Building Concepts: Ratios and Proportional Relationships Overview

Building Concepts: Ratios and Proportional Relationships consists of a sequential, coordinated series of interactive dynamic TI-Nspire™ activities designed to support a developmental trajectory of concepts related to ratios, proportions, and proportional reasoning. The approach, described in the Common Core State Standards (CCSS) focuses on interpreting a ratio, $a:b$, as a pair of values representing two (or more) quantities that vary in the same relative relationship. Each ratio $a:b$ is associated with a rate of $\frac{a}{b}$ units of ***a*** per every 1 unit of ***b***. This fraction $\frac{a}{b}$ associated with the ratio $a:b$ is also interpreted as the value of the ratio $a:b$, where ***a*** is $\frac{a}{b}$ times as much as ***b***. The development stresses equivalent ratios, connecting collections of equivalent ratios to points on a line leading to proportions and proportional reasoning.

These lessons are intended for educators at any level who are teaching or learning ratios and proportions or reviewing the concepts to help others develop a foundation for reasoning about ratios and proportions. These lessons may be used on grade level for middle grades or as enrichment at any level, including elementary, middle school, high school, developmental mathematics courses at the post-secondary level, and/or pre-service mathematics education university courses.

TI-Nspire™ Activities

The TI-Nspire™ activities are compatible with the following technologies:

TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

The activities can be adapted for different classroom environments. One teacher might use a whiteboard to demonstrate the activity to a large group of students, while another might have students working individually or in small groups using a TI-Nspire™ handheld. The delivery method used depends on the technology available in the classroom. The estimated activity time for students working at grade level is 50-90 minutes, depending on the delivery method of the activity.

Each TI-Nspire™ activity is accompanied by Teacher Notes and a Student Activity sheet, both of which help to facilitate the use of the TI-Nspire™ activity in the classroom.

Teacher Notes

Teacher Notes are intended to provide a starting point for how to use the activities with students. The activities themselves are typically robust enough to be adapted for learning objectives not mentioned in the guides. The activities are designed to help students investigate ratios and proportions that will allow them to develop the necessary fluency with proportional reasoning for their continuing work in mathematics, not to provide drill and practice.

Teacher Notes are designed to serve as a guide for implementation in classrooms and include:

- a description of the mathematics that underlies the activity including alignment of the mathematics to the CCSS;
- a description of the activity and how to use it;
- possible mathematical objectives for student learning;
- discussion questions for student investigation; and
- a set of assessment questions typical of questions on the National Assessment of Educational Progress or a high stakes state test.

The sample questions for students vary in difficulty; instructors might choose to use only a subset of the questions rather than working through all of them.

Student Activity

The Student Activity sheet is an optional resource that may be used as a tool to keep students engaged and on track throughout the lesson. This resource can be used in a small-group instruction scenario.

Although teachers can successfully work through the activities without using the Student Activity, the resource can prove to be valuable. The ratios and proportions concept aligns with the math standards for grades 6-8, and the activity sheet has been created to match this range.

Activity Series

The series consists of the following 15 activities. Each activity should take 50-90 minutes to complete with students, though teachers may choose to extend lessons, as needed.

1. *What is a Ratio?*

   A *ratio* is a pair of non-negative numbers, $a:b$, which are not both 0. This interactive activity allows students to set a ratio between a number of circles and a number of squares, and then generate equivalent ratios. Students will observe that *equivalent ratios* arise by multiplying or dividing each measurement in a ratio pair by the same positive number. Thus, $a:b$ is equivalent to $c:d$ if $a = k \cdot c$ and $b = k \cdot d$ where k is a positive number. This is reflected in the arrangement of circles and squares in each equivalent ratio, with the pairing of the original $a:b$ duplicated in each row of an equivalent ratio.

2. *Introduction to Rates*

   Every ratio has an associated rate. By investigating how to give out bags of dog food in order for each dog to have the same amount, students explore the concept of a unit rate: how many bags of dog food for 1 dog. They become familiar with the language of unit rate such as “per,” “for every,” or “for each” dog. They can choose from 1 to 6 dogs and up to 8 bags of dog food per dog, including halves, thirds, and fourths of a bag.

3. *Building a Table of Ratios*

Students generate a set of equivalent ratios and the corresponding table of values for each ratio. They should observe the multiplicative aspect of the table: the values in each row of the table are a multiple of the values in the original ratio; and the recursive aspect: the entries in each column can be obtained by adding the original values in the ratio to the previous row.

4. *Ratio Tables*

A ratio table can be powerful tool for solving problems involving ratios. In this activity, students generate a table of equivalent ratios and look for relationships among the entries, noting that the sum of two equivalent ratios is another equivalent ratio. This "additive" property of equivalent ratios can be used to solve both multiplication and division problems by creating appropriate and "easy" equivalent ratios.

5. *Comparing Ratios*

Students investigate how mixing different amounts of red and yellow paint will affect which mixture will be more of one color, i.e., a redder mixture. By generating tables of equivalent ratios, students can reason from some common value (for example, the same number of cans of red paint for two different ratios or the same total number of cans of paint). The display shows the original ratio for a given mixture, a table of some of the equivalent ratios for the given mixture, and the physical number of cans of paint for each color for any of the ratios in the table. This serves as a precursor for solving proportions by creating equal numerators or equal denominators.

6. *Ratios and Fractions*

In this lesson, we investigate how ratios are different from fractions. A ratio may be associated with a value; the *value* of a ratio $a:b$ is the quotient $\frac{a}{b}$ (if *b* is not 0). Students investigate possible ratios for pairs of congruent rectangles of two different colors and then explore the corresponding value or fraction for each ratio. They should recognize that the ratio 6:15 involves more parts of the same size than the ratio 2:5, while the corresponding value of the ratio, the fraction $\frac{6}{15}$, involves more parts of smaller sizes than $\frac{2}{5}$.

7. *Double Number Lines*

Double number lines are used as tools to organize and solve problems involving equivalent ratios, where three of the four values for a pair of equivalent ratios are known. With an eye on the ratio with the missing value, students generate collections of equivalent ratios on double number lines, visually displaying the relative sizes of the two different quantities. They divide the known ratio by a common factor and locate multiples of the result on the number lines until they find the missing value in the ratio equivalent to the original.

8. *Connecting Ratios to Graphs*

The activity builds on students' prior knowledge of plotting points in a coordinate grid by associating a ratio with an ordered pair of values. Students enter a ratio into a ratio table and create an equivalent ratio much as in Activity 3, *Building a Table of Ratios*. The ratio is displayed in a traditional "t-table," and horizontal and vertical arrows on the grid show the location of the associated point. A **Draw** command displays the line through the collection of equivalent ratios. Students move a point to set a unit rate and use the unit rate to graph the line, which allows them to investigate the concept of a "slope triangle" and its relationship to the original ratio.

9. Ratios and Rational Numbers

Students revisit Activity 3, *Building a Table of Ratios*, expanding the ratios they can enter into the table to rational numbers. Generating equivalent ratios allows them to reason about ratios such as $\frac{3}{4}$ of a cup of flour to $\frac{2}{3}$ cup of milk and compare it to 9 cups of flour to 8 cups of milk or relate the associated fraction $(\frac{3}{4})/(\frac{2}{3})$ to $\frac{9}{8}$, connecting division of a fraction by a fraction to the concept of ratio.

10. *Connecting Ratios to Equations*

In Activity 8, *Connecting Ratios to Graphs*, students graphed collections of equivalent ratios and generated the associated line. In this activity, students use almost the same graph, but the line is related to its equation, where the slope of the line is the unit rate or rate of change. Parallel grids allow students to compare the tables, graphs, and equations for different unit rates. Students enter a constant of proportionality (or rate), *k*; observe the resulting graph; and reflect on the relationship between the graph and the value of *k*.

11. *Proportional Relationships*

    In the first part of the activity, randomly generated points associated with a ratio allow students to explore the equations of proportional relationships and the collection of points represented by the equations and on the line. In the second part of the activity, students move one of two given points that do not represent equivalent ratios to create lines representing proportional relationships and distinguish the graphs of proportional relationships from those that are not.

12. *Solving Proportions*

    In the first part of the activity, students can use the line representing a proportional relationship and its equation as tools to solve problems involving proportions. They move one of two points on the graph of a line representing a proportional relationship and observe the impact on the equation of the line. In the second part of the activity, students are given an added constraint involving the two quantities in the ratio, which introduces them to solving a system of equations where one of the equations represents a proportional relationship.

13. *Adding Ratios*

    In this activity, students investigate the graph of the sum of two ratios, reinforcing from earlier work the fact that the sum of two equivalent ratios will be associated with another point on a line determined by the two equivalent ratios. If the ratios are not equivalent the sum will lie between the graphs of the two lines containing the collection of points representing each of the ratios. Students also use the activity to investigate Simpson's paradox.

14. *Connecting Ratios and Scaling*

    In this activity, students investigate the relationship between the sides of two pentagons, one of which has been scaled from the other. Students relate scale factor to the ratios of the corresponding sides and use it to solve problems involving missing side lengths in the shapes.

15. *Ratios Within and Between Scaled Shapes*

    Students are given two rectangles on a grid, one an enlarged copy of the other. They can move a dot on the corner of one rectangle and then change the scale factor to enlarge or shrink the corresponding rectangle. In the second part of the activity, students explore the connection among scale factor and side lengths of two rectangles, one an enlarged copy of the other, building a sense of how ratios across the two shapes behave and of how ratios between two sides of one shape compare to the ratios between the corresponding two sides of the other shape. Students also investigate the ratio between the areas of two shapes when one is an enlarged copy of the other.

Grade Level Alignment

It is recommended that the activities be used in sequential order at any grade level. Each activity builds upon a concept that has been presented in a previous activity.

The following table shows how the activities align with typical grade-level expectations.

	Grade 6	Grade 7	Grade 8
Activity 1 *What is a Ratio?*	√ CCSS: 6.RP.1, 6.RP.3		
Activity 2 *Introduction to Rates*	√ CCSS: 6.RP.2, 6.RP.3b		
Activity 3 *Building a Table of Ratios*	√ CCSS: 6.RP.2, 6.RP.3a		
Activity 4 *Ratio Tables*	√ CCSS: 6.RP.1, 6.RP.3		
Activity 5 *Comparing Ratios*	√ CCSS: 6.RP.3a		
Activity 6 *Ratios and Fractions*	√ CCSS: 6.RP.2, 6.RP.3b		
Activity 7 *Double Number Lines*	√ CCSS: 6.RP.2, 6.RP.3 6.NS.4		
Activity 8 *Connecting Ratios to Graphs*	√ CCSS: 6.RP.3a, 6.RP.3b		√ CCSS: 8.EE.6
Activity 9 *Ratios and Rational Numbers*	•	√ CCSS: 7.RP.1, 7.RP.2 7.RP.3	√ CCSS: 8.EE.5, 8.EE7b

Activity 10 *Connecting Ratios to Equations*	•	√ CCSS: 7.RP.1, 7.RP.2b, 7.RP.2c, 7.RP.3	√ CCSS: 8.EE.5
Activity 11 *Proportional Relationships*	√ CCSS: 6.EE.9	√ CCSS: 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d, 7.RP.3	√ CCSS: 8.EE.5
Activity 12 *Solving Proportions*	•	√ CCSS: 7.RP.2a, 7.RP.2c, 7.RP.3	√ CCSS: 8.EE.5, 8.EE7b
Activity 13 *Adding Ratios*	•	√ CCSS: 7.RP.2, 7.RP.3	√ CCSS: 8.EE.5
Activity 14 *Connecting Ratios and Scaling*	•	√ CCSS: 7.RP.2, 7.RP.3, 7.G.2	√ CCSS: 8.EE7b
Activity 15 *Ratios Within and Between Scaled Shapes*	•	√ CCSS: 7.G.1	

Key: √ = on grade level
• = remediation

References

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

Progressions for the Common Core State Standards in Mathematics. *Tools for the Common Core Standards.* The Common Core Standards Writing Team, 26 December 2011. Web. 05 Aug. 2014 *http://commoncoretools.me/2011/09/12/progression-on-ratios-and-proportional-reasoning/*