



Building Concepts: Visualizing Quadratic Expressions

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students manipulate geometric figures to explore equivalent expressions that can be expressed in the form $x^2 + bx + c$ where b and c are positive integers.



Using geometric shapes as tools to visualize the algebraic structure of expressions can help in thinking about equivalent expressions involving quadratics.

Learning Goals

1. Connect the distributive law to an area model;
2. recognize that quadratic expressions can be written in many different equivalent forms;
3. understand that the product of two binomials can be found by multiplying “each times every” (each term in one sum times every term in the other sum)

Prerequisite Knowledge

Visualizing Quadratic Expressions builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *What Is a Variable? What Is an Equation?* and *Equations and Operations*. Students should understand:

- the concept of the distributive property;
- how to calculate the area of a square and a rectangle.

Vocabulary

- **rectangle:** a quadrilateral with four right angles and opposite sides of equal length
- **square:** a quadrilateral with four right angles and all sides of equal length
- **area:** the measurement of the surface of a figure or an object

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Visualizing Quadratic Expressions_Student.pdf
- Visualizing Quadratic Expressions_Student.doc
- Visualizing Quadratic Expressions.tns
- Visualizing Quadratic Expressions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.



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Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.



Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

Interpreting and creating illustrations for algebraic expressions can help students visualize the meaning of algebraic symbols, and the illustrations can be useful tools for building numerical fluency, algebraic generalizations, and building connections to geometric representations. In *Visualizing Linear Expressions*, students used geometric representations to explore equivalent expressions that could be expressed in the form $ax + b$, where a and b are rational numbers. They considered the role properties play in creating equivalent expressions, e.g., the distributive property of multiplication over addition, and identified and rewrote expressions as sums, as products, or as a combination of sums and products. In this lesson, students use geometric representations to explore equivalent expressions that can be expressed in the form $x^2 + bx + c$ where b and c are positive integers. The emphasis is on using an area model to represent the product of two binomials, continuing to build on the interpretation of the distributive property of multiplication over addition as “each times every”: given the product of two binomials, each term in the sum of the first binomial multiplies every term in the sum of the second binomial.

To connect the properties of multiplication and addition to rewriting expressions, the area of a rectangle is described as the product of the width (base) and the height of the rectangles on the screen. Thus, if the width is 1 and the height is x , the area is the product $1x$, which can be written as x by the identity property of multiplication.

The questions are designed to confront student misconceptions such as $(x + y)^2 = x^2 + y^2$, $(x + 4)(x + 2) = x^2 + 8$, and $x + x = x^2$.



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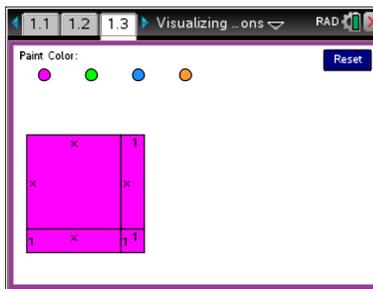
Part 1, Page 1.3

Focus: Virtual manipulatives help connect the structure of equivalent expressions.

On page 1.3, the arrow keys or grabbing a point on the upper right corner of the interior large square will change the size of x .

Select a color and select a shape to color a shape, which will separate the shape from the others.

Note that shapes of the same color will connect to form a new shape.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the colors.

P, b, g, o can be used as shortcut keys to select colors.

esc deselects the colors.

ctrl del Resets the page.



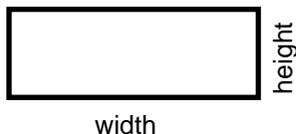
Class Discussion

Teacher Tip: In this TNS activity, an object all one color should be expressed as the area of the rectangle or, if it is not a complete rectangle, as the area of a rectangle minus the area of a smaller rectangle that has been removed. Every figure of the same color will have an area that can be expressed either as a product or as the difference of two products. Note that x is considered the length of a segment and $1(x)$ the area of the segment with dimensions 1 and x . In writing expressions, $x(x+1) + x + 1$ would be considered as representing the area of three different shapes, while $x(x+1) + 1(x+1)$ would represent the area of two shapes.

Have students...

Look for/Listen for...

Look at the figure on page 1.3. Remember that the area of a rectangle can be written as the product of the width (base) and height.





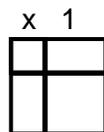
Class Discussion (continued)

- *Let x be the length of the side of the large square. Write the area of the entire shaded rectangle as the product of two expressions involving x .*
- *Grab the upper right corner of the large interior square and make the original figure larger. Does your answer to the question above change?*
- *Make a sketch of what you think the figure would look like if x were less than 1. Be ready to explain your reasoning.*

Answer: $(x+1)(x+1)$ or $(x+1)^2$

Answer: It does not change; x represents the length of one side of the square, and the expression gives the area of the square no matter how large or small the value of x is.

Answer:



Color each of the sections of the figure on page 1.3 a different color.

- *Describe what happens.*
- *Is the total area of the four pieces the same as the area of the original square? Why or why not?*
- *Write the area of each colored shape as the product of its width and height (note that some of the linear dimensions are 1 unit.)*
- *Use your answer for your description above to write an expression for the total area of the colored pieces.*

Answer: The original square decomposes into four shapes; two rectangles and a large and small square.

Answer: Yes because the four pieces can be arranged to make the original square.

Answer: x^2 , $1x$, $x1$, $1(1)$

Answer: $x^2 + 1x + x1 + 1(1)$

In earlier work you described the distributive property as “each times every”.

- *Describe what this means for an expression of the form $x(x+1)$.*
- *Describe how you could relate the areas of the four different colored shapes to the area of the original large square to illustrate the distributive property.*

Answer. The x outside of the parentheses is multiplied times each of the terms in the sum, the x and the 1 .

Answer: The original area was $(x+1)^2 = (x+1)(x+1)$. The x in the first $()$ multiplied the x and the 1 in the second $()$, and the 1 in the first $()$ multiplies the x and the 1 in the second $()$ to get the areas of the four shapes: $(x^2 + x1) + (1x + 1(1))$.



Class Discussion (continued)

- **How can the distributive property be used to combine $1x + 1x$?** Answer: The common factor x is distributed out of each term to get $(1+1)x$.
- **Why can $1(x)$ be written as $(x)(1)$ or as x ?** Answer: The identity property of multiplication (some may also mention the commutative property of multiplication).

This question emphasizes that the area is the width times the height and the use of 1 as a dimension in order to help students interpret the areas of different shapes. The last part of the question is important in recognizing that different shapes are separated by “+” signs and to find the total area you add the individual areas of each shape in the decomposition.

Reset. Color the vertical rectangle and the small square green.

- **Write the area of each colored piece as a product of its two dimensions (width times height) using x and 1.** Answer: $x(x+1)$ and $1(x+1)$
- **Write an expression that would give the total area of the two pieces together.** Answer: $x(x+1) + 1(x+1)$
- **Shauna wrote $(x+1)(x) + (x+1)(1)$ to represent the total area of the two shapes. Do you agree with her? Why or why not?** Answer: Yes because she found the area by changing the order and using the product of the height and the width not the width and the height. These are the same by the commutative property of multiplication.
- **Charla wrote the expression as three terms: $x(x+1) + x + 1$. Would her expression represent the same arrangement of the shapes? Why or why not?** Answer: Her representation would really describe three shapes, the rectangle that is $x(x+1)$, a small rectangle x or $1(x)$ and the small square $1(1)$ or just 1.



Student Activity Questions—Activity 1

1. **Reset. Write an expression for the total area of the shapes using the product of the width and height for each area.**
 - a. **Color the horizontal rectangle and the small square green.**
Answer: $(x+1)x + (x+1)1$
 - b. **Choose blue and color the vertical rectangle.**
Answer: $x^2 + 1(x) + (x+1)1$
 - c. **Color the horizontal rectangle blue.**
Answer: $x^2 + 2(x) + 1(1)$



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Student Activity Questions—Activity 1 (continued)

2. a. How are the total areas described in each problem so far related? Explain your reasoning.

Answer: They are the same because they all are made of the same shapes displayed in different ways. Each arrangement would have the same total area as the original figure.

- b. How are the expressions related to the total areas for each problem related? Use the distributive property to support your answer.

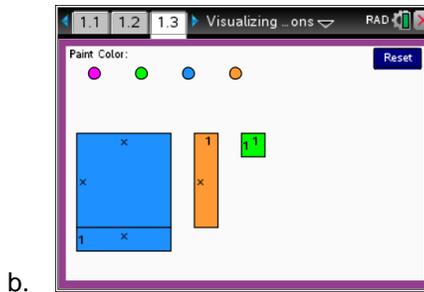
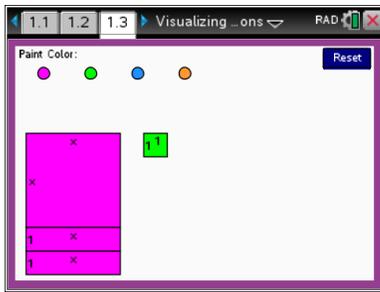
Answer: They are equivalent. By using the distributive property to expand and collect terms, each of these is equivalent to $x^2 + 2x + 1$. You may want different students to show exactly how each is equivalent to the expression $x^2 + 2x + 1$.

3. Find an arrangement for the shapes that would illustrate each of the following expressions. Sketch your arrangement.

a. $x(x+2)+1(1)$

b. $x(x+1)+1(1)$

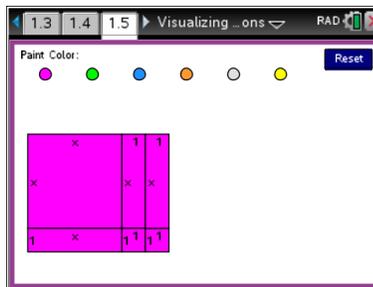
Answer:



Part 2, Pages 1.3 and 1.5

Focus: Some expressions can be represented using the area of irregular shapes.

Page 1.5 functions in the same way as page 1.3.



TI-Nspire Technology Tips

w and y are added to the color shortcut keys.



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Class Discussion

The following questions involve finding the area of irregular shapes by enclosing them in a larger region (rectangle) and subtracting the unneeded area(s). Students may have used this strategy for finding area of irregular shapes in earlier grades.

Have students...

Look for/Listen for...

Reset page 1.3 and color the large square green.

- Think about the rectangle formed by the pink irregular shape and the dotted lines. What are the dimensions of this rectangle?
- What is the area of the pink irregular shape? Explain your thinking.
- Show that the sum of the expressions for the areas of the green and pink shapes is equivalent to the expression for the area of the original figure.

Answer: the width is 2 and the height is $x + 1$

The area would be $2(x + 1) - 1$ because you have to subtract the extra area of the small square.

The sum of the two expressions for the areas of the green and pink shapes would be $x^2 + 2(x + 1) - 1$. By distributing the 2 and combining like terms, you will get $x^2 + 2(x) + 2(1) - 1 = x^2 + 2(x) + 1$, which is the same expression as the original area.

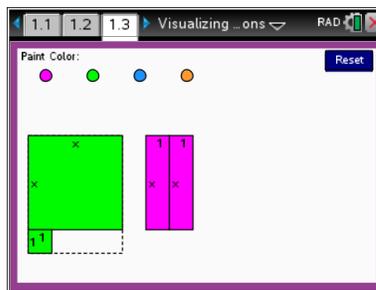
Without resetting, color the small square and one of the vertical rectangles green.

- Find the area of the irregular green shape. Explain your reasoning.
- Show that the sum of the expressions for the areas of the green and pink shapes is equivalent to the expression for the area of the original figure.
- Find an arrangement for the shapes that would illustrate $x(x + 1) - (x - 1) + 2x$.

Answer: $(x + 1)(x + 1) - 1(x)$ because the rectangle enclosing the shape has area $(x + 1)(x + 1)$. Then you have to subtract the extra area from the dotted rectangle, which is $1(x)$.

Answer: $(x + 1)(x + 1) - 1x + 1x$ is equivalent to $(x + 1)(x + 1)$ because $-1x + 1x = 0$.

Answer: An arrangement such as the one below will work.





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Class Discussion (continued)

- **Show that your answers to the questions above are equivalent to the expressions in Student Activity 1, question 1b.**
- **Move to page 1.5. Write an expression for the area of the shape on page 1.5.**
- **Decompose the shape into six separate pieces by using the different colors. Write an expression for the total area of these pieces.**
- **Combine the congruent shapes and write an expression that shows the sum of the areas for all of the combined shapes.**
- **Color one vertical strip a different color than any of the colors used for the other shapes. Describe how this arrangement of the shapes supports the “each times every” approach to finding the product of $(x+1)(x+2)$.**

Answer: Using the distributive property, both will be equivalent to $x^2 + 2x + 1$ and so equivalent to the original expression.

Answer: $(x+2)(x+1)$

Answer: $x^2 + x + x + x + 1 + 1$

Answer: $x^2 + 3x + 2$

Answer: “Each times every” produces $(x)(x) + 2x + 1(x) + 1(2)$. The large square has an area represented by the expression $(x)(x)$; the area of the joined rectangles by $2(x)$; $1(x)$ represents the area of the single rectangle and $1(2)$ the area of the rectangle made of the two unit squares.

Reset. Answer each of the following:

- **Color the vertical rectangle and small square on the right orange. Write an expression for the total area of the two shapes.**
- **Color one more small square orange and write an expression for the total area of the two shapes.**
- **Show that the expressions you wrote in the questions above are equivalent expressions.**

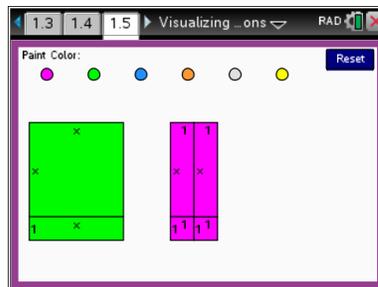
Answer: $(x+1)^2 + 1(x+1)$

Answer: $x(x+2) + 1(x+2)$

Answer: They all can be rewritten as $x^2 + 3x + 2$

Decompose the rectangles into shapes that would match the given expressions:

- $x(x+1) + 2(x+1)$





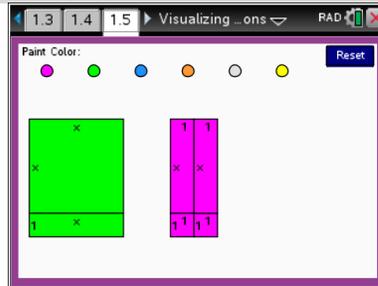
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Class Discussion (continued)

- $x^2 + 3(x+1) - 1$



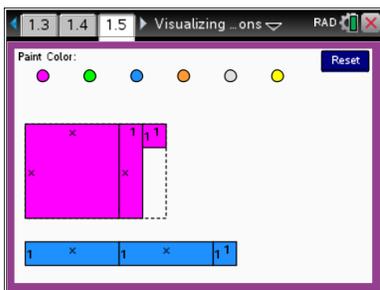
Reset. Create two different rearrangements of the rectangles and squares in the original shape. Write an expression for the total area of the shapes for each of your arrangements. Give your expressions to a partner and see if they can create an arrangement for each expression.

Answers will vary.



Student Activity Questions—Activity 2

1. Use the diagram below to decide whether statements a–d are true or false, then answer e. Explain your thinking in each case.



- a. The area of the long rectangle can be represented by $1(x+1)$.

Answer: False because the width of the rectangle is $2x+1$.

- b. $x(x+2)$ represents the area of the rectangle enclosing the irregular shape.

Answer: True, the width is $x+2$ and the height is x .

- c. The area represented by $(x+2)(x)$ is the same as the area represented by $x(x+2)$.

Answer: True, by the commutative property; one is the width times the height, the other is the height times the width and both will give the same area.

- d. The total area represented by the two shapes will be $x(x+1) + (2x+1)(1)$.

Answer: False because the area of the irregular shape will be $x(x+2) - (x-1)$ so the total area will be $x(x+2) - (x-1) + (2x+1)(1)$.



Student Activity Questions—Activity 2 (continued)

- e. Show that the total area represented by the two shapes can be expressed as $(x+1)(x+2)$.

Answer: By distributing, $x(x+2) - (x-1) + (2x+1)(1)$ becomes $x(x) + 2x - 1x + 1 + 2x + 1$. Collecting like terms gives $x^2 + 3x + 2$ which factors to become $(x+1)(x+2)$.

2. Answer each of the following questions.

- a. Sally said that no matter how you rearrange the squares and rectangles in the original figure on page 1.5, the corresponding expressions will always be equivalent to the original product. Do you agree with Sally? Why or why not?

Answer: Sally is right because the total area is conserved or stays the same no matter where it is located or how the shapes are combined (as long as the shapes don't overlap).

- b. Find at least two ways to explain why $(x+2)(x+1) \neq x^2 + 2$.

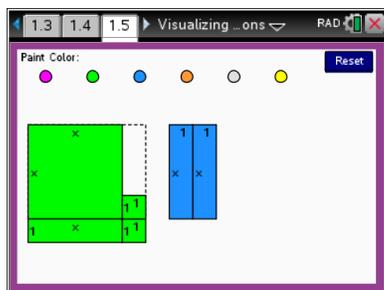
Answers may vary. One explanation is: the area model for the product of $(x+2)(x+1)$ has an x^2 and a 2, but also contains more parts, so the product must be greater than just the sum of x^2 and 2; another might be: when you use “each times every” to expand the multiplication you get $x^2 + 2x + 1x + 2$ which is not equal to $x^2 + 2$.

- c. Color all of the non-square rectangles and one small square blue. Explain why the area represented by the blue shapes could be the expression $3(x+1) - 2$ or the expression $3x + 1$.

Answers may vary. The width of the blue region is 3 and the height is $x+1$, so the area of the rectangle enclosing the irregular shape with dotted lines is $3(x+1)$. But the extra area has width 2 and height 1, so you have to subtract $2(1)$ to get $3(x+1) - 2$. If you colored the small square green, you could write the area represented by the three rectangles as $3x$ and then add the small square to have $3x+1$.

- d. Find at least two ways to explain why $(x+1)^2 - ((x+1)-2) + (2x)$ is equivalent to $x^2 + 3x + 2$.

Answers may vary. Expanding and collecting terms using the distributive property, when appropriate, produces $x^2 + 3x + 2$. A second way is to show how the area model justifying the relationship looks like the following:





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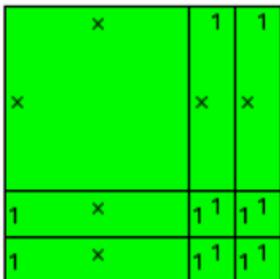


Student Activity Questions—Activity 2 (continued)

3. Answer each of the following. Give a reason for your thinking.

a. Sketch the diagram that would represent $(x + 2)(x + 2)$.

Answer: See diagram below. It needs to have an x and two 1s for each dimension.



b. Use the sketch in a to help you decide which of the following would be equivalent to x .

i. $x^2 + 4$

ii. $x^2 + 2x + 4$

iii. $x^2 + 4x + 4$

Answer: iii is equivalent because you can break the figure into a square, x^2 , a block of 4 rectangles or $4x$, and a small square that would be $2(2)$.

c. Use what you know about the distributive property to help you decide which of the following will be equivalent expressions. Check your thinking using the sketch.

i. $(x + 1)^2 + 1(x + 2) + 1(x + 1)$

ii. $x(x + 2) + 2(x + 2)$

iii. $(x + 2) + (x + 1) - 1(x + 1) + 2x + 3$

iv. $(x)^2 + x(x + 2) + 1(x + 4)$

Answer: the first three are equivalent because they are all equal to $(x + 2)^2$



Deeper Dive

Go to page 1.3. An identity is a statement that two expressions are equal.

- Write an equivalent expression for $(x + 1)^2 - x^2$ and state the relationship as an identity.

Answer: $(x + 1)^2 - x^2 = 2x + 1$

- Substitute 10 for x and write the result.

Answer: $11^2 - 10^2 = 21$ or $121 - 100 = 21$

- Try three more whole numbers for x .

Answers will vary.

Carley announced that all of her answers for the question above were odd and she thinks that she will always get an odd number. Do you agree with her? Why or why not?

Answer: Carley is correct. If x is a whole number, $2x + 1$ will always be odd because $2x$ is even, so adding 1 to an even number will make it odd.



Deeper Dive (continued)

Use your work in the first set of questions above to write each of the following as the difference in squares.

- 19
- 23
- 99

Answer: $19 = 10^2 - 9^2 = 100 - 81$

Answer: $23 = 12^2 - 11^2 = 144 - 121$

Answer: $99 = 50^2 - 49^2 = 2,500 - 2,401$

Give an argument that justifies the following statements for any whole number x :

- Every odd number can be expressed as the difference of the squares of two consecutive numbers.
- $(x + 1)^2 - x^2 - 1$ will be even.

Answers may vary. Every odd number can be expressed as $2x + 1$, where x is a whole number. Two consecutive integers would be x and $x + 1$. The difference of the squares of these integers would be $(x + 1)^2$, which is equivalent to $2x + 1$, so every odd number can be written as the difference of the squares of two consecutive integers.

Answer: $(x + 1)^2 - x^2 = 2x + 1$. Add -1 to both sides and you have $(x + 1)^2 - x^2 - 1 = 2x$. For any whole number x , $2x$ is even.

Suppose you have the following:



Step 1 Step 2 Step 3

- Assuming the pattern for the dots continues in the same way, write the number of dots in each of the steps including the number of dots in Steps 4 and 5.
- The expression describing the figure on page 1.5 is $(x + 1)(x + 2)$. Let x equal 0, 1, 2, 3, and 4 and find the value of the expression for each.
- How are your answers to the two previous questions related?

Answer: 1, 3, 6, 10, 15, ...

Answer: 2, 6, 12, 20, 30, ...

Answer: The numbers in the answer above are twice the numbers in the answer before it.



Deeper Dive (continued)

- ***The numbers in the first set of answers are called Triangular Numbers. Find an explicit rule for the n th triangular number. How does this explain your answer for the question above?***

Answer may vary. This is the classic problem of finding the sum of the first n whole numbers and can be answered many different ways: One way is straighten the dots to form a right triangle, complete the shape to get a rectangular array of dots of dimensions n by $n+1$, then divide by 2 because the diagonal has been double counted.

This gives the rule as $\frac{n(n+1)}{2}$. If n is $x+1$, then

$\frac{n(n+1)}{2}$ becomes $\frac{(x+1)(x+2)}{2}$, half of the original expression.



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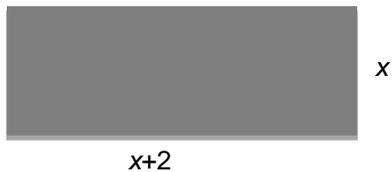
Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which of the following expressions are equivalent to $(x+4)^2$?
 - a. $x^2 + 16$
 - b. $x^2 + 4x + 16$
 - c. $(x+2)^2 + 4(x+3)$
 - d. $(x+5)^2 - 4(x+1)^2$

Answer: c. $(x+2)^2 + 4(x+3)$

2. What is the area of this rectangle?



- a. $x^2 + 2$
- b. $x^2 + 2x$
- c. $2x + 2$
- d. $4x + 4$

Answer b. $x^2 + 2x$

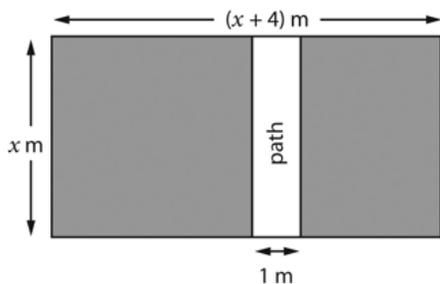
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3.



This is a diagram of a rectangular garden. The white area is a rectangular path that is 1 meter wide. Which expression shows the area of the shaded portion of the garden in m^2 ?

- a. $x^2 + 3x$
- b. $x^2 + 4x$
- c. $x^2 + 4x - 1$
- d. $x^2 + 3x - 1$

TImSS ID: M052173 Grade 8 2011

Answer: a. $x^2 + 3x$

4. Identify which of the expressions are equivalent.

- a. $(x + 4)(x - 2)$ and $x^2 - 8$
- b. $(x + 4)(x - 2)$ and $x^2 - 6x - 8$
- c. $(x - 4)(x + 2)$ and $x(x + 2) - 4(x + 2)$
- d. $(x - 4)(x + 2)$ and $x^2 - 2x - 8$

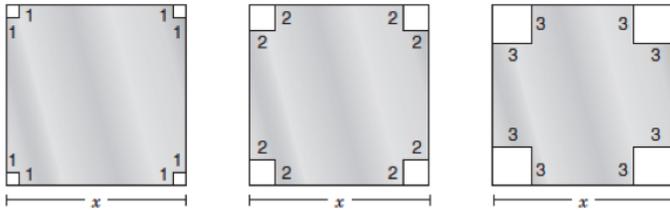
Answer: c. $(x - 4)(x + 2)$ and $x(x + 2) - 4(x + 2)$ and d. $(x - 4)(x + 2)$ and $x^2 - 2x - 8$.



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5. Four squares are cut from the corners of a square sheet of metal. As the size of the small squares increases, the remaining area decreases, as shown below.



If this pattern continues, what will be the difference between the first square's shaded area and the fifth square's shaded area?

- a. 4 square inches
- b. 24 square inches
- c. 49 square inches
- d. 96 square inches

TAKS 2006, Grade 11

Answer: d. 96 square inches



Building Concepts: Visualizing Quadratic Expressions

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Student Activity Solutions

In these activities you will use geometric figures to explore quadratic expressions. After completing the activities, discuss and/or present your findings to the rest of the class.



Activity 1 [Page 1.3]

1. Reset. Write an expression for the total area of the shapes using the product of the width and height for each area.

- a. Color the horizontal rectangle and the small square green.

Answer: $(x+1)x + (x+1)1$

- b. Choose blue and color the vertical rectangle.

Answer: $x^2 + 1(x) + (x+1)1$

- c. Color the horizontal rectangle blue.

Answer: $x^2 + 2(x) + 1(1)$

2. a. How are the total areas described in each problem so far related? Explain your reasoning.

Answer: They are the same because they all are made of the same shapes displayed in different ways. Each arrangement would have the same total area as the original figure.

- b. How are the expressions related to the total areas for each problem related? Use the distributive property to support your answer.

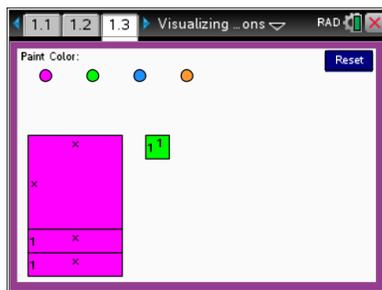
Answer: They are equivalent. By using the distributive property to expand and collect terms, each of these is equivalent to $x^2 + 2x + 1$. You may want different students to show exactly how each is equivalent to the expression $x^2 + 2x + 1$.

3. Find an arrangement for the shapes that would illustrate each of the following expressions. Sketch your arrangement.

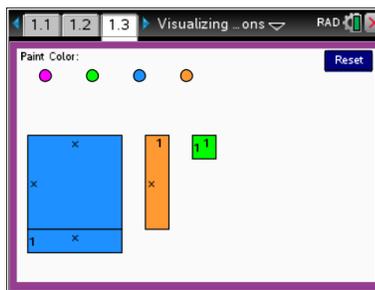
- a. $x(x+2) + 1(1)$

- b. $x(x+1) + 1(1)$

Answer:



a.



b.



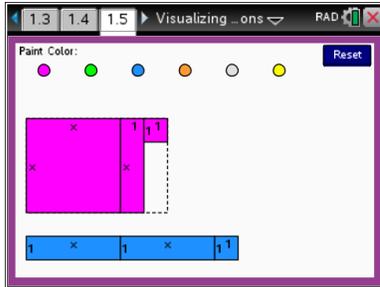
Building Concepts: Visualizing Quadratic Expressions

TEACHER NOTES



Activity 2 [Page 1.5]

1. Use the diagram below to decide whether statements a–d are true or false, then answer e. Explain your thinking in each case.



- a. The area of the long rectangle can be represented by $1(x+1)$.

Answer: False, because the width of the rectangle is $2x+1$.

- b. $x(x+2)$ represents the area of the rectangle enclosing the irregular shape.

Answer: True, the width is $x+2$ and the height is x .

- c. The area represented by $(x+2)(x)$ is the same as the area represented by $x(x+2)$.

Answer: True, by the commutative property; one is the width times the height, the other is the height times the width and both will give the same area.

- d. The total area represented by the two shapes will be $x(x+1) + (2x+1)(1)$.

Answer: False, because the area of the irregular shape will be $x(x+2) - (x-1)$ so the total area will be $x(x+2) - (x-1) + (2x+1)(1)$.

- e. Show that the total area represented by the two shapes can be expressed as $(x+1)(x+2)$.

Answer: By distributing, $x(x+2) - (x-1) + (2x+1)(1)$ becomes $x(x) + 2x - 1x + 1 + 2x + 1$.

Collecting like terms gives $x^2 + 3x + 2$ which factors to become $(x+1)(x+2)$.



Building Concepts: Visualizing Quadratic Expressions

TEACHER NOTES

2. Answer each of the following questions.

- a. Sally said that no matter how you rearrange the squares and rectangles in the original figure on page 1.5, the corresponding expressions will always be equivalent to the original product. Do you agree with Sally? Why or why not?

Answer: Sally is right because the total area is conserved or stays the same no matter where it is located or how the shapes are combined (as long as the shapes don't overlap).

- b. Find at least two ways to explain why $(x+2)(x+1) \neq x^2 + 2$.

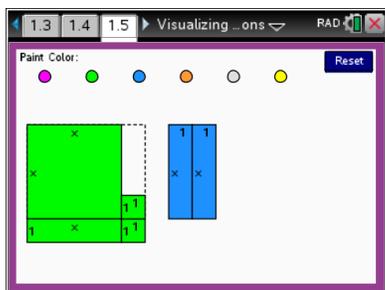
Answers may vary. One explanation is: the area model for the product of $(x+2)(x+1)$ has an x^2 and a 2, but also contains more parts, so the product must be greater than just the sum of x^2 and 2; another might be: when you use "each times every" to expand the multiplication you get $x^2 + 2x + 1x + 2$ which is not equal to $x^2 + 2$.

- c. Color all of the non-square rectangles and one small square blue. Explain why the area represented by the blue shapes could be the expression $3(x+1) - 2$ or the expression $3x + 1$.

Answers may vary. The width of the blue region is 3 and the height is $x + 1$, so the area of the rectangle enclosing the irregular shape with dotted lines is $3(x + 1)$. But the extra area has width 2 and height 1, so you have to subtract $2(1)$ to get $3(x + 1) - 2$. If you colored the small square green, you could write the area represented by the three rectangles as $3x$ and then add the small square to have $3x + 1$.

- d. Find at least two ways to explain why $(x+1)^2 - ((x+1) - 2) + (2x)$ is equivalent to $x^2 + 3x + 2$.

Answers may vary. Expanding and collecting terms using the distributive property, when appropriate, produces $x^2 + 3x + 2$. A second way is to show how the area model justifying the relationship looks like the following:



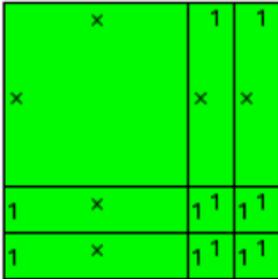


Building Concepts: Visualizing Quadratic Expressions

TEACHER NOTES

3. Answer each of the following. Give a reason for your thinking.
- a. Sketch the diagram that would represent $(x+2)(x+2)$.

Answer: See diagram below. It needs to have an x and two 1 s for each dimension.



- b. Use the sketch in a to help you decide which of the following would be equivalent to x .
- i. $x^2 + 4$ ii. $x^2 + 2x + 4$ iii. $x^2 + 4x + 4$

Answer: iii is equivalent because you can break the figure into a square, x^2 , a block of 4 rectangles or $4x$, and a small square that would be $2(2)$

- c. Use what you know about the distributive property to help you decide which of the following will be equivalent expressions. Check your thinking using the sketch.
- i. $(x+1)^2 + 1(x+2) + 1(x+1)$
- ii. $x(x+2) + 2(x+2)$
- iii. $(x+2) + (x+1) - 1(x+1) + 2x + 3$
- iv. $(x)^2 + x(x+2) + 1(x+4)$

Answer: the first three are equivalent because they are all equal to $(x+2)^2$.