

Lesson Overview

This TI-Nspire™ lesson allows students to reason about ratios and proportions and to use their knowledge of multiplication tables to investigate properties of ratio tables. They learn that ratio tables are one strategy to solve problems involving ratios. Students should note the role of multiplication and division in how the entries in the table are related to each other. They should also recognize that “adding” equivalent ratios produces an equivalent ratio because the two ratios have a common factor.



The sum of two equivalent ratios is another equivalent ratio. This “additive” property of equivalent ratios can be used to solve both multiplication and division problems by creating appropriate and “easy” equivalent ratios.

Learning Goals

1. Generate tables of equivalent ratios beginning with the ratio of two positive whole numbers;
2. find missing values in ratio tables;
3. use ratio tables to solve problems;
4. understand how addition and multiplication relate to the values in a ratio table;
5. recognize that the sum of equivalent ratios is another equivalent ratio.

Prerequisite Knowledge

Ratio Tables is the fourth lesson in a series of lessons that explore the concepts of ratios and proportion. This lesson builds on the concepts of the previous lessons: *What is a Ratio?*, *Introduction to Rates*, and *Building a Table of Ratios*. Prior to working on this lesson, students should understand:

- the concepts of ratios and equivalent ratios;
- how to complete multiplication tables.

Vocabulary

- **distributive property:** the rule that says that multiplying a number by a group of numbers added together is the same as performing each multiplication separately.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Ratio Tables_Student.pdf
- Ratio Tables_Student.doc
- Ratio Tables.tns
- Ratio Tables_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the Activity:



Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.



Additional Discussion: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In this TI-Nspire™ lesson, students investigate properties of ratio tables. The lesson engages students in reasoning about ratios and proportions using their knowledge of multiplication tables. They learn that ratio tables are one strategy to solve problems involving ratios. For example, if a fruit punch has a mixture of grape juice and cranberry juice in a ratio of 3 to 2, students can find how many cups of grape juice to mix with 8 cups of cranberry juice by setting up a ratio table and generating equivalent ratios until they find the pair with 8 cups of cranberry juice. Students should note the role of multiplication and division in how the entries in the table are related to each other. They should also recognize that “adding” equivalent ratios produces an equivalent ratio because the two ratios have a common factor: 6 for every 9 of some quantity is the same as 8 for every 12 or 14 for every 21 of the same quantity. These ratios are all equivalent to the ratio 2:3 because the values in each of these ratio pairs are multiples of values of the same ratio pair, 2:3. So, 6 for every 9 is 3 times the values in the ratio 2 for every 3; and 8 for every 12 is the same as 4 times the values in the ratio pair of 2 for every 3. Be sure that students pay attention to the units or labels in setting up and working the problems. Ratios only make sense in context and the units involved in the context are important in setting up a ratio and interpreting any equivalent ratio.

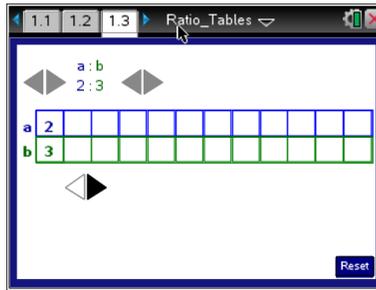
At first, students often use skip counting to generate the elements in ratio tables with only positive whole number entries (note that 0:0 is not a ratio). One goal is to begin to use entries that are large whole numbers, fractions or decimals, or those made from finding sums of existing entries.

Part 1, Page 1.3

Focus: Students generate positive whole number multiples of a given ratio.

Page 1.3 displays a ratio pair and a table. The arrows on the top (for $a:b$, both a and b are ≤ 20). Each time the arrows below the table are selected, the next multiple of the original ratio will be generated.

The **Reset** button returns to the original screen.



TI-Nspire Technology Tips

The **Reset** button or **ctrl del** returns the screen to the original setting.

Use the **tab** key to toggle between the sets of arrows on the page.

Use the arrows on top of the screen to set the ratio, $a:b$

Use the arrows below the table to complete ratio table.

◀▶ Arrow keys on the handheld can also be used to change these values.

Class Discussion

Have students...

Use the set of arrows located below the table to generate equivalent ratios shown in the table.

- *How are the values in the top row related to each other? In the bottom row?*
- *How are the columns related?*
- *Will one of the columns ever be 28 to 42? Why or why not?*

Look for/Listen for...

Answer: They are all multiples of 2 in the top row and multiples of 3 in the bottom row.

Answer: As ratios, they are all equivalent to the ratio 2:3.

Answer: Yes because $14 \times 2 = 28$ and $14 \times 3 = 42$, so 28:42 is equivalent to 2:3.

Class Discussion (continued)

Reset page 1.3 and use the arrows at the top to set the ratio 15:7. Consider the following situation:

- **A machine produced 15 items in 7 minutes. Predict what the values will be in the second and third columns. Check your prediction using the TNS activity.**

Answer: 30:14 and 45:21

- **How many items will the machine produce in 112 minutes?**

Answer: 240 items

- **How long will it take to produce 90 items?**

Answer: 42 minutes



Student Activity Questions—Activity 1

1. **Find at least two ratios that will generate a table that contains a ratio of 24 and 42 other than 24:42. Explain your thinking.**

Possible answers: 4:7 or 7:4; 6 is a factor of both 24 and 42, so I divided each by 6. 8:14; once you have 4:7, any multiple of 4:7 in the table would work as the beginning ratio.

Part 2, Page 2.2

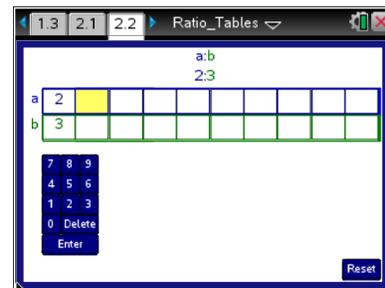
Focus: Students generate a table of equivalent ratios in any order and that are any whole number multiple of the original ratio.

Students investigate using sums and differences of the columns in a ratio table as a strategy for solving problems.

On page 2.2, selecting any cell in either row highlights that cell.

Use the keypad to enter a value in the cell and select **Enter** to create a ratio that is equivalent to the original ratio for that value.

The entries are restricted to whole number multiples of the original ratio. To enter a new ratio, highlight one of the numbers in the first column, press **Delete** and enter a new value using the keypad on the screen or the keypad on the handheld. Use the arrow keys on the handheld to move from cell to cell in the table.



Class Discussion

Set the original ratio to 2:3; with 2 in the top row and 3 in the bottom row.

Use the keypad to enter 6 in the top row of the second column.



- **How are the numbers shown in the table related?**

Possible answer: The ratios 2:3 and 6:9 are equivalent; 2:3 is multiplied by 3 to obtain 6:9.

- **Use the keypad to enter 54 in the bottom row of the third column. Predict the corresponding value in the top row. How are the numbers related?**

Possible answer: The ratio will be 36:54, which is equivalent to the other two ratios. The common factor is 18.

- **Make a prediction about the value you will get in the second row of the next column if you enter 50 in the top row. Check your prediction using the TNS activity.**

Answer: 75



Student Activity Questions—Activity 2

On page 2.2 of the TNS activity, set the original ratio to 2:3; with 2 in the top row and 3 in the bottom row.

1. **Tiani claimed if you added two ratios in the table, you will get another ratio that could be in the same ratio table.**

- a. **Play with values in the table and decide whether you agree with Tiani.**

Answer: Tiani is right. If you take the ratio 4:8 and the ratio 6:12 and add you will get 10:20. This always seems to work.

- b. **Provide an example to explain how the distributive property works.**

Possible Answer: One example could be $3(5+11) = 3(5) + 3(11) = 15 + 33 = 48$.

- c. **How might the distributive property help you explain why she is right or wrong?**

Answer: Any two equivalent ratios are a number multiplied by a common ratio. If you add the equivalent ratios, the common ratio can be distributed out and the sum will be the two multipliers. For example, adding 10:6, which is $2 \times 5:2 \times 3$ and 15:9, which is $3 \times 5:3 \times 3$ gets 25:15, which is $(2 \times 5) + (3 \times 5): (2 \times 3) + (3 \times 3)$ or $(2 + 3) \times 5: (2 + 3) \times 3$.

2. **Cases of Fresh Fruits juice contain 15 bottles of juice. Create a ratio that represents the number of full cases to the number of bottles of juice contained in them. Create a ratio table so that the sum of two of the ratios in the table results in the number of bottles of Fresh Fruits juice per case. How many cases would you have all together?**

- a. 75 bottles b. 165 bottles c. 1575 bottles d. 330 bottles

Answers will vary. Possible answers:

- a. 2:30 and 3:45 – 75 bottles would fill 5 cases
- b. 6:90 and 5:75 – 165 bottles would fill 11 cases
- c. 5: 75 and 100:1500 – 1575 bottles would fill 105 cases
- d. 2:30 and 20:300 – 330 bottles would fill 22 cases.

3. Cases of Fresh Fruits juice contain 15 bottles of juice. Create a ratio that represents the number of full cases to the number of bottles of juice contained in them. Create a ratio table so that the difference of two of the ratios in the table results in the number of bottles of Fresh Fruits juice per case. What is the difference in the number of bottles for each?

- a. 8 cases b. 6 cases c. 13 cases d. 99 cases

Answers will vary. Possible answers:

- a. 30:2 and 150:10 – the difference in 8 cases would be 120 bottles
 b. 105:7 and 195:13 – the difference in 6 cases would be 90 bottles
 c. 285:19 and 60:6 – the difference in 13 cases would be 225 bottles
 d. 15:1 and 1500:100 – the difference in 99 would be 1485 bottles



Class Discussion

Cases of Fresh Fruits juice contain 15 bottles of juice. Create a ratio that represents the number of full cases to the number of bottles of juice contained in them.

- **Will any two ratios that have the same difference in the number of cases have the same number of bottles? Explain.**

Answer: Yes because if the difference is a cases and each case has 15 bottles then number of bottles will always be $15a$.

- **Will any two ratios that have the same total number of bottles fill the same number of cases? Explain.**

Answer: Yes because if the total number of bottles is some number, a , then the number of cases would be a divided by 15.



Additional Discussion

Have students...

Look for/Listen for...

Use the TNS activity to help explain reasoning.

An advertisement states: 9,000 bottles of mineral water in 450 cases. Suppose you did not need 9,000 bottles.

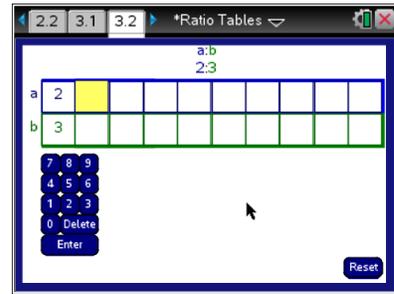
- **Find at least three equivalent ratios that would let you buy a smaller number of cases.**
- **What operation was involved in finding your answer and why does it work?**

Possible answer: 900 bottles per 45 cases, 100 bottles per 5 cases, and 20 bottles to 1 case.

Possible answer: Divide 9000 by a factor that is common to both 450 and 9000. It works because equivalent ratios are formed by multiplying or dividing each value in the pair by a common non-zero number. This multiplier could be a fraction such as $\frac{1}{100}$.

Part 3, Page 3.2

Focus: Students generate a table of equivalent ratios in any order and that are any fraction or whole number multiple of the original ratio. In addition to solving problems by finding the sum/ difference of the columns in a ratio table, they use the notion of unit rate as a strategy to find solutions to problems involving proportions.



Page 3.2 functions in the same way as page 2.2, but students can enter any number and ratios involving fractions are displayed.

Class Discussion

A ratio table is useful for solving problems. Choose a beginning ratio and generate a ratio table that involves each strategy: Multiplying by 2, dividing by 2, multiplying by 10, multiplying by a particular number, adding columns, subtracting columns. Explain to a partner how the strategy is visible in your tables.

Answers will vary. Possible answer:

50	25	500	525	475	100	450
18	9	180	189	171	36	162
	Divide by 2	Multiply 50:18 by 10	Add 25:9 and 500:180	Subtract 25:9 from 500:180	Multiply 50:18 by 2	Multiply 50:18 by 9

Teacher Tip: Give students time to create their own tables of ratios with a partner or in small groups. Have the students list each strategy and show each step in their calculations beneath the strategy. Monitor the student work and select groups to explain each strategy. Have them show their beginning ratio and describe how they used that strategy to complete a column in their table.

Have students...

Focus on page 3.2 of the TNS activity.

Enter 1 in the top row of the second column.

- **Explain why the ratio in the second column is equivalent to the original ratio.**
- **If a represents the number of circles and b represents the number of squares, interpret the ratio in the second column in terms of circles and squares.**

Look for/Listen for...

Answer: The new ratio is $1:\frac{3}{2}$. It is equivalent to 2:3 because if you divide each value in 2:3 by 2, you get the ratio $1:\frac{3}{2}$.

Answer: You would have 1 circle for every $\frac{3}{2}$ or 1 and $\frac{1}{2}$ squares.



Class Discussion (continued)

- Using your answer from above, predict how many squares you would have for 3 circles. Check your answer using the TNS activity.**

Answer: If there is 1 circle for every $1\frac{1}{2}$ squares, then for 3 circles, you would have $3(\frac{3}{2}) = 3(1 + \frac{1}{2}) = 3 + 1\frac{1}{2} = \frac{9}{2}$ or $4\frac{1}{2}$ squares.



Student Activity Questions—Activity 3

Reset page 3.2. Enter 1 in the bottom row of the second column.

- Predict the value in the top row of the second column and explain your thinking. Check your answer using the TNS activity.**

Answer: To get a 1 in the bottom row, you need to divide 3 by 3, so the 2 would have to be divided by 3 as well. The ratio would be $\frac{2}{3}:1$.

- Remember that a unit rate is a ratio of the form $c:1$. If a is the number of cups of water and b is the number of batches of a pancake mix, explain what the unit rate tells you.**

Answer: $\frac{2}{3}$ cup of water for every batch of the mix.

- Reset page 3.2. Fill out the top row so that you have consecutive, odd integers, starting with 1. Let a represent the number of circles and b represent the number of squares.

- Describe the pattern for the numbers in the bottom row of the ratio table.**

Answer: The number of squares in the bottom row for each column increases by $\frac{6}{2}$ or 3, which is reasonable because you are adding two $\frac{3}{2}$ or $\frac{6}{2}$ or 3 squares as you move from cell to cell across the bottom row of the ratio table.

- Explain how to use the results in the table to find the number of squares if you have 8 circles.**

Answer: You can add the number of squares for 3 circles to the number of squares for 5 circles, which would be $\frac{9}{2} + \frac{15}{2} = \frac{24}{2} = 12$ squares.

Class Discussion

Have students...

Reset page 3.2. Fill out the cells in the ratio table by using consecutive whole numbers in the bottom row of the table beginning with 1.

- **Use the table to answer how many cups of water you will need for 8 batches of mix.**
- **Explain how the ratio 6:9 fits into the pattern in the table.**
- **Suppose you had to mix 11 batches for a pancake breakfast. How many cups of water do you need? Explain how you found your answer.**

Solve each problem, explain your reasoning, and then check your answer using the TNS activity.

It costs \$480 for every 36 feet of pipe.

- **How much would it cost for 12 feet of pipe?**
- **Henri spent \$240. How many feet of pipe did he buy?**
- **The pipe comes in two colors, both the same price. How can you use the table to help answer the question: If he bought 24 feet of one color and 60 feet of another, how much would it cost?**

Look for/Listen for...

Answer: $\frac{16}{3} = 5\frac{1}{3}$ cups

Answer: It would be $\frac{18}{3} : 9$ because there are $\frac{2}{3}$ cup for every 1 batch and so there would be $9(\frac{2}{3}) : 9(1)$ for 9 batches.

Answer: You could multiply $11(\frac{2}{3})$ to get an equivalent ratio $11(\frac{2}{3}) : 11(1)$ for $\frac{22}{3}$ or $7\frac{1}{3}$ cups of water; you add $\frac{2}{3}$ to the amount of water for 9 batches to get the amount of water for 10 batches and add $\frac{2}{3}$ once more to get the amount of water for 11 batches.

Possible answer: From \$480:36 feet, you need to multiply 36 by something that will produce 12. $\frac{1}{3}$ of 36 is 12, so $\frac{1}{3}$ of \$480, will be \$160.

Possible answer: 480:36 is equivalent to 40:3 which is equivalent to 6(40):6(3) or 240:18. He bought 18 feet of pipe.

Answer: Create two ratios where the number of feet is 24 and 60; 320:24 and 800:60. Adding will give 1120:84, so the price for 84 feet of pipe would be \$1,120.

 **Additional Discussion**

Gliders typically have a glide ratio, the vertical drop to the horizontal movement. So a glide ratio of 4:10 would describe a glider that dropped 4 feet for every 10 feet it goes horizontally. Two gliders start from the same height, one with a glide ratio of 25:185 and the other with a glide ratio of 20:155. Use the TNS activity to decide which glider has the greater horizontal movement.

Possible answer: Create a ratio table using 25: 185. Then create the ratios 5: 37 and 20: 148. This glider will drop 20 feet and travel 148 feet horizontally, while the glider with the glide ratio 20:155 will travel 155 feet horizontally, so that one goes farther horizontally. Another way is to make two ratio tables and get both of them to drop 100 feet. Or start with the 20:155, 4:31; 24:186. This one goes farther because in a 24-foot drop it has already gone 186 feet.



Sample Assessment Items

After completing the activity, students should be able to answer the following types of questions. If students understand the concepts involved in the activity, they should be able to answer the following questions without using the TNS activity.

- Sara is making a fruit punch by combining grape and orange juice. Which of the following is a ratio table? Justify your reasoning.

a.

Cups grape	3	5	7	
Cups orange	5	7	9	

b.

Cups grape	4	8	12	20	
Cups orange	9	18	27	45	

c.

Miles	3750	375	125	250
Weeks	30	3	1	2

d.

Miles	420	210	840	1050	
Weeks	12	6	24	30	

Answers:

- Not a ratio table, no common multiplier for the columns (3:5 and 5:7).**
 - Yes, every column is a number times 4:9.**
 - Yes, every column is a multiple of 125:1.**
 - Yes, every column is a number times 210:6 (or 70:3).**
- A car can travel 250 miles on 6 gallons of gasoline. How far can it travel on 9 gallons?
Answer: 375 miles.
 - A 1-meter strip of wood weighs 2.5 kg. How much would 3 meters weigh?
Answer: 7.5 kg
 - Sally bikes for 5 hours every 7 days. How many hours would she bike over 42 days?
Answer: 30 hours

Student Activity Solutions

In these activities you will work together to generate tables of equivalent ratios and solve problems using equivalent ratios. After completing each activity, discuss and/or present your findings to the rest of the class.



Activity 1 [Page 1.3]

- Find at least two ratios that will generate a table that contains a ratio of 24 and 42 other than 24:42. Explain your thinking.

Possible answers: 4:7 or 7:4; 6 is a factor of both 24 and 42, so I divided each by 6. 8:14; once you have 4:7, any multiple of 4:7 in the table would work as the beginning ratio.



Activity 2 [Page 2.2]

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- Play with values in the table and decide whether you agree with Tiani.

Answer: Tiani is right. If you take the ratio 4:8 and the ratio 6:12 and add you will get 10:20. This always seems to work.

- Provide an example to explain how the distributive property works.

Possible Answer: One example could be $3(5+11) = 3(5) + 3(11) = 15 + 33 = 48$.

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Answer: Any two equivalent ratios are a number multiplied by a common ratio. If you add the equivalent ratios, the common ratio can be distributed out and the sum will be the two multipliers. For example, adding 10:6, which is $2 \times 5:2 \times 3$ and 15:9, which is $3 \times 5:3 \times 3$ gets 25:15, which is $(2 \times 5)+(3 \times 5): (2 \times 3) + (3 \times 3)$ or $(2 + 3) \times 5: (2 + 3) \times 3$.

- Cases of Fresh Fruits juice contain 15 bottles of juice. Create a ratio that represents the number of full cases to the number of bottles of juice contained in them. Create a ratio table so that the sum of two of the ratios in the table results in the number of bottles of Fresh Fruits juice per case. How many cases would you have all together?

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Activity 3 [Page 3.2]

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- a. Predict the value in the top row of the second column and explain your thinking. Check your answer using the TNS activity.

Answer: To get a 1 in the bottom row, you need to divide 3 by 3, so the 2 would have to be divided by 3 as well. The ratio would be $\frac{2}{3} : 1$.

- b. Remember that a unit rate is a ratio of the form $c:1$. If a is the number of cups of water and b is the number of batches of a pancake mix, explain what the unit rate tells you.

Answer: $\frac{2}{3}$ cup of water for every batch of the mix.

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