

Building Concepts: Fractions Overview

Building Concepts: Fractions consists of a series of interactive dynamic TI-Nspire™ files designed to develop concepts related to fractions and operations with fractions. The activities, which are based on the *Progressions for the Common Core State Standards in Mathematics, Grades 3-5 Number and Operations – Fractions* (Common Core Standards Writing Team, 2013), are designed to give students ways to think about fractions and fraction operations that will allow them to develop the necessary fluency with fractions for their continuing work in mathematics. The development, based on the approach outlined in the Common Core State Standards and other state standards (e.g., Texas, 2012) uses the number line and an area model to help students visualize the concept of a fraction as a number. The concept of a unit fraction is the building block used to develop concepts and operations with fractions (Wu, 2013).

The activities are created for those who are teaching or learning fractions or reviewing fraction concepts at any level (elementary, middle school, high school, remedial mathematics at the post-secondary level, preservice mathematics education university courses), parents, or anyone interested in learning or helping others learn about fractions. The activities are designed to develop students' expertise in the processes and proficiencies that serve as the foundation for the Standards for Mathematical Practice described in the Common Core State Standards.

TI-Nspire Files

The TI-Nspire files are compatible with the following technologies:



TI-Nspire™ CX Handhelds,



TI-Nspire™ Apps for iPad®,



TI-Nspire™ Software

The activities can be adapted for different classroom environments. One teacher might use a whiteboard to demonstrate the file to a large group of students, while another might have students working individually or in small groups using a TI-Nspire handheld. The delivery method used depends on the technology available in the classroom. The estimated activity time for students working at grade level is 30-50 minutes, depending on the delivery method of the activity.

Each TI-Nspire file is accompanied by Teacher Notes and a Student Activity sheet to help facilitate the use of the TI-Nspire file in the classroom.

Teacher Notes

The Teacher Notes are intended to provide a starting point for how these activities can be used with students. It is important to note that the TI-Nspire activities themselves are typically robust enough to be adapted for learning objectives not mentioned in the Notes. The activities are designed to give students ways to think about fractions and fraction operations that will allow them to develop the necessary fluency with fractions for their continuing work in mathematics. Although the activities are not created to provide drill and practice, they can be used that way.



Teacher Notes are designed to serve as a guide for implementation in classrooms and include:

- descriptions of the mathematics that underlies each activity, including alignment of the mathematics to the Common Core State Standards (CCSS), the Common Core Standards for Mathematical Practice, and to the CCSS progressions document for fractions
- a description of the activity and how to use it
- mathematical objectives for student learning
- discussion questions for teacher-led classroom exploration or individual student investigation
- assessment questions that vary in difficulty. These questions might be found on the National Assessment of Educational Progress mathematics assessments or a high stakes state test. Instructors may choose to use all of the suggested questions or select a few from those presented.

Student Activity

The Student Activity sheet is an optional resource that may be used as a tool to keep students engaged and on track throughout the lesson. This resource is particularly valuable in a large-group instruction scenario.

Although teachers can successfully work through the activities without using the Student Activity, the resource can prove to be valuable. The fractions concept aligns with the math standards for grades 3-5 and the activity sheet has been created to match this range. The Student Activity is provided as an editable word document and therefore may be modified to use with middle-school students or higher, increasing its value.

Activity Series

The series consists of the following 15 activities. Each activity should take 30-50 minutes to complete with students, though teachers may choose to extend lessons, as needed.

1. *What is a Fraction?*

This activity uses a dynamic action/consequence document to help students understand and visualize a fraction as a number represented as a point on a number line. In particular, a fraction $\frac{a}{b}$ is a length of $\frac{1}{b}$, where **b** indicates the unit segment 0 to 1 partitioned into **b** equal intervals and **a** represents the number of copies of those intervals positioned end to end on the number line beginning at 0 (CCSS). Students will learn that the denominator indicates the number of equal parts in each whole unit, and the numerator indicates the number of those parts.

2. *Equivalent Fractions*

Students investigate equivalent fractions and ordering fractions using number lines. Two fractions are considered equivalent if they are located at the same point on a number line. By seeing the movement of a point along two number lines divided into different unit fractions, students learn what it means to have different names for the same point. They compare fractions by thinking about them in terms of unit fractions and by locating them on a number line. Students recognize that, as with whole numbers, the larger fraction is located farther to

the right on the number line. Students should be familiar with the terms *numerator*, *denominator*, and *unit fraction* and should have completed the activity *What is a Fraction?*

3. *Fractions and Unit Squares*

This activity, essentially a dynamic geoboard, connects fractions represented on a number line to a unit square, developing the concept of a fraction as a part of the area of a unit square. Students learn that the fraction $\frac{a}{b}$ is **a** regions of area $\frac{1}{b}$, where **b** indicates the number of regions of equal area into which the unit square has been divided and **a** represents the number of those regions that are shaded. Students reason about the area of a shaded region to decide what fraction of the whole it represents (MP3). The activity can also be used to illustrate the need to clearly specify the unit before a fraction can be interpreted meaningfully.

4. *Creating Equivalent Fractions*

Both interactive number lines and area models are used to develop the principle that the numerator and denominator of a fraction can be multiplied or divided by the same non-zero whole number, resulting in a new fraction equal to the value of the original fraction. Multiplying the numerator and denominator of a fraction by the same number, *n*, corresponds physically to partitioning each unit fraction into *n* smaller pieces. The whole is partitioned into *n* times as many pieces, and there will be *n* times as many smaller unit fraction pieces as in the original fraction.

5. *Adding and Subtracting Fractions with Like Denominators*

Interactive number lines are used to help students investigate addition and subtraction of fractions. Students build on the concept that the fraction $\frac{a}{b}$ is **a** copies of $\frac{1}{b}$. Therefore, adding the fraction $\frac{c}{b}$ would be joining **c** copies of $\frac{1}{b}$ to **a** copies of $\frac{1}{b}$ on the number line. Such an addition results in **a + c** copies of $\frac{1}{b}$. Students are taught to consider a fraction as the sum of unit fractions, which enables them to compose and decompose fractions with the same denominator. The activity can also be used to investigate the number properties for addition. Students should be familiar with the basic concepts of fractions before they start the activity.

6. *Mixed Numbers*

In this activity students rewrite improper fractions as mixed numbers. The development of this skill depends on the recognition that an improper fraction can be converted to the sum of a whole number and a fraction less than one. Converting a mixed number to a fraction can be thought of as fraction addition. Students should have experience with the concepts in *Fraction as Division* and *Adding Fractions with Unlike Denominators* before starting this activity.

7. *Adding Fractions with Unlike Denominators*

Building on the strategies and understandings from *Equivalent Fractions*, *Creating Equivalent Fractions* and *Adding and Subtracting Fractions with Like Denominators*, students recognize that fractions with unlike denominators need to be re-expressed in terms of equivalent fractions with a new common denominator. Area models help students visualize the concept of a common denominator. At this stage, students should be able to make and test conjectures about how to create denominators that might enable them to add the fractions.

8. *Fractions as Division*

In this activity students explore the division of two whole numbers by a fraction (i.e., $\frac{7}{4}$ can be thought of as 7 copies of $\frac{1}{4}$ and as 7 whole units divided into 4 parts). It is important for students to recognize that this concept shows two different ways to think about the same fraction. This understanding is an important precursor to successfully working with mixed numbers. Once students can see the connection between fractions and division they will understand that $\frac{5}{3}$ is one third of 5, which leads to the meaning of multiplication by a unit fraction.

9. *Multiplying Fractions by a Whole Number*

The activity uses a number line to develop multiplication of a fraction by a whole number. It builds on the earlier concept that the product of two whole numbers r and s is r sets, each containing s elements. Using the same reasoning in the context of a number line, the product of c and the fraction $\frac{a}{b}$ would be c copies of $\frac{a}{b}$, which is equivalent to ca copies of $\frac{1}{b}$. Students learn how to mark a “unit” such as $\frac{2}{3}$ on a number line (e.g., a product is sold in units of $\frac{2}{3}$ of a pound) and visualize the value of 5 copies of $\frac{2}{3}$ (e.g., the amount you would have with five units). Students should have experience with the activities *What is a Fraction?* and *Equivalent Fractions* before working on this activity.

10. *Fraction Multiplication*

The concept of fraction multiplication is developed by considering the area of a unit square that has been partitioned into rectangles with dimensions $\frac{1}{n}$ by $\frac{1}{m}$. Building on earlier work of finding the area of a rectangle by multiplying side lengths, students investigate the fraction of an area shaded in a unit square. This leads to the generalization that the product of two fractions is the product of the numerators and the product of the denominators. Students should have prior experience with *Fractions and Unit Squares* before working with this activity.

11. *Dividing a Fraction by a Whole Number*

Students use the relationship between division and multiplication to solve problems involving dividing fractions by whole numbers. The activity uses a unit square to develop an understanding of the meaning of dividing a fraction by a whole number. By observing the partitioning of unit squares students experience a visual representation of the process.



12. *Dividing a Whole Number by a Fraction*

Unit squares are used to illustrate the division of a whole number, c , by a unit fraction $\frac{1}{b}$.

Once this concept has been established, students build on this thinking to analyze representations of dividing the whole number, c , by the fraction $\frac{a}{b}$. The activity also allows students to approach the task by considering the relationship between multiplication and division and identifying a missing factor. This approach builds on the ideas in *Fraction Multiplication*.

13. *Dividing a Fraction by a Fraction*

Number lines are used along with equivalent fractions to help students develop a method for dividing fractions. Once equivalent fractions with common denominators have been identified, focus on the relationship between the numerators in the two fractions, regardless of the denominators. Students need experience in creating equivalent fractions and multiplying fractions.

14. *Units Other than Unit Squares*

Students learn how to use an area model to examine differences in the size of unit squares. Their investigation leads to the understanding that fractions make sense only in reference to a particular scale or unit. Students consider how one half of a unit square can represent different areas depending on the size of the unit square. Building on the activity *Fractions and Unit Squares*, students investigate fractions of shapes where the unit is not a unit square (e.g., the “unit” is one fourth of a fraction). NOTE: This activity may require 60 minutes for developing the concepts and ensuring that students understand and can explain the mathematics.

15. *Comparing Units*

Students investigate how the size of a fraction can vary with the scale used to define the unit fraction. The concept of the whole is critical to developing an understanding of fractions (e.g., $\frac{1}{2}$ foot is larger than $\frac{1}{2}$ inch but smaller than $\frac{1}{2}$ meter). The activity builds students’ awareness of the role played by a unit’s scale or size when working with fractions. This activity leads into the concept of ratios and equivalent ratios, but might be used earlier to develop the concept of different sized unit fractions depending on the original scale.



Grade Level Alignment

It is recommended that the activities be used in sequential order at any grade level. Each activity builds upon a concept that has been presented in a previous activity.

The following table shows how the activities align with typical grade-level expectations.

	Grade 3	Grade 4	Grade 5	Grade 6-8	Grade >8
Activity 1 <i>What is a Fraction?</i>	√ CCSS: 3.NF.1, 3.NF.2a, 3.NF.2b, 3.NF.3c	•	•	•	•
Activity 2 <i>Equivalent Fractions</i>	√ CCSS: 3.NF.3a, 3.NF.3b, 3.NF.3c, 3.NF.3d	•	•	•	•
Activity 3 <i>Fractions and Unit Squares</i>	√ CCSS: 3.NF.1, 3.NF.3a, 3.NF.3b, 3.G.2	√ CCSS: 4.NF.1	•	•	•
Activity 4 <i>Creating Equivalent Fractions</i>		√ CCSS: 4.NF.1	√ CCSS: 5.NF.5b	•	•
Activity 5 <i>Adding and Subtracting Fractions with Like Denominators</i>		√ CCSS: 4.NF.3a, 4.NF.3b, 4.NF.3d	•	•	•
Activity 6 <i>Mixed Numbers</i>		√ CCSS: 4.NF.4c	√ CCSS: 5.NF.1 5.NF.2	•	•
Activity 7 <i>Adding Fractions with Unlike Denominators</i>			√ CCSS: 5.NF.1	•	•



Activity 8 <i>Fractions as Division</i>			√ CCSS: 5.NF.3	•	•
Activity 9 <i>Multiplying Fractions by Whole Numbers</i>		√ CCSS: 4.NF.4b 4.NF.4c	√ CCSS: 5.NF.4a	•	•
Activity 10 <i>Fraction Multiplication</i>			√ CCSS: 5.NF.4a 5.NF.4b 5.NF.5a 5.NF.5b 5.NF.6	•	•
Activity 11 <i>Dividing a Fraction by a Whole Number</i>			√ CCSS: 5.NF.7a 5.NF.7c	•	•
Activity 12 <i>Dividing a Whole Number by a Fraction</i>			√ CCSS: 5.NF.7b	•	•
Activity 13 <i>Dividing a Fraction by a Fraction</i>			√ CCSS: 5.NF.7	√ CCSS: 6.NS.1	•
Activity 14 <i>Units Other than Unit Squares</i>	√ CCSS: 3.NF.3d	√ CCSS: 4.NF.2 4.NF.3d	√ CCSS: 5.NF.2	•	•
Activity 15 <i>Comparing Units</i>		√ CCSS: 4.NF.2 4.NF.3a 4.NF.3d	•	•	•

Key: √ = on grade level
• = remediation

References

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

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Wu, H. (2011, August 5, Revised 2014, February 8). Teaching fractions according to the Common Core Standards. Retrieved February 24, 2014 from <http://math.berkeley.edu/~wu/CCSS-Fractions.pdf> Wu, H. (2011). *Understanding numbers in elementary school mathematics*. Providence RI: American Mathematical Society.