

### Lesson Overview

This TI-Nspire™ lesson focuses on the relationship between the unit rate or slope of a line and the equation. The graph of a proportional relationship passes through the origin, and as a consequence, the graph is completely determined by one point that defines the collection of equivalent ratios.



A proportional relationship ( $y = kx$  with  $k > 0$ ) is often referred to as **direct variation**; the variable  $y$  varies directly with the variable  $x$ .

### Learning Goals

1. Represent proportional relationships between quantities using tables, graphs, diagrams, and equations;
2. explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of a context, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate;
3. identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships;
4. determine whether two quantities are in a proportional relationship;
5. identify the dependent and independent variables for a given context and understand how they are interpreted in a graph.

### Prerequisite Knowledge

*Proportional Relationships* is the eleventh lesson in a series of lessons that explore the concepts of ratios and proportional relationships. The lesson builds on students' prior knowledge using an equation to describe a proportional relationship. Prior to working on this lesson, students should have completed *Connecting Ratios to Graphs* and *Connecting Ratios to Equations*. Students should understand that:

- the members of any set of equivalent ratios have the same unit rate;
- the unit rate is a measure of the steepness of the slope;
- the constant of proportionality,  $k$ , is the unit rate or the slope of a line through the origin.

### Vocabulary

- **direct variation**: a proportional relationship where the variable  $y$  varies at a constant rate with the variable  $x$ .
- **dependent variable**: the output variable that is plotted on the vertical axis.
- **independent variable**: the input variable that is plotted on the horizontal axis.

# Building Concepts: Proportional Relationships

TEACHER NOTES

## Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Proportional Relationships\_Student.pdf
- Proportional Relationships\_Student.doc
- Proportional Relationships.tns
- Proportional Relationships\_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.



**Additional Discussion:** These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

**Mathematical Background**

This TI-Nspire™ lesson focuses on the relationship between the unit rate or slope of a line and the equation. The graph of a proportional relationship passes through the origin, and as a consequence, the graph is completely determined by one point that defines the collection of equivalent ratios, often the point representing the unit rate,  $(1, \frac{B}{A})$ . Students might reason as follows: two points determine a line, and when one of the points has to be the origin, the other point determines the line. Ordered pairs that satisfy equations of the form  $y = ax + b$  with  $b \neq 0$  will not lie on a line through the origin and so will not belong to a collection of equivalent ratios.

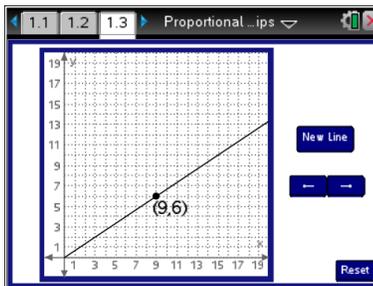
A proportional relationship ( $y = kx$  with  $k > 0$ ) is often referred to as **direct variation**; the variable  $y$  varies directly with the variable  $x$ . Or as  $x$  increases, so does  $y$  at a constant rate. In a real-world problem that change in relationship to one another, a central concept is to clarify which variable is the dependent variable, in terms of the other quantity, thought of as the independent variable. In a graph, the independent variable is located on the horizontal axis and the dependent variable on the vertical axis. The relationship between the dependent and independent variables can be analyzed using graphs, tables, and equations.

The previous lesson, *Connecting Ratios to Equations*, focused on developing basic tools for thinking about a proportional relationship: beginning with a collection of equivalent ratios, linking them to sets of ordered pairs, and graphing the corresponding points. An equation of the form  $y = kx$ , where  $k$  is a positive constant, often called a *constant of proportionality*, was used to describe the graph, which is a ray beginning at the origin. For a ratio  $A:B$ , the constant of proportionality,  $k$ , is equal to the  $\frac{B}{A}$ , which is also the slope of the line associated with the collection of equivalent ratios. The unit rate,  $\frac{B}{A}$ , associated with the ratio  $A:B$ , can be used to generate points on the line or entries in the corresponding table.

## Part 1, Page 1.3

Focus: How can you tell whether a line represents a proportional relationship?

In Part 1, students revisit 1) writing an equation to describe a proportional relationship, paying attention to the role of the independent and dependent variable for a given context and 2) the role of the unit rate or constant of proportionality in the equation and in generating ordered pairs in the proportion. They investigate patterns in tables of ordered pairs in a proportional relationship, finding a recursive pattern (how the next row in the table can be generated from the preceding row) and finding the slope or rate of change for any two ordered pairs in the table.



### TI-Nspire Technology Tips

Use the arrows to move the point along the line.

Select **New Line** or press  to generate a new line.

**Reset** returns to the original screen or press   to reset.

On page 1.3, students can grab and drag the point along the ray, or they can use the right and left arrows on the keypad if using the handheld. Using **New Line** (or the up or down arrows on the keypad) will generate a new ray with a new given point. Selecting **Reset** returns the page to the original screen with a ray through the point (9, 6).

### Class Discussion

Have students...

*The band was practicing for a marching show and lined up on a floor with large tiles. They lined up so there were 9 students to every 6 tiles. Using the TNS activity,*

- **Identify the two variables and indicate which variable you think will be the independent or input variable and which one will be the dependent or output variable. Explain your reasoning.**
- **Look at the graph on page 1.3. Interpret the ordered pair in terms of the two variables.**

Look for/Listen for...

Answer: You have a given number of students to line up so that would be the input or independent variable. Once you know the number of students, you can use the ratio or graph to find out the number of tiles, which will be the dependent or output variable.

Answer: The ordered pair (9, 6) means that to line up nine students, you will need 6 tiles.



# Building Concepts: Proportional Relationships

TEACHER NOTES



## Class Discussion (continued)

Have students...

- *Use the arrows to move the point to find the unit rate. Explain what this point means in terms of the context.*

Look for/Listen for...

Answer: The unit rate will be at the point  $(1, \frac{6}{9})$  or  $(1, \frac{2}{3})$ . It means that  $\frac{2}{3}$  of a tile is used per student in lining up.



## Student Activity Questions—Activity 1

1. Label the columns in the table below. Then record the point for the unit rate from the last question in the Class Discussion.
  - a. Using the right arrow on the keypad or touchpad, move the point 1 unit and record the new coordinates in the table. Repeat moving and recording until you have at least five points in the table.

x- _____ [answer: students]	y- _____ [answer: tiles]
1	[answer: $\frac{6}{9} = \frac{2}{3}$ ]
2	[answer: $\frac{12}{9} = \frac{4}{3}$ ]
3	[answer: $\frac{18}{9} = 2$ ]
4	[answer: $\frac{24}{9} = \frac{8}{3}$ ]
5	[answer: $\frac{30}{9} = \frac{10}{3}$ ]
6	[answer: $\frac{36}{9} = 4$ ]

- b. For the point whose x-coordinate is 5, explain what the point means in terms of the context in the Class Discussion problem.

Answer: When there are 5 students, you need  $\frac{30}{9}$  or 3 and  $\frac{1}{3}$  of a tile for them to line up.

- c. Describe the pattern in the table and how it relates to the graph.

Answer: Each x variable increases by 1 student and each y variable increases by  $\frac{6}{9}$  or  $\frac{2}{3}$  of a tile.



## Student Activity Questions—Activity 1 (continued)

- d. Write the equation of the proportional relationship between the number of students and the number of tiles. Explain in words what the equation means.

Answer: If  $s$  = number of students and  $t$  = number of tiles, the equation would be  $t = \frac{2}{3}s$ . The number of tiles is  $\frac{2}{3}$  times the number of students.

2. Select New Line on page 1.3. Record the original coordinate of the point on the ray. Then move the point and record the coordinates of at least five points in the table.

Answer: Responses for this question and a-d will vary depending on the initial point.

$x$	$y$
14	20
12	$\frac{240}{14}$
7	10
4	$\frac{40}{7}$
2	$\frac{20}{7}$

- a. Pick two of the points and find the ratio of the change in the  $x$ -values to the change in the  $y$ -values.

Answer: For the values above, there would be a 10-unit change in  $y$  for every 7-unit change in  $x$  or any other equivalent ratio.

- b. Pick another pair of points from the table and find the ratio of the change in the  $x$ -values to the change in the  $y$ -values.

Answer: For the values above, there would be the same as for part a, a 10-unit change in  $y$  for every 7-unit change in  $x$  or any other equivalent ratio.

- c. Make a conjecture about what the ratio of the change in the  $x$ -values to the change in the  $y$ -values will be for any two points on the line. Explain your reasoning.

Answer: The ratio will always be equivalent to a 10-unit change in  $y$  for every 7-unit change in  $x$ , which is reasonable because all of the points lie on the same line through  $(0, 0)$  and so should belong to the same set of equivalent ratios.



## Student Activity Questions—Activity 1 (continued)

- d. **Locate the point containing the unit rate. What is the unit rate and how is it related to the ratios?**

Sample answer: The unit rate is given by the point  $(1, \frac{10}{7})$  and will be  $\frac{10/7}{1}$ , or for every one unit over, the vertical change will be  $\frac{10}{7}$  of a unit.

3. **What is the equation of the proportional relationship containing the points from the previous question? Give at least two explanations you could use to convince someone why this equation makes sense for the line.**

Answer based on the values from question 3. The equation would be  $y = \frac{10}{7}x$ . One argument might be that all of the points are equivalent to the ratio 7:10, and equivalent ratios lie on a line. Another might be that the points can be generated using the unit rate. A third might be that all of the points are solutions for the equation.



## Class Discussion

**Use ‘New Line’ to generate another line. Move the point on the new line to the point where the x-coordinate equals 1 and record the value in the table.**

- **Use the right arrow to move the point along the line in increments of 1 and record the values of the coordinates of each point in the table.**

Responses will vary. An example is below.

x	y
1	3
2	6
3	9
4	12

- **How are the x-coordinates of the values in the table related? the y-coordinates?**

Responses will vary. For the example above, the x-coordinates increase by 1, and the y-coordinates increase by 3.

- **What is the constant of proportionality and how is it related to the points on the line?**

Answer: The constant of proportionality is  $\frac{3}{1}$  or 3. It is the rate of change that describes the vertical rise of the line for every 1 unit of horizontal change.



# Building Concepts: Proportional Relationships

TEACHER NOTES



## Class Discussion (continued)

Use the TNS activity to find examples that either support the statement or show why each of the following is not correct.

- **If the constant of proportionality is greater than 1, the line is will be steeper than a line with a constant of proportionality less than 1.**

Answer: The statement is true; examples will vary. One might be  $y = 3x$  and  $y = \frac{1}{3}x$ .

- **If a relationship is proportional, a point with a large x-coordinate and a small y-coordinate will determine a line that rises very slowly.**

Answer: True. One example might be to compare the relationships determined by (8, 1) and by (1, 8). The first proportion will produce a line with a slope that is much “flatter” than the second.

- **A proportion is sometimes called direct variation. If y varies directly with x in a proportional relationship, a point with a small x-coordinate and a large y-coordinate will determine a line that increases slowly.**

Answer: Not true. Consider (1, 10), which will be a fairly steep line.



## Additional Discussion

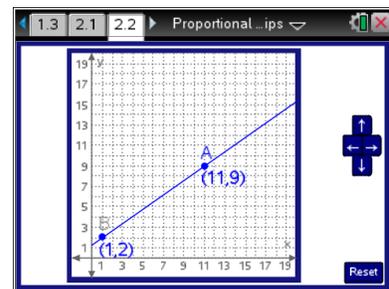
**Select New Line. Make up a context that can be represented by the line and pose a problem that can be solved using the line.**

Answers will vary.

## Part 2, Page 2.2

Focus: Lines that do and do not represent proportional relationships.

On page 2.2, use the arrow keys to move each point on the line. Use the **tab** key to toggle between points A and B.



**Teacher Tip:** To find a line representing a particular proportional relationship, students might first identify two points in the associated ratio and move the points on the screen to these points using the horizontal and vertical arrows.



## Class Discussion

Have students...

*Move the points on the screen to graph the corresponding line. Decide if the line represents a proportion and explain why or why not in each case.*

- *(2, 3) and (6, 5)*
- *(12, 9) and (8, 6)*
- *(0, 4) and (2, 6)*
- *(4, 1) and (12, 3)*

*Select Reset. Note the points on the screen and think about your answers to the previous question.*

- *Does the line on the screen represent a proportion? Why or why not?*
- *Sandi says that the line representing a proportional relationship will always go through the origin. Do you agree with Sandi? Why or why not?*

Look for/Listen for...

Answer: No because the ratios, 2:3 and 6:5, associated with the points are not equivalent and a proportion is a collection of equivalent ratios.

Answer: Yes because the ratios, 12:9 and 8:6, associated with the points are equivalent.

Answer: No because the ratios, 0:4 and 2:6, associated with the points are not equivalent, and a proportion is a collection of equivalent ratios.

Answer: Yes because the ratios, 4:1 and 12:3, associated with the points are equivalent.

Answer: No, because the line through a proportion always goes through (0, 0).

Answer: Yes, because every situation is grounded by 0 of one quantity and 0 of the other in any given ratio, although 0:0 does not belong to any set of equivalent ratios.



## Student Activity Questions—Activity 2

1. Consider each of the following meal plans. Find two ordered pairs that would represent each plan (number of meals, cost) and graph each set of ordered pairs. Does the plan describe a proportional relationship? Explain your thinking.

- a. \$5 each month plus \$4 a meal.

Answer: Two ordered pairs might be (1, 9) and (2, 13). The plan is not a proportional relationship because the graph does not go through (0, 0) (You would pay the \$5 even if you did not eat a meal). Also the ratios, 1:9 and 2:14, associated with the ordered pairs are not equivalent.



## Student Activity Questions—Activity 2 (continued)

- b. **A prepaid option where 2 meals cost \$11 and 4 meals cost \$20.**

Answer: The ordered pairs would be (2, 11) and (4, 20). The plan is not a proportional relationship because the graph does not go through (0, 0). Also the ratios, 2:11 and 4:20, associated with the ordered pairs are not equivalent.

- c. **A special where 1 meal costs \$5 and 3 meals cost \$15.**

Answer: The ordered pairs would be (1, 5) and (3, 15). This would be a proportional relationship because the graph goes through (0,0), and the ratios, 1:5 and 3:15 associated with the ordered pairs are equivalent.

2. **Find two points that satisfy each equation and use them to create the line on the TNS page representing the equation. Which of the following represent a direct variation? Explain your reasoning.**

a.  $y = \frac{3}{4}x$

b.  $y = 4 + 3x$

c.  $y = 5x$

d.  $y = 5x + 1$

Answer: Choices a and c represent proportional relationships because they lie on a line through the origin. The points that satisfy each of the two equations are associated with equivalent ratios.

3. **The science class built solar-powered robots and raced them in the gym. One of the robots can travel 5 meters for every 2 seconds.**

- a. **Identify the variables and explain which is the independent variable and which is the dependent variable.**

Answer: The independent variable will be the number of seconds, and the dependent variable the distance traveled.

- b. **Find two points that will be in the set of ratios equivalent to 2:5. Use these two points to create the graph of the proportional relationship showing the distance in meters the robot traveled after a given number of seconds.**

Answers will vary: Possible points are (2, 5) and (4, 10).

- c. **Carol says that the ratios of the number of seconds the robot traveled to the number of meters it traveled are equivalent. Do you agree with her? Why or why not?**

Answer: Yes. The rate is constant and therefore the ratios are equivalent.

- d. **Are the fractions associated with the ratios in (c) equal? Explain your reasoning.**

Answer: Yes. The fractions associated with equivalent ratios are equal.



## Student Activity Questions—Activity 2 (continued)

- e. Write an equation to describe the distance,  $d$  in meters, the robot travels for some time,  $t$  in seconds? How can you use the graph to answer the question?

Possible answer:  $d = \frac{5}{2}t$  You can use the graph to find the slope or constant of proportionality, the horizontal change and the vertical change as you move from one point on the graph to another.



## Additional Discussion

*Nia and Trey both had a sore throat, so their mom told them to gargle with warm salt water. Nia mixed 1 teaspoon salt with 3 cups water. Trey mixed  $\frac{1}{2}$  teaspoon of salt with  $1\frac{1}{2}$  cups of water. Nia tasted Trey's salt water. She said, "I added more salt so I expected that mine would be more salty, but they taste the same."*

Have students...

- **Explain why the salt water mixtures taste the same.**
- **Identify the variables. Which variable is the independent variable and which is the dependent variable?**

Look for/Listen for...

Answer: The ratios are equivalent, so the balance of salt and water is the same in both mixtures.

Answer: The variables are the amount of water and the amount of salt. Either variable could be the input or independent variable. Typically you would start with the water and add the salt, which would make the amount of water the independent variable.

**Using the salt solution context from the problem above, find an ordered pair that satisfies each of the following equations, then graph the equations. Interpret the ordered pair  $(w, s)$  in terms of the context, salt and water.**

**Use the graph to determine if the equation that relates  $s$ , the number of teaspoons of salt, with  $w$ , the number of cups of water, for the mixtures represent the proportional relationship in the problem above. Explain why or why not.**

*(Adapted from Illustrative Mathematics.)*

- $s = 3w$

Possible answer:  $(2, 6)$ , which would mean that for 2 cups of water you would add 6 teaspoons of salt. This is not the same as the proportional relationship given in problem 13 because the point  $(2, 6)$  does not lie on the same line through the origin as the point  $(3, 1)$ .

 **Additional Discussion (continued)**

- $s = \frac{1}{3}w$

Possible answer:  $(2, \frac{2}{3})$  which would mean that for 2 cups of water you would use  $\frac{2}{3}$  teaspoon of salt. This is the same as the proportion in problem 13 as the point lies on the line determined by  $(3, 1)$ .

- $s = 1\frac{1}{2}w$

Possible answer:  $(2, 3)$ , which would mean that for 2 cups of water you would add 3 teaspoons of salt. This is not the same as the proportional relationship given in problem 13 because the point  $(2, 3)$  does not lie on the line determined by  $(3, 1)$ .

- $w = 3s$

Possible answer: The ordered pair would be  $(6, 2)$  for (water, salt), which indicates 2 teaspoons of salt for 6 cups of water. This is the same proportion because the original ratio is 3 cups water to 1 teaspoon salt, which would be equivalent to 6 cups water to 2 teaspoons salt. The points  $(3, 1)$  and  $(6, 2)$  lie on the same line.

- $w = \frac{1}{3}s$

Possible answer: For  $s = 3$ ,  $w = 1$ , 3 teaspoons of salt for 1 cup of water, the ordered pair would be  $(1, 3)$ . This is not equivalent to the original ratio represented by  $(3, 1)$  and would produce a different line.



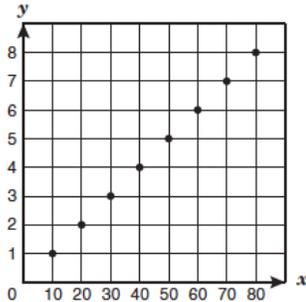
# Building Concepts: Proportional Relationships

TEACHER NOTES

## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. Look at the graph below.



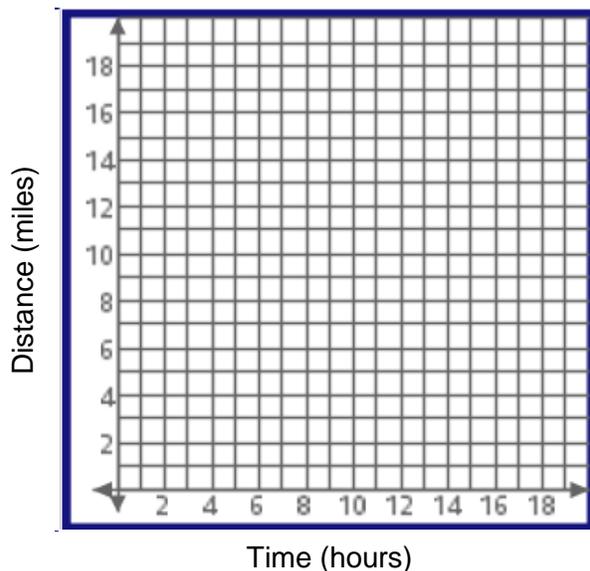
Which is the best interpretation of this graph?

- a. Jorge earns \$20 for each hour worked.
- b. For every 10 pieces of candy Stacey buys, she pays \$1.
- c. For every 10 students at a dance, 2 teachers are needed as chaperones.
- d. A runner runs at a constant rate of 2 miles every 30 minutes.

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**Answer: b) For every 10 pieces of candy Stacey buys, she pays \$1.**

2. Jayne participated in a walk-a-thon to raise money for school. She recorded the total distance she walked at several different points in time, but the paper tore and can no longer be read in some places. (Adapted from Illustrative Mathematics.)





# Building Concepts: Proportional Relationships

TEACHER NOTES

## Sample Assessment Items (continued)

- a. Assume Jayne walked at a constant speed. Complete the table and plot Jayne's progress in the coordinate plane.

Time (hours)	Distance (miles)
1	
2	6.4
	8
5	

**Answer:**

Time (hours)	Distance (miles)
1	<b>3.2</b>
2	6.4
<b>2.5</b>	8
5	<b>16.0</b>

- b. What was Jayne's walking rate in miles per hour? How long did it take her to walk 1 mile? Where do you see this information on the graph?

**Answer: In 1 hour, she walked 3.2 miles. This is the point (1, 3.2) on the graph.**

- c. Write an equation for the distance  $d$ , in miles, that Jayne walked in  $n$  hours.

**Answer:  $d = 3.2n$**

- d. Next year Jayne is planning to walk for 7 hours. If she walks at the same speed next year, how many miles will she walk?

**Answer: 22.4 miles**

3. Which of the following will graph as a proportional relationship? The line determined by

- a. (4, 3) and (8, 7)      b. (6, 9) and (8, 12)      c.  $(1, \frac{3}{2})$  and  $(2, \frac{1}{2})$       d. (2, 3) and (3, 2)

**Answer: b) (6, 9) and (8, 12)**



## Building Concepts: Proportional Relationships

TEACHER NOTES

### Sample Assessment Items (continued)

4. Cost in dollars,  $c$ , and amount of fruit in pounds,  $p$ , are related by the equation  $c = 2.5p$ . Which of the following is true about the proportional relationship?
- The cost increases by \$1 for every increase of 2.5 pounds.
  - The cost increases \$2 for every half-pound increase in weight.
  - The cost increases by \$2.50 for every 1-pound increase in weight.
  - There is not enough information to determine how the cost will increase.

**Answer: c) The cost increases by \$2.50 for every 1-pound increase in weight.**



# Building Concepts: Proportional Relationships

TEACHER NOTES

## Student Activity Solutions

In these activities you will work together to identify and graph proportional relationships to solve problems. After completing each activity, discuss and/or present your findings to the rest of the class.



### Activity 1 [Page 1.3]

1. Label the columns in the table below. Then record the point for the unit rate from the last question in the Class Discussion.
  - a. Using the right arrow on the keypad or touchpad, move the point 1 unit and record the new coordinates in the table. Repeat moving and recording until you have at least five points in the table.

x- _____ [answer: students]	y- _____ [answer: tiles]
1	[answer: $\frac{6}{9} = \frac{2}{3}$ ]
2	[answer: $\frac{12}{9} = \frac{4}{3}$ ]
3	[answer: $\frac{18}{9} = 2$ ]
4	[answer: $\frac{24}{9} = \frac{8}{3}$ ]
5	[answer: $\frac{30}{9} = \frac{10}{3}$ ]
6	[answer: $\frac{36}{9} = 4$ ]

- b. For the point whose x-coordinate is 5, explain what the point means in terms of the context in the Class Discussion Problem.

*Answer: When there are 5 students, you need  $\frac{30}{9}$  or 3 and  $\frac{1}{3}$  of a tile for them to line up*

- c. Describe the pattern in the table and how it relates to the graph.

*Answer: Each x variable increases by 1 student and each y variable increases by  $\frac{6}{9}$  or  $\frac{2}{3}$  of a tile.*



# Building Concepts: Proportional Relationships

## TEACHER NOTES

- d. Write the equation of the proportional relationship between the number of students and the number of tiles. Explain in words what the equation means.

*Answer: If  $s$  = number of students and  $t$  = number of tiles, the equation would be  $t = \frac{2}{3}s$ . The number of tiles is  $\frac{2}{3}$  times the number of students.*

2. Select **New Line** on page 1.3. Record the original coordinate of the point on the ray. Then move the point and record the coordinates of at least five points in the table.

x	y
14	20
12	$\frac{240}{14}$
7	10
4	$\frac{40}{7}$
2	$\frac{20}{7}$

- a. Pick two of the points and find the ratio of the change in the x-values to the change in the y-values.

*Answer: For the values above, there would be a 10-unit change in y for every 7-unit change in x or any other equivalent ratio.*

- b. Pick another pair of points from the table and find the ratio of the change in the x-values to the change in the y-values.

*Answer: For the values above, there would be the same as for part a, a 10-unit change in y for every 7-unit change in x or any other equivalent ratio.*

- c. Make a conjecture about what the ratio of the change in the x-values to the change in the y-values will be for any two points on the line. Explain your reasoning.

*Answer: The ratio will always be equivalent to a 10-unit change in y for every 7-unit change in x, which is reasonable because all of the points lie on the same line through (0, 0) and so should belong to the same set of equivalent ratios.*

- d. Locate the point containing the unit rate. What is the unit rate and how is it related to the ratios?

*Sample answer: The unit rate is given by the point  $(1, \frac{10}{7})$  and will be  $\frac{10/7}{1}$ , or for every one unit over, the vertical change will be  $\frac{10}{7}$  of a unit.*

*Answer: Responses for this question and a-d will vary depending on the initial point.*



## Building Concepts: Proportional Relationships

### TEACHER NOTES

3. What is the equation of the proportional relationship containing the points from the previous question? Give at least two explanations you could use to convince someone why this equation makes sense for the line.

*Answer based on the values from question 3. The equation would be  $y = \frac{10}{7}x$ . One argument might be that all of the points are equivalent to the ratio 7:10, and equivalent ratios lie on a line. Another might be that the points can be generated using the unit rate. A third might be that all of the points are solutions for the equation.*



### Activity 2 [Page 2.2]

1. Consider each of the following meal plans. Find two ordered pairs that would represent each plan (number of meals, cost) and graph each set of ordered pairs. Does the plan describe a proportional relationship? Explain your thinking.

- a. \$5 each month plus \$4 a meal.

*Answer: Two ordered pairs might be (1, 9) and (2, 13). The plan is not a proportional relationship because the graph does not go through (0, 0) (You would pay the \$5 even if you did not eat a meal). Also the ratios, 1:9 and 2:14, associated with the ordered pairs are not equivalent.*

- b. A prepaid option where 2 meals cost \$11 and 4 meals cost \$20.

*Answer: The ordered pairs would be (2, 11) and (4, 20). The plan is not a proportional relationship because the graph does not go through (0, 0). Also the ratios, 2:11 and 4:20, associated with the ordered pairs are not equivalent.*

- c. A special where 1 meal costs \$5 and 3 meals cost \$15.

*Answer: The ordered pairs would be (1, 5) and (3, 15). This would be a proportional relationship because the graph goes through (0,0), and the ratios, 1:5 and 3:15 associated with the ordered pairs are equivalent.*

2. Find two points that satisfy each equation and use them to create the line on the TNS page representing the equation. Which of the following represent a direct variation? Explain your reasoning.

- a.  $y = \frac{3}{4}x$       b.  $y = 4 + 3x$       c.  $y = 5x$       d.  $y = 5x + 1$

*Answer: Choices a and c represent proportional relationships because they lie on a line through the origin. The points that satisfy each of the two equations are associated with equivalent ratios.*



## Building Concepts: Proportional Relationships

### TEACHER NOTES

3. The science class built solar-powered robots and raced them in the gym. One of the robots can travel 5 meters for every 2 seconds.

- a. Identify the variables and explain which is the independent variable and which is the dependent variable.

*Answer: The independent variable will be the number of seconds, and the dependent variable the distance traveled.*

- b. Find two points that will be in the set of ratios equivalent to 2:5. Use these two points to create the graph of the proportional relationship showing the distance in meters the robot traveled after a given number of seconds.

*Answers will vary: Possible points are (2, 5) and (4, 10).*

- c. Carol says that the ratios of the number of seconds the robot traveled to the number of meters it traveled are equivalent. Do you agree with her? Why or why not?

*Answer: Yes. The rate is constant and therefore the ratios are equivalent.*

- d. Are the fractions associated with the ratios in (c) equal? Explain your reasoning.

*Answer: Yes. The fractions associated with equivalent ratios are equal.*

- e. Write an equation to describe the distance,  $d$  in meters, the robot travels for some time,  $t$  in seconds? How can you use the graph to answer the question?

*Possible answer:  $d = \frac{5}{2}t$  You can use the graph to find the slope or constant of proportionality, the horizontal change and the vertical change as you move from one point on the graph to another.*