a new event: having a new baby and so each outcome (girl: A or boy: B) could happen  $\frac{1}{2}$  of the time if a girl was born in the first stage or a boy was born in the first stage.



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## Class Discussion (continued)

- **Notice the table in the upper left. How many cells are in the table?**
- **Select the dot for the branch in the tree that is farthest to the left. How is that branch related to the table and to a family with two children?**

Answer: 4

Answer: The branch is connected to the cell with AA, which would represent the family with two girls.

**Remember that probability is the number of successful outcomes over the total number of possible outcomes.**

- **What do you think is the probability that a two-child family will have two girls? Explain your thinking.**
- **Which cell in the table represents having a boy first and then a girl. Select a dot at the end of the branch in the tree to check your thinking. In how many ways can you have a boy first and then a girl?**
- **What is the probability of having a boy first and then a girl?**
- **Describe the families represented by the diagram.**
- **Select a branch that shows having two children, one a boy and the other a girl. What is the probability of having a boy and a girl in a two-child family? Explain your reasoning.**

Answer:  $\frac{1}{4}$  because there are four possible outcomes represented by the four dots at the end of the tree or by the four cells in the table.

Answer: The cell BA or the third dot from the left on the bottom represents boy first, then girl and that can happen in one way.

Answer:  $\frac{1}{4}$

Answer: The branches represent two girls (AA), a girl and then a boy (AB); a boy and then a girl (BA), or two boys (BB).

Answer: The middle two branches both show a girl and a boy but in different birth orders. The probability is  $\frac{1}{2}$  because there are 2 ways for this to happen out of the 4 possible ways.

**Ann, Brady, Carola, and David put their names in a hat to see who would represent the class at a meeting. Reset page 1.3. Select menu > Outcomes-Stage 1 > Four, then menu > Probability and select the probability that any one of the four will be chosen. Move the probability to the letters on each branch of the tree. Highlight the branches that represent each situation to help you explain your answers.**



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## Class Discussion (continued)

- **What is the probability that Carola will be chosen?**

Answer:  $\frac{1}{4}$  because she is one of the four possible options.
- **What is the probability that Carola or David will be chosen?**

Answer:  $\frac{1}{2}$  because they are two of the possible options, so it would be  $\frac{2}{4}$  or  $\frac{1}{2}$ .
- **What is the probability that Ann will not be chosen?**

Answer:  $\frac{3}{4}$  because there are 3 ways for the others to be chosen out of the 4.
- **Greg said that he figured out that Ann will be chosen by adding  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$  and subtracting the sum from 1. What would you say to Greg?**

Answers will vary. He is right because the sum of the probabilities is 1, but he is doing it in a very complicated way.



## Student Activity Questions—Activity 1

1. A group of students was trying to decide which two colors they should use for the border of their poster and for the color of the text: Avocado, Blue, or Canary Yellow. They decided to put the three colors in a bag and have the teacher draw one for the color of the border, then replace the color and draw again for the color of the text.

- a. Predict what the tree will look like for this situation.

Answers will vary. Some students might suggest two levels with each branch will have three parts so there would be nine outcomes. Others might think there will be three levels with different numbers of outcomes.

Select  > Outcomes-Stage 1 > Three. Select the correct probability and mark the branches representing the colors for the border with the correct probabilities Then select  > Stage 2 > Stage 2 and mark the branches representing the colors for the text with the correct probabilities.

- b. If *A* stands for Avocado, *B* for Blue, and *C* for Canary Yellow, highlight the branches that show the border as Blue and the text as Canary Yellow. Describe the cells that correspond to those colors and the probability of getting that outcome.

Answer: The cell *BC* will correspond to a blue border and canary yellow text. The probability of that outcome is  $\frac{1}{9}$ .



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## Student Activity Questions—Activity 1 (continued)

- c. **What is the probability that the two colors on the poster are Avocado and Canary Yellow? Explain your reasoning.**

Answer:  $\frac{2}{9}$  because there are nine possible two color choices (nine cells in the table or nine dots at the bottom of the tree diagram) and two of those nine involve Avocado and Canary Yellow.

- d. **Ryan says that the probability that Blue will be one of the colors is  $\frac{1}{3}$ . What would you say to Ryan?**

Answers may vary. If you highlight all of the branches with a B or if you count the cells in the table with a B, you get 5 out of the 9 that are possible counting the one that is BB so it is  $\frac{5}{9}$ .

- e. **What is the probability that the two colors are Blue and Canary Yellow? Explain your reasoning.**

Answer:  $\frac{2}{9}$  because two cells and two branches have both blue and canary yellow.

- f. **What is the probability that exactly one of the colors (the border or the text) is either Blue or Canary Yellow? Explain your reasoning.**

Answer:  $\frac{4}{9}$  because there are four ways to get one blue or 1 canary yellow: avocado then blue; avocado then canary yellow; blue then avocado; and canary yellow then avocado.

- g. **What is the probability that neither of the choices is canary yellow? Explain your reasoning.**

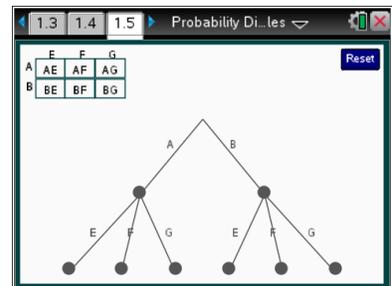
Answer:  $\frac{4}{9}$  because four of the nine cells in the table do not have a C in them.

## Part 2, Page 1.5

Focus: Tables and tree diagrams can be used to organize all of the outcomes of a chance process that consists of two events, one following the other.

Page 1.5 shows the first event of two compound events where the second event can be different from the first.

Page 1.5 has the same functionality as 1.3 only now there can be a different number of outcomes in stage one and stage two. Students assign probabilities and highlight the dots in the same way as they did on page 1.3.





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## Class Discussion

Have students...

*A coin has an apple on one side and a banana on the other. A bag has three cards one with a picture of an elephant, another with a picture of a frog, and the third has a picture of a giraffe. In order to earn points in a game, you toss the coin and then draw 1 card from the bag.*

- *Explain how a tree diagram and cells in the table might relate to the situation.*
- *What probability should be assigned to A and B and to E, F, and G? Explain why.*
- *What is the probability that you get a banana and the frog?*
- *What is the probability that you get a frog?*
- *Using the coin and cards from the scenario above, rank the following outcomes in terms of most probable to least probable. Explain your reasoning.*  
*a) an apple and the elephant; b) the elephant; c) either the frog or the giraffe; d) a banana; e) no giraffe and no banana*
- *How would your answers to the questions above change if you had to draw one of the cards first and then toss the coin?*

Look for/Listen for...

Answer: The tree diagram would have a first level with two branches for tossing the coin and a second level with three branches for the three cards. The table would have two rows for the coin toss and three columns for drawing a card.

Answer: The probability of A and B would each be  $\frac{1}{2}$  because there are two equally likely outcomes from tossing the coin, and the probability of E, F, and G would each be  $\frac{1}{3}$  because there are three equally likely outcomes from drawing a card.

Answer:  $\frac{1}{6}$  because there is one path out of the six possible paths to get a banana and the frog (or one cell out of six that has a BF).

Answer:  $\frac{2}{6} = \frac{1}{3}$  because there are two paths out of the six possible paths to get a frog (or two cells out of the six that have an F).

Answer: the probability of either the frog or giraffe is  $\frac{4}{6}$  or  $\frac{2}{3}$ ; banana is  $\frac{1}{2}$ ; an elephant and no giraffe and no banana are tied at  $\frac{2}{6}$  or  $\frac{1}{3}$ ; an apple and the elephant is  $\frac{1}{6}$ . So the rankings are: c), d), b), e); and a).

Answer: The answers will not change. The order did not seem to make a difference.



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## Student Activity Questions—Activity 2

1. Student can choose three classes for 1st period: Art, Biology, Civics; they can choose from four different classes for 2<sup>nd</sup> period: English, French, Geometry, Health. Set up the table and tree and use either to find the probability a student has

- a. Art 1<sup>st</sup> period then Geometry

Answer:  $\frac{1}{12}$

- b. English 2<sup>nd</sup> period

Answer:  $\frac{3}{12}$  or  $\frac{1}{4}$

- c. no Biology

Answer:  $\frac{2}{3}$

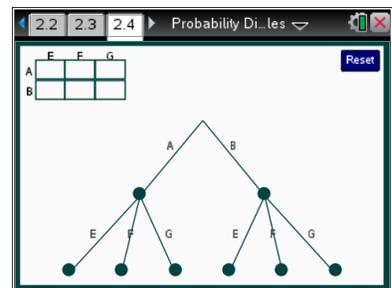
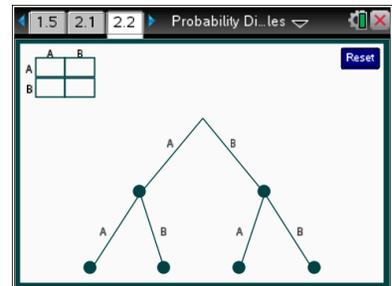
- d. Civics and English or French

Answer:  $\frac{2}{12}$  or  $\frac{1}{6}$ .

### Part 3, Pages 2.2 and 2.4

Focus: The probability of a given outcome from a compound event, where the events are independent, is the product of the probabilities of each individual outcome.

Pages 2.2 and 2.4 function in the same way as pages 1.3 and 1.5, respectively, only now the area of each region in the table changes to match the assigned probabilities.





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## Class Discussion

*This set of questions reviews multiplication of fractions, which students should remember from their earlier work, the fraction of the shaded area to the area of the whole figure is the same as using the formula for finding the area of a rectangle, that is, finding the product of the base and height.*

Have students...

Look for/Listen for...

*Page 2.2 is similar to page 1.3. Fill in the probabilities if branch A in the tree represents having a girl and branch B represents having a boy.*

- *Select the branch for a two-child family with two girls. Note the diagram in the upper left corner. How much of the rectangle does the shaded area represent?*
- *If the rectangle is a square with sides equal to 1 unit, how long is the vertical side of the shaded region? The horizontal side of the shaded region? Explain your reasoning.*
- *The area of a square can be found by multiplying the base and the height of the square. What is the product of the base and the height of the shaded area?*
- *Select the branches that represent a two-child family with at least 1 boy. What fraction of the whole rectangle is represented by the shaded regions? What is the probability of at least 1 boy?*

Answer:  $\frac{1}{4}$

Answer: both sides of the shaded region are  $\frac{1}{2}$  because they represent two equally likely outcomes and the total length is 1.

Answer:  $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

Answer: the shaded regions are  $\frac{3}{4}$  of the whole square, and the probability of at least 1 boy is also  $\frac{3}{4}$ .

*The following question establishes the product rule: if two compound outcomes are independent, the probability that both of the outcomes will occur is the product of the probabilities that each one will occur.*



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## Class Discussion (continued)

- ***Cary figured that to find the probability of one event followed by another one, you multiply the probability of each event. Does the area model on page 2.2 support her thinking? Why or why not?***

Answer: It does because each of the cells in the rectangle is  $\frac{1}{4}$  of the total rectangle and each of the branches in the tree diagram represents  $\frac{1}{4}$  of the possible outcomes. The cells in the rectangle map onto the tree diagram—and you can find the area of the cells by taking the product of the lengths of the two sides because that is how you find area for a rectangle.



## Student Activity Questions—Activity 3

1. Go to page 2.4. The diagram represents a compound event where the first event is having a boy or a girl and the second event is drawing the name for the baby from a hat containing the names Eden, Francis, or Geert. Fill in the probabilities for each outcome.
  - a. Select the branch for having a boy named Francis. What fraction of the whole rectangle is the area of the shaded region? Explain your reasoning.

Answer: There are six congruent small rectangles so the shaded region is  $\frac{1}{6}$  of the total area.

- b. How else could you find the area of the shaded rectangle?

Answer: The base of the shaded rectangle is  $\frac{1}{3}$  and the height is  $\frac{1}{2}$  so the area is the product of the base and height or  $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$ .

- c. What is the probability of a boy named Francis?

Answer:  $\frac{1}{6}$

- d. Suppose you were interested in representing a girl named either Geert or Eden in the large rectangle. Make a conjecture about which of the smaller rectangles will be shaded and what fraction of the total area that will be. Then explain how you would use the tree diagram to find the probability of a girl named Geert or Eden.

Answer: The small rectangles for AE and AG would be shaded and the area would be  $\frac{2}{6}$  or

$\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{3}$ . To use the tree diagram you count the total number of outcomes at the end of the

second stage, which would be six and two of the branches would be AE and AG so it would be  $\frac{2}{6}$ .



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## Student Activity Questions—Activity 3 (continued)

2. A hospital survey showed that the most popular names for a baby girl were Sophia, Emma, Olivia, and Ava. The most popular middle names were Grace, Rose and Elizabeth. Answer the following questions; use the TNS activity to check your reasoning. If the parents at the hospital choose from the most popular first and second names, what is the chance their daughter

a. will be named Ava Elizabeth?

Answer:  $\frac{1}{12}$

b. will have Grace as a second name?

Answer:  $\frac{1}{3}$

c. will be named Sophia Rose or Olivia Rose?

Answer:  $\frac{2}{12} = \frac{1}{6}$

d. will not have either Rose or Grace as a second name?

Answer:  $\frac{1}{3}$



## Deeper Dive — Page 1.3

*Suppose a family had three children. Set up a table or a tree diagram to organize the possible arrangements of boys and girls.*

- *What is the most likely outcome, 1 boy, 2 boys, or 3 boys?*

Answer: The table will have three stages, each with two outcomes. Both 1 boy and 2 boys will have the same chance of occurring,  $\frac{3}{8}$ .

- *What is the least likely outcome, no girls, 1 girl, 2 girls or 3 girls?*

Answer: Either no girls or 3 girls—both  $\frac{1}{8}$ .

*Tomas tossed two coins at once and Serina tossed one coin twice.*

- *Will they have two different tree diagram or tables recording the outcomes? Why or why not?*

Answer: The tree diagrams and the tables will be the same because the two events have the same number of outcomes and whether you get there by doing one after the other or both at once will not make a difference.

- *What is the probability of getting two heads in one toss of two coins?*

Answer:  $\frac{1}{4}$

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## Deeper Dive — Page 1.3 (continued)

- *What is the probability of getting two heads, when you toss one coin twice?* Answer:  $\frac{1}{4}$

## Deeper Dive —Page 2.4

*The class is having a two-stage lottery: in the first stage, the chance of getting an A is  $\frac{1}{4}$ , a B is  $\frac{1}{4}$  and a C is  $\frac{1}{2}$ . In the second stage, the chance of getting E is  $\frac{1}{3}$ , F is  $\frac{1}{6}$ , and G is  $\frac{1}{2}$ . Make a conjecture about each of the following probabilities, then use the TNS activity to check your thinking.*

- *What is the probability of getting AE?* Answer:  $\frac{1}{12}$
- *What is the probability of getting CG?* Answer:  $\frac{1}{4}$
- *What is the probability of getting BF?* Answer:  $\frac{1}{24}$

## Deeper Dive —Page 2.4 (continued)

- *If you toss a coin once and get a heads and toss the coin again and get a heads and then a third time and fourth time and get a heads each time, is your chance of getting a heads on the fifth toss greater than  $\frac{1}{2}$ ,  $\frac{1}{2}$ , or less than  $\frac{1}{2}$ ? Explain your reasoning.* Answers may vary. If the coin is fair, the chance remains  $\frac{1}{2}$  every time you toss the coin. The coin cannot remember what happened before. If you make a list or use a tree diagram, you will get the answer  $\frac{1}{2}$ .



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## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. A nickel, a dime, and a quarter are flipped at the same time. Each coin can land either heads up (H) or tails up (T). List all the different possible outcomes for this event in the chart below. The list has been started for you.

Nickel	Dime	Quarter
H	H	H
H	H	T

NAEP, Grade 8 2013

**Answer:**

<i>Nickel</i>	<i>Dime</i>	<i>Quarter</i>
<i>H</i>	<i>H</i>	<i>H</i>
<i>H</i>	<i>H</i>	<i>T</i>
<i>H</i>	<i>T</i>	<i>H</i>
<i>H</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>H</i>	<i>H</i>
<i>T</i>	<i>H</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>H</i>
<i>T</i>	<i>T</i>	<i>T</i>



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2. Graysen has a green number cube and a white number cube. The faces of the cubes are numbered 1 through 6. Graysen will roll each cube one time. What is the probability that the green cube will land with an even number face up and the white cube will land with a number greater than 2 face up?

- a.  $\frac{1}{9}$       b.  $\frac{1}{36}$       c.  $\frac{1}{3}$       d.  $\frac{1}{6}$

Texas STAAR 2013

**Answer: c)  $\frac{1}{3}$**

3. A math game is played with 2 fair number cubes. A green cube is numbered 1 through 3, and each number is marked on 2 faces. A red cube has faces numbered 1 through 6. If both cubes are tossed, what is the probability of rolling a 3 on the green cube and a 6 on the red cube?

- a.  $\frac{1}{36}$       b.  $\frac{1}{18}$       c.  $\frac{1}{9}$       d.  $\frac{1}{3}$

Texas TAKS Grade 8 2010

**Answer: b)  $\frac{1}{18}$**

4. Mrs. Sandoval's knitting basket contains 3 balls of blue yarn, 2 balls of white yarn, and 3 balls of red yarn. One evening Mrs. Sandoval randomly selected a ball of yarn from her basket and replaced it after she finished knitting. The next evening she randomly selected another ball of yarn from the basket. What is the probability that on the first evening the selected ball of yarn was white and that on the second evening the selected ball of yarn was red?

- a.  $\frac{9}{64}$       b.  $\frac{5}{8}$       c.  $\frac{3}{32}$       d.  $\frac{3}{28}$

Texas TAKS Grade 10 2008

**Answer: c)  $\frac{3}{32}$**



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5. Mr. Martin is holding a trivia contest. The 13 students who are participating randomly draw cards that are numbered with consecutive integers from 1 to 13.
- The student who draws number 1 will be the host.
  - The students who draw the other odd numbers will be on the Red Team.
  - The students who draw the even numbers will be on the Blue Team.
- a. What is the probability a student will be on the Blue team?
- i.  $\frac{5}{13}$       ii.  $\frac{5}{12}$       iii.  $\frac{6}{12}$       iv.  $\frac{6}{13}$

**Answer: iv)  $\frac{6}{13}$**

- b. One student has already drawn a card and is on the Blue Team. If Carlos is the next student to draw a card, what is the probability that he will be on the Red Team?
- i.  $\frac{1}{13}$       ii.  $\frac{1}{12}$       iii.  $\frac{6}{13}$       iv.  $\frac{6}{12}$

Florida FCAT Released Items 2006

**Answer: iv)  $\frac{6}{12}$**

# Building Concepts: Probability, Diagrams, and Tables

TEACHER NOTES

## Student Activity Solutions

In these activities you will find the probability of an outcome involving compound events. After completing the activities, discuss and/or present your findings to the rest of the class.



### Activity 1 [Page 1.3]

1. A group of students was trying to decide which two colors they should use for the border of their poster and for the color of the text: Avocado, Blue, or Canary Yellow. They decided to put the three colors in a bag and have the teacher draw one for the color of the border, then replace the color and draw again for the color of the text.

- a. Predict what the tree will look like for this situation.

*Answers will vary. Some students might suggest two levels with each branch will have three parts so there would be nine outcomes. Others might think there will be three levels with different numbers of outcomes.*

Select  > **Outcomes-Stage 1 > Three**. Select the correct probability and mark the branches representing the colors for the border with the correct probabilities. Then select  > **Stage 2 > Stage 2** and mark the branches representing the colors for the text with the correct probabilities.

- b. If  $A$  stands for Avocado,  $B$  for Blue, and  $C$  for Canary Yellow, highlight the branches that show the border as Blue and the text as Canary Yellow. Describe the cells that correspond to those colors and the probability of getting that outcome.

*Answer: The cell  $BC$  will correspond to a blue border and canary yellow text. The probability of that outcome is  $\frac{1}{9}$ .*

- c. What is the probability that the two colors on the poster are Avocado and Canary Yellow? Explain your reasoning.

*Answer:  $\frac{2}{9}$  because there are nine possible two color choices (nine cells in the table or nine dots at the bottom of the tree diagram) and two of those nine involve Avocado and Canary Yellow.*

- d. Ryan says that the probability that Blue will be one of the colors is  $\frac{1}{3}$ . What would you say to Ryan?

*Answers may vary. If you highlight all of the branches with a  $B$  or if you count the cells in the table with a  $B$ , you get 5 out of the 9 that are possible counting the one that is  $BB$  so it is  $\frac{5}{9}$ .*

- e. What is the probability that the two colors are Blue and Canary Yellow? Explain your reasoning.

*Answer:  $\frac{2}{9}$  because two cells and two branches have both blue and canary yellow.*



# Building Concepts: Probability, Diagrams, and Tables

TEACHER NOTES

- f. What is the probability that exactly one of the colors (the border or the text) is either Blue or Canary Yellow? Explain your reasoning.

*Answer:  $\frac{4}{9}$  because there are four ways to get one blue or 1 canary yellow: avocado then blue; avocado then canary yellow; blue then avocado; and canary yellow then avocado.*

- g. What is the probability that neither of the choices is canary yellow? Explain your reasoning.

*Answer:  $\frac{4}{9}$  because four of the nine cells in the table do not have a C in them.*



## Activity 2 [Page 1.5]

1. Student can choose three classes for 1<sup>st</sup> period: Art, Biology, Civics; they can choose from four different classes for 2<sup>nd</sup> period: English, French, Geometry, Health. Set up the table and tree and use either to find the probability a student has

- a. Art 1<sup>st</sup> period then Geometry

*Answer:  $\frac{1}{12}$*

- b. English 2<sup>nd</sup> period

*Answer:  $\frac{3}{12}$  or  $\frac{1}{4}$ .*

- c. no Biology

*Answer:  $\frac{2}{3}$*

- d. Civics and English or French

*Answer:  $\frac{2}{12}$  or  $\frac{1}{6}$ .*



## Activity 3 [Page 2.4]

1. Go to page 2.4. The diagram represents a compound event where the first event is having a boy or a girl and the second event is drawing the name for the baby from a hat containing the names Eden, Francis, or Geert. Fill in the probabilities for each outcome.

- a. Select the branch for having a boy named Francis. What fraction of the whole rectangle is the area of the shaded region? Explain your reasoning.

*Answer: There are six congruent small rectangles so the shaded region is  $\frac{1}{6}$  of the total area.*



## Building Concepts: Probability, Diagrams, and Tables

### TEACHER NOTES

- b. How else could you find the area of the shaded rectangle?

*Answer: The base of the shaded rectangle is  $\frac{1}{3}$  and the height is  $\frac{1}{2}$  so the area is the product of the base and height or  $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$ .*

- c. What is the probability of a boy named Francis?

*Answer:  $\frac{1}{6}$*

- d. Suppose you were interested in representing a girl named either Geert or Eden in the large rectangle. Make a conjecture about which of the smaller rectangles will be shaded and what fraction of the total area that will be. Then explain how you would use the tree diagram to find the probability of a girl named Geert or Eden.

*Answer: The small rectangles for AE and AG would be shaded and the area would be  $\frac{2}{6}$  or*

*$\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{3}$ . To use the tree diagram you count the total number of outcomes at the end of the*

*second stage, which would be six and two of the branches would be AE and AG so it would be  $\frac{2}{6}$ .*

2. A hospital survey showed that the most popular names for a baby girl were Sophia, Emma, Olivia, and Ava. The most popular middle names were Grace, Rose and Elizabeth. Answer the following questions; use the TNS activity to check your reasoning. If the parents at the hospital choose from the most popular first and second names, what is the chance their daughter

- a. will be named Ava Elizabeth?

*Answer:  $\frac{1}{12}$*

- b. will have Grace as a second name?

*Answer:  $\frac{1}{3}$*

- c. will be named Sophia Rose or Olivia Rose?

*Answer:  $\frac{2}{12} = \frac{1}{6}$*

- d. will not have either Rose or Grace as a second name?

*Answer:  $\frac{1}{3}$*