Building Concepts: Visualizing Equations Using Mobiles

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students explore interactive mobiles for modeling one or more equations. Students are given specific constraints and use solution preserving moves to reinforce the understanding of how equations work.

Mobiles can be used as a visual/physical model to leverage student reasoning about an equation involving two variable expressions in terms of balance between weights on the mobiles.

Learning Goals

1. Write an algebraic expression, using variables, that represents the relationship among the weights in a mobile;
2. associate moves that preserve balance in a mobile to algebraic moves that preserve the equivalence of two expressions;
3. find solutions to equations that can be converted to the form $ax = b$ and $x + a = c$.

Prerequisite Knowledge

Visualizing Equations Using Mobiles is the seventh lesson in a series of lessons that explore the concepts of expressions and equations. In this lesson students use interactive mobiles to explore equations. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *Equations and Operations* and *Using Structure to Solve Equations*. Students should understand:

- the associative and commutative properties of addition and multiplication;
- how to associate an addition problem with a related subtraction problem and a multiplication problem with a related division problem.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **equation:** a statement in which two expressions are equal
- **variable:** a letter that represents a number in an expression
- **solution:** a number that makes the equation true when substituted for the variable
- **identity:** when the expression on the left is equivalent to the expression on the right no matter what number the variables represent

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

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Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Visualizing Equations Using Mobiles_Student.pdf
- Visualizing Equations Using Mobiles_Student.doc
- Visualizing Equations Using Mobiles.tns
- Visualizing Equations Using Mobiles_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

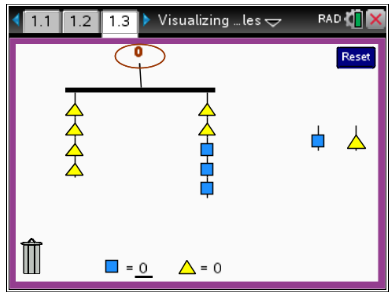
Lesson 5, *Equations and Operations,* focused on “solution preserving moves”, operations that can be performed on an equation such that the new equation has the same solution set as the original equation. Lesson 6, *Using Structure to Solve Equations*, extended the strategies students might use to solve an equation of the form $ax+b=c$ or $c=ax+b$, in particular emphasizing how to reason about the structure of an equation and of the expressions on either side of the equation as a way to think about the equation as one that can be solved easily. This lesson combines the two earlier strategies by using a mobile to represent one or more equations. Given a mobile with two arms, each containing a number of shapes, students experiment with different weights for the shapes that will produce a target weight for a balanced mobile. The relationship among the shapes on the mobiles can be represented algebraically; for example, if three triangles on one arm must balance a triangle and a circle on the other arm where each of the arms must have the same total weight, 18 in order to balance. The relationship among the shapes on the mobile can be mapped to the equations $3T=T+C$ and $3T=18=T+C$ (leading to the solution, $C=12$ and $T=6$).

After students become familiar with reasoning about the mobiles and how different configurations can be made to balance, they move to a mobile for which some of the values are specified and try to find values for the other shapes that will maintain the balance. In this phase, students revisit the solution-preserving moves developed in *Equations and Operations* in the context of the mobile, adding or taking away the same shape from both sides to keep the mobile balanced. They also experiment with a mobile that has multiple shapes on each arm. These can be formally visualized as equations of the form $\left(ax\right)+b=c$, which can be solved by the highlight method from *Using Structure to Solve Equations*. Reasoning about the structure of the mobile and how to keep it balanced allows students to solve equations of the form, where the expressions on the sides of the equation map to the arms of the mobile.

Part 1, Page 1.3

Focus: Identifying the relationship among shapes on a mobile can be used to determine weights that will balance the arms of a mobile.

Weights can be assigned to the shapes on page 1.3 by entering a value in the blank next to the shape or using tab to highlight the blank. The circle at the top shows the total weight of the shapes on the mobile.



Set Mobile Values> shapes saves the current configuration of the mobile including weights of the shapes.

Set Mobile Values> reload shapes reloads the last configuration saved by set mobile values> shapes.

Clear Mobile clears all shapes from the mobile.

Tab Key chooses whether tab cycles through the shapes or the blanks.

Reset returns to original mobile.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the shapes or the blanks.

Up/Down arrows move the tab between shapes and blanks.

Right/Left arrows move highlighted shapes among the arms and to the trash.

ctrl del resets the page.

Class Discussion

In the following questions, students investigate how the mobile works. Students should be encouraged to try different values for the weights to get a sense of how the mobile behaves.

On page 1.3, the shapes on the mobiles each have weight 0.

- ***Why is the mobile balanced?***

  Answer: Because the weights on both arms of the mobiles total 0.

- ***If the triangles each had weight 10 and the square had weight 5, make a conjecture about how the mobile would change. Then fill in the blanks with those values to check your conjecture.***

  Answers may vary. The arm on the left with the four triangles will be lower than the arm on the right because the left arm would have weight 40 and the right arm weight 35.

- ***Explain what the value in the circle at the top of the mobile means.***

  Answer: The value is 75, which comes from six triangles each weighing 10 and three squares each weighing 5 or $60+15=75$.

Class Discussion (continued)

• ***Find a weight for the triangle such that when the weight of a square is five, the total weight of the left arm is less than the total weight of the right arm.***	Answers will vary. Any positive integer less than 7 works.

Reset.

• ***Make the weight of the square 8. Try to find some value for the weight of the triangle that makes the mobile balance. Is there more than one value that works? Why or why not?***	Answers may vary. Triangle has to be 12 because any weight less than that will make the left side too light and any weight more than that makes the right side too light.
• ***Reset. Make the weight of the triangle 21. Explain how the value of the number at the top of the mobile is calculated.***	Answer: The value is 126 because the mobile has six triangles, each worth 21, and the squares do not weigh anything yet.
• ***Try to find a weight for the square that makes the mobile balance. Is there more than one answer? Why or why not?***	Answers may vary. The square has to be 14 because the weight of the four triangles on the left arm is 84. The two arms have to balance so the total weight on the right arm is 84 as well and it has two triangles or 42, so the three squares have to add to 42 to get 84. That makes each square 14 and it is the only number that will work.

The next few questions ask students to find the values of the shapes that will keep the mobile balanced as well as reach a certain target weight for all of the shapes.

Reset.

Suppose you wanted the value in the oval to be 72.

• ***Find weights for the square and the triangle that will achieve that goal. Explain your reasoning.***	Answer: The weight of the triangle is 9, and the weight of the square is 6. The two arms have to weigh the same, which will be half of 72 or 36. So the triangle has to weigh 9 because four triangles make 36. This makes the weight of the two triangles on the other arm 18, but the total weight of that arm has to be 36 as well, so the weight of the three squares is 18, and one square has a weight of 6.
• ***Make a conjecture about the weights of the square and triangle that will achieve a goal of 144. Check your conjecture using the mobile.***	Answer: 144 is twice 72, so if the triangle and square each weigh twice as much, the mobile should balance. This makes the triangle weigh 18 and the square 12.

Class Discussion (continued)

- ***Tamara wrote the following as a shorthand record of the mobile: $4T = 2T + 3S$, where T was the weight of the triangle and S the weight of the square. Do you agree or disagree with what she wrote?***

  Answer: She is correct because the two arms of the mobile have to balance each other and because 4T is the same as $T + T + T + T$; 2T is the same as $T + T$; and 3S is the same as $S + S + S$.

> **Teacher Tip:** Have students share their reasoning; some may reason about the numerical value for each expression and how they must be the same, while others might reason by identifying numbers that make the mobile unbalanced and in which direction. Students connect the mobile and its shapes to symbolic representations of expressions and equations. This question is critical in making this connection; be sure that students understand that to find the total weight on one arm you would add the weights of the shapes on that arm and that 3T is the same as $T + T + T$.

- ***Heli disagreed with Tamara and wrote: $4T = 36$ and $2T + 3S = 36$. What would you say to him?***

  Answer: Tamara wrote an equation that describes the whole mobile. Heli is describing the weight of each arm when the total weight is 72 since each arm weighs 36.

Students might make a connection to proportional relationships in the next question, if they reduce the mobile to an expression of the form $a = kb$ for shapes with weights *a* and *b* with *k* as the constant of proportionality. Generating a set of equivalent ratios that satisfy this relationship can produce a solution to the original problem (i.e., $na + nkb$ has the desired total value). This approach might become a typical strategy for some as they work through these tasks and those on the ensuing pages.

Reset. You can create a new mobile by dragging the triangles and squares at the right to the arms of the mobile. Create a mobile whose left arm can be represented by the expression $2T + 2S$ and whose right arm by the expression $4S + T$.

- ***Find values for the weights of the triangle and square so the total weight of the mobile is 48. Be ready to explain your thinking.***

  Answer: Triangle weighs 8; square weighs 4. Each arm has to weigh 24 so a triangle and a square together weigh 12 because $2T + 2S = 24$. So a square and a triangle on the other arm is 12, but the whole arm is 24, and that makes $3S = 12$ so $S = 4$.

- **Suppose the total weight is 24. Make a conjecture about weights of the triangle and square that would make the mobile balance. Check your conjecture using the mobile.**

  Answer: The weight of the triangle is 4, of the square 2. Because 24 is half of 48, taking half of the weight of the triangle and half of the weight of the square should make a balanced mobile with a total weight of 24.

Class Discussion (continued)

• ***Find at least two other pairs of values for the weights of the triangle and square that make the arms balance.***	Answers may vary. Any pair where the weight of triangles is twice the number of squares will work.
• ***Compare your answers to the question above with your classmates. Write an equation using S and T that could be used to generate all of your answers.***	Answer: $2S = T$
• ***Create a mobile and assign a total weight. Give your mobile to your partner to solve. (Be sure you know the answer.)***	Answers will vary. Students should be sure the mobile they create has a solution. Students might want to share their mobiles with the class.

Student Activity Questions—Activity 1

1. Create a mobile with three triangles and one square on the left arm and one triangle and two squares on the right arm.

a. If the total weight goal is 10, find the weight of a triangle and the weight of a square.

Answer: A triangle has a weight of 1, and a square has a weight of 2.

b. If you double the number of triangles and squares on each side of the mobile, what is the new total weight?

Answer: 20

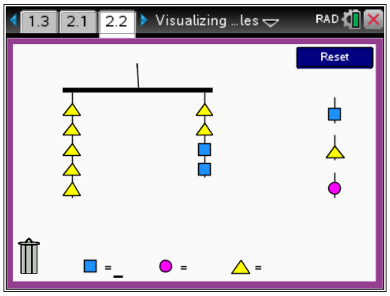
c. Which of the following equations could be associated with the original mobile? Explain your thinking in each case.

i. $3T + S = T + 2S$ ii. $3TS = 2TS$ iii. $T + T + T + S = T + S + S$

Answer: Both i and iii can be associated with a mobile having three triangles and a square on one arm and a triangle and two squares on the other arm because writing something like 3T is the same as adding T three times. ii is not correct because it looks like multiplication and if you multiplied 3(1)(2) and 2(1)(2) the values would be different and the mobile would not balance.

Part 2, Page 2.2

Focus: Mapping a balanced mobile to an equation provides a foundation for reasoning about strategies for solving equations, including the highlight method and solution preserving operations on equations.



The mobiles on page 2.2 are set up to model finding a solution for an equation. In these questions, students enter given values for one or two shapes, then submit their choices. The TNS activity assigns a value to the missing shape.

The commands related to the shapes function in the same way as those on page 1.3.

Submit submits the entered weight(s) and create a mystery weight for the other shape.

Check shows the original mobile when a correct weight is entered for the missing shape.

Selecting a shape in the mobile a shape shows the assigned weight for each shape.

Class Discussion

Note that the mobile on page 2.2 does not have an oval at the top of the mobile for the total weight of the two arms. The problems for this page have to do with a "mystery" weight that you have to find given information about the weights of some of the shapes.

- ***Assign the weight of 6 to the square and Submit. Note that the mobile is balanced after you submit. Is the weight of the triangle 2? Why or why not?***

  Answer: When the weight of the triangle is 2, the right arm is heavier, which means the triangle has to weigh more than 2.

- ***Try several weights for the triangle. What weight for the triangle will make the mobile balance? Explain your reasoning. (Note that you can add shapes or take them off of an arm.)***

  Answer: The triangle has to have weight 4 because anything more than 4 makes the left side too heavy and anything less than 4 makes it too light.

- ***Saundra said she made an easier mobile by taking two triangles off of each arm. How can this help her reason about the weight of the triangle that will make the mobile balance?***

  Answer: Since the three triangles on the left arm are balanced with the two squares on the right arm, the weight of three triangles has to be 12. If three triangles have a weight of 12, the weight of one triangle is 4.

- ***Try Saundra's method, then use the check button and do the arithmetic to make sure the weights on both sides of the original mobile are the same. (Note that selecting each of the shapes on the mobile shows the weight of the shape.)***

  Answer: The original mobile has 5(4) on the left arm and $2(4)+2(6)$ on the right arm, which makes 20 on both arms. So choosing 4 for the weight of the triangle makes the original mobile balance.

Class Discussion (continued)

Reset.

- ***Form an equation by writing an expression for each arm of the mobile.***

  Answer: $5T = 2T + 2S$

- ***Assign the triangle a weight of 30, then Submit. Use Saundra's method to figure out what weight for the square will balance the mobile. Be ready to explain your reasoning.***

  Answer: The value of the square is 45 because taking two triangles from each side leaves three triangles worth 90 on the left arm and two squares on the right arm, so the value of each square has to be 45.

- ***Write an expression for each arm of the mobile after you used Saundra's method in the question above. What equation do these represent?***

  Answer: An expression for the left arm is 3T or 90; for the right arm 2S. The equation to balance the mobile will be $90 = 2S$.

- ***Petra found the weight for the triangle by writing $150 = 60 + 2T$ and then thinking what would I add to 60 to get 150. That would be 90, so two triangles have to weigh 90 and so each triangle will weigh 45. What do you think of Petra's strategy?***

  Answer: Petra is correct; she is reasoning about the structure of the addition problem using the "chunking" or "highlight" method.

Reset. Shaq submitted 30 as the weight for the square instead of the triangle.

- ***Find a weight for the triangle that will make Shaq's mobile balance. Explain your thinking.***

  Answer: A weight of 20 for the triangle will balance Shaq's mobile. Explanations may vary. Some may take two triangles off each arm and consider what weight for the three remaining triangles on the left arm would make 60 (a highlight method). Others might try numbers until they find a weight that works.

- ***Visually seeing the mobile balance suggests you have found the right value for the missing weight. How else can you know you have the correct weight? Explain how this would work for your answer to the question above.***

  Answer: Do the arithmetic with the weights for your original mobile: Using the weight of the triangle as 20 and of the square 30, you get $20 + 20 + 20 + 20 + 20 = 20 + 20 + 30 + 30$ or $100 = 100$.

- ***Write an equation that could represent Shaq's original mobile. Explain your reasoning.***

  Answer: $5T = 2T + 2(30)$ or $5T = 2T + 60$ because each square weighs 30 so two squares would weigh 60, and the mobile has 5 triangles on the left arm and 2 triangles and the two squares on the right arm.

Class Discussion (continued)

• ***Some equations have the same solutions. Which of the following equations have the same solutions as your answer in the question above?*** ***i.*** $4T = T + 60$ ***ii.*** $2T + 60 = 5T$ ***iii.*** $3T = 60$	Answers may vary: Some students might suggest that all three have the same solutions because i) and iii) were obtained by solution preserving moves of taking the same number of triangles from each side and that ii) is the same mobile but with the weights on the arms reversed. Others might note that $T = 20$ makes all three equations true.
Reset. Timon submitted the weight of the square as 12.Then he moved both of the squares from the right arm and three of the triangles from the left arm into the trash.	
• ***Write an equation that describes the new mobile.***	Answer: $2T = 2T$
• ***Try a weight for T. Describe what happened to the mobile. Then check.***	Answer: The mobile stayed balanced until the check, when it became unbalanced.
• ***Try several more weights for T. What do you notice?***	Answer: All the weights balance the mobile when it has two triangles on each side but never balance the original mobile that has both triangles and squares. When the squares are removed, you are not counting the weight of the square as 12.
• ***The equation you wrote for the first question is an identity, which happens when the expression on the left is equivalent to the expression on the right no matter what number the variables represent. Was Timon's original mobile an identity? Why or why not?***	Answer: His original mobile was not an identity because when he assigned the weight of 12 to the square there is only one weight for the triangle that will balance the original. The only weight for the triangle when the square weighs 12 is 8. To be an identity all the numbers have to work.
Reset. Add two circles to the left arm.	
• ***Write an equation representing the mobile.***	Answer: $2C + 5T = 2T + 2S$, where C is the weight of a circle, T the weight of a triangle and S the weight of a square.
• ***Give the circle a weight of 15 and the square a weight of 45 and Submit. Write a new equation representing the mobile with those values.***	Answer: $2(15) + 5T = 2(45) + 2T$ which yields $30 + 5T = 90 + 2T$

Class Discussion (continued)

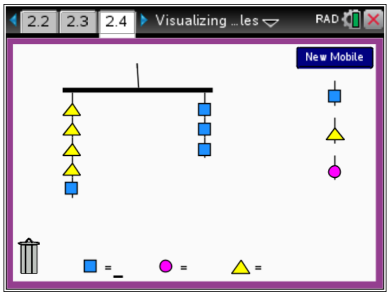
- ***What can you remove from both arms to keep the mobile balanced? Write the new equation.***

  Answers will vary: You can take one or two triangles from each side to get either $30+4T=90+T$ or $30+3T=90$. Other answers might be possible.

- ***Find the value of 3T. Use your work in Lesson 6,* Using Structure to Solve Equations *to explain your reasoning.***

  Answer: By thinking about what you add to 30 to get 90, you have $3T=60$, then what is multiplied by 3 to make 60 means $T=20$.

Part 2, Page 2.4



Focus: Mapping a balanced mobile to an equation provides a foundation for reasoning about strategies for solving equations, including the highlight method and solution preserving operations on equations.

Page 2.4 displays randomly generated mobiles. The commands related to the shapes function in the same way as those on page 1.3.

New Mobile will generate mobiles with different shapes and numbers of shapes.

A weight can be submitted for a shape; the commands function in the same way as those on page 2.2.

Student Activity Questions—Activity 2

1. Generate a new mobile.

a. Write down an equation for your mobile.

Answers vary due to random generation.

b. Assign a weight to one of the shapes. Can you find the weight of the other shapes in your mobile? Write down the solution if you can.

Answers vary due to random generation.

c. Generate a new mobile. Repeat steps a and b. Do this until you have at least two mobiles for which you can find the weight of the other shape(s).

Answers vary due to random generation.

d. Exchange your equations for your two mobiles with a partner. Then go to page 2.4 and build the mobiles. Find the missing weight(s). Check with your partner to see if your answers agree.

Answers will vary.

Student Activity Questions—Activity 2 (continued)

2. **Go to page 2.5. Work with a partner to create a mobile that models each situation using *at least* one triangle, one circle and one square. Assign weights to give the desired outcome. Be sure you check your work using your mobile.**

   a. **Determine weights for the square and circle that makes the triangle have to have a weight of 16.**

   Answers will vary. An example: a mobile that has a triangle and a circle with weight 8 on one arm and a square that has weight 24 on the other, that is, $T+8=24$. Another example: a mobile that has a triangle on one arm and a circle of weight 4 and a square of weight 12 on the other, that is $T=12+4$.

   b. **Assign weights to the triangle and the circle so that no whole number weight for the square will make your mobile balance.**

   Answers will vary. An example: $2T+C=2S$ where $T=5$, $C=9$. 2S would have to be 19 and S would be 9.5, which is not a whole number.

   c. **Assign weights to the square and to the triangle so than any weight for the circle will make the mobile balance.**

   Answers will vary. A square, two triangles and a circle on the left arm and two squares and a circle on the right arm will produce an identity where any weight for the circle can be used if the square has weight 10 and the triangle has weight 5.

   d. **Assign weights to the square and the triangle so that the only weight for the circle is 0.**

   Answers will vary. An example would be $2T=S+C$, where $T=2$ and $S=4$, which would map to the equation $4=4+C$.

3. **Suppose you have a balanced mobile. Decide which move will always keep the mobile balanced. Explain your reasoning in each case.**

   a. **removing the same shape from both arms**

   b. **removing the same number of shapes from each arm, regardless of what type they are**

   c. **adding an additional shape to both arms**

   d. **interchanging the shapes on the two arms**

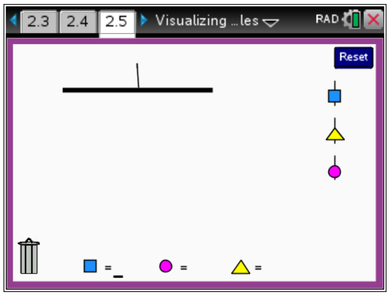
   e. **doubling the numbers of shapes of each kind on each arm**

   f. **adding the same shape to both arms**

   Answer: a, d, e, and f will keep the mobile balanced as those moves are solution preserving moves and will produce equations that have the same solutions. The other moves are not solution preserving because they do not specify that the same shape must be added or removed from both arms of the mobile to preserve the balance.

Part 3, Page 2.5

Focus: Thinking about the relationships among variables in a contextual problem using the mobiles as a model can provide a way to reason about a solution.



Class Discussion

Have students…	Look for/Listen for…
Use page 2.5 to make mobiles to model each of the following situations. Write down a description of your mobile, then answer the question.	
Markus bought 4 tulips for his mother. Tina bought 3 tulips and 2 sunflowers. They both spent the same amount, and together they spent $16.	
• ***Describe a mobile that would represent the problem.***	Answers may vary: Using page 1.3, the mobile could be one whose arms satisfy $4T = 3T + 2S$.
• ***How much did a tulip cost?***	Answer: A tulip costs $2 and a sunflower $1 because when you take 3 tulips from each side, you are left with $T = 2$.
At a different market, Jack bought 5 bouquets of tulips and a bouquet of carnations for the same amount of money that Suki paid for 3 bouquets of tulips and 4 bouquets of carnations.	
• ***Set up a mobile to model this situation.***	Answer: $5T + C = 3T + 4C$
• ***Can you find the cost of a bouquet of tulips or the cost of a bouquet of carnations? Why or why not?***	Answer: you need more information to figure out the cost of either bouquet.
• ***Find some prices that satisfy the relationship.***	Answer may vary: This will work when $T = 6$ and $C = 4$; $T = 9$ and $C = 6$; $T = 3$ and $C = 2$. In other words, whenever the cost of two bouquets of tulips is the same as the cost of three bouquets of carnations.
• ***If the carnations cost $4, how much did the tulips cost?***	Answer: Tulips cost $6 a bouquet.

Student Activity Questions—Activity 3

1. **Al and Bert lived in a large urban area and had to use some kind of transportation to get to their workplace.**

   a. **Bert found that it took the same amount of time for two shuttles on the weekend as for the total time it took to make the trip by train five times. If the shuttle trip takes 70 minutes, how much time would it take Bert to make the commute by train?**

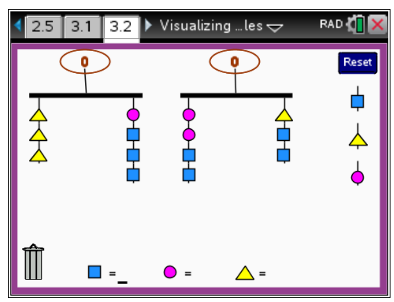
   Answer: $2S = 5T$, and if $S = 70$. $140 = 5T$, so $T = 28$. It would take Bert 28 minutes to commute by train.

   b. **One week they both worked seven days and spent the same total amount of time commuting. Al went by car every day, and Bert took the train five days and the shuttle the other two (the train did not work on the weekend). How long did it take Al to make the trip by car?**

   Answer: $7C = 5T + 2S$ would model the situation. From a), the trip by train was 28 and S was 70, so $7C = 5(28) + 2(70) = 280$. So C would be 40; it took Al 40 minutes to make the trip by car.

Part 4, Page 3.2

Focus: Thinking about the relationships among variables in a contextual problem using the mobiles as a model can provide a way to reason about a solution.



Page 3.2 shows two mobiles. The weight used for a shape in one of the mobiles will be the weight for that shape in the other mobile. The circle at the top of a mobile shows the total weight of the two arms for that mobile. The commands behave as they did on page 1.3.

Class Discussion

Teacher Tip: This section introduces relationships specified using at least three different constraints. The questions could be used as extensions or as a way to continue developing student reasoning about the relationships visible in mobiles and expressing what they see visually using symbols.

Have students…	Look for/Listen for…
Go to page 3.2.	
• ***What relationships do you see among the shapes in the two mobiles?***	Answers may vary. One relationship might be that a triangle has the same weight as two circles.

Class Discussion (continued)

- ***Suppose the total weight goal for the mobile on the left is 36 and for the mobile on the right is 32. What weights for the triangle, square and circle will make the two mobiles balance?***

Answer: Since $3T = C + 3S$ and $3T + C + 3S = 36$, $3T = 18$ which means $T = 6$. Since $T = 2C$, $C = 3$ which makes $S = 5$. Triangle should have weight 6, circle 3 and square 5.

- ***If you ignore the total weights for each mobile, what other weights for the shapes, if any, make the mobiles balance?***

Answers may vary. Any multiple of ($T = 6$, $S = 5$, $C = 3$) will work.

- ***Abbi claims that if the total weight on the left mobile is 144, she can figure out the total weight on the other mobile. Do you believe her? Why or why not?***

Answers may vary. Abbi is right because 144 is four times the original total weight, which was 36. So the total weight for the other mobile would be four times 32, or 128, and the weights would be triangle 24, square 20, and circle 12.

Reset. Think about the equations represented in the mobile assuming the original goal weights of 36 and 32.

- ***Find an equation in that mobile you think is "easy to solve". Explain why you think it is easy to solve.***

Answers may vary. One equation might be $3T = 18$.

- ***What other equations can be solved easily if you know the solution for the equation you gave for the question above?***

Answers may vary. Once you know the value of T, you could use the highlight method to solve $2S + T = 16$.

- ***Cody claims that once he knows the value of S, he can use the equation $S + C = 8$ to find the value of C. What would you say to Cody? Use the TNS activity to support your thinking.***

Answers may vary. Cody is correct. The equation $2S + 2C = 16$ has the same solutions as $T + C = 8$, so the weights that make the mobiles balance should be the same as the solutions for the equations.

Create two mobiles where the right mobile is $2S = 2C + 2T$, and the left mobile is $3C = S + 2T$.

- **What relationships can you see among the three shapes?**

Answers may vary. One response might be that the weight of one square is the same as the sum of the weights of a circle and a triangle.

Class Discussion (continued)

• ***Suppose the total weight on the left mobile is 36. Find the total weight on the right mobile and the weights of each of the shapes. Explain your reasoning.***	Answers may vary. (S, C, T) = (10, 6, 4). If the total weight of the mobile on the left is 36, each arm has weight 18, and three circles weigh 18 so the weight of a circle is 6. Since $2S = 2C + 2T$, then $S = C + T$ so you can replace the square by a triangle and a circle and then take a circle from each side. This means three triangles weigh the same as two circles so $3T = 12$ and $T = 4$. Two triangles and two circles will then weigh 20, so a triangle and a circle weigh 10, which is the same as a square. These weights make the total weight of the mobile on the right equal to 40.
• ***Write a set of equations that would describe how you figured out the weights of the shapes.***	Answers will vary. Some examples might be $3C = 18$; $3T = 12$, $T + C = S$.

Deeper Dive — Page 2.2

Go to page 2.2 and Reset. Make the square weighs 17.

• ***Is there a weight for the triangle that will balance the mobile?***	Answer: No whole number weight will make the mobile balance. When triangle weighs 11, the left arm is lighter; when the triangle weighs 12, the right arm is lighter.
• ***Reset, then change the weight of the square to 11. Is there a weight for the triangle that will balance the mobile?***	Answer: No whole number weight will make the mobile balance. It might balance with some number between 7 and 8.
• ***In general for what weights for the square will you be able to find a whole number of triangles to balance the mobile? Explain your reasoning.***	Answer: Any weight that is a multiple of three will create a mobile that will have a whole number for the weight of the triangle to balance the mobile. This is because you will always have three triangles that have to equal a number on the right. So to get a whole number for the weight of a triangle, the right side has to be divisible by 3. The weights must be ratio 2:3 (S:T)

Deeper Dive — Page 2.4

Go to page 2.4 and create a mobile that can be represented by the equation $2T = T + 3C$.

- ***If the circle has weight 18, what is the weight of the triangle?***

  Answer: The triangle would have weight $3(18) = 54$

- ***If the triangle has weight 27, what is the weight of the circle?***

  Answer: The circle would have weight 9.

- ***Suppose you doubled the number of triangles and circles on each side of the mobile. What would the new mobile look like?***

  Answer: The equation representing the new mobile would be $4T = 2T + 6C$.

- ***How would your answers to the first two questions change? Explain your reasoning.***

  Answer: The answers would stay the same because you are just multiplying both arms by 2, which is a solution-preserving move.

Deeper Dive — Page 2.5

The solution to $5a = 30$ and $2a + b = 38$ is also a solution for $7a + b = 68$. Why?

Answer: The sum of the first two equations is the third equation. The specific solution to the first two equations ($a = 6$ and $b = 26$) also makes the $7a + b = 68$. But many other solutions exist for $7a + b = 68$ ($a = 0$, $b = 68$ for example) and no other values for a and b will make both of the first two equations true. Adding the two equations preserves the solution for the first two but introduces many other possibilities for making the result true.

Deeper Dive — Page 3.2

For each of the following, use all three shapes to create two balanced mobiles that satisfy the stated conditions. Give your problem to a partner to find the missing weights for the shapes. (Be sure you can find the weights of the shapes from your mobile first.)

- ***The triangle weighs twice as much as a circle, and the square weighs five times as much as a circle. The total weight has to be 30 for one of the mobiles.***

  Answers may vary. One example is having mobiles $2C = T$ and $5C = S$. In this case $S = 30$, $C = 6$, and $T = 12$

  Another example is $C = 5$; $T = 10$, $S = 25$ with mobiles $2T + 2C + S = 5T + C$ and $3C = T + C$.

Deeper Dive — Page 3.2 (continued)

• ***The sum of the weights of a triangle and a square is 15. The weight of a circle is two more than the weight of a triangle and the total weight on one mobile is 30.***	Answers will vary: $(C=5, T=3, S=12)$ with conditions $2S+T=3C+4T$ and $3C=T+S$
• ***The triangle is twice the weight of a square, and the triangle is half the weight of the circle. Each arm on both mobiles is an odd number.***	Answers will vary. One might be $(S=5, T=10, \mathrm{C}=20)$ with conditions $2T+S=S+C$ and $3S=T+S$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. TJ could buy four magazines for $28 or two magazines and a book for $42. Which of the equations below model the situation?

   a. $4m + b = 70$

   b. $6m = 70$

   c. $4m = 28$ and $2m + b = 42$

   d. $m + b = 42$ and $4m = 28$

   Answer: c. $4m = 28$ and $2m + b = 42$

2. Four campers can cross the lake in one canoe. Six campers can go in one boat. If *c* is the number of canoes and *b* the number of boats, which expression models the number of campers that can go across the lake in *c* canoes and *b* boats?

   a. $6c + 4b$

   b. $6b + 4c$

   c. $b + c$

   d. $10cb$

   Answer: b. $6b + 4c$

3. If $3c + b = 2b$ and $b = 18$, what is *c*?

   Answer: c = 6

4. If $3c + b = 2b$ and $c = 32$, what is *b*?

   Answer: b = 96

5. If $2xy + y = 18$ and $y + z = 20$ and $y = 8$, what is $x + z$?

   Answer: x = 5 and z = 12, so x + z = 17.

Building Concepts: Visualizing Equations Using Mobiles

TEACHER NOTES

Student Activity Solutions

In these activities you will use interactive mobiles to explore equations. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Create a mobile with three triangles and one square on the left arm and one triangle and two squares on the right arm.

   a. If the total weight goal is 10, find the weight of a triangle and the weight of a square.

   Answer: A triangle has a weight of 1, and a square has a weight of 2.

   b. If you double the number of triangles and squares on each side of the mobile, what is the new total weight?

   Answer: 20

   c. Which of the following equations could be associated with the original mobile? Explain your thinking in each case.

   i. $3T + S = T + 2S$ ii. $3TS = 2TS$ iii. $T + T + T + S = T + S + S$

   Answer: Both i and iii can be associated with a mobile having three triangles and a square on one arm and a triangle and two squares on the other arm because writing something like 3T is the same as adding T three times. ii is not correct because it looks like multiplication and if you multiplied 3(1)(2) and 2(1)(2) the values would be different and the mobile would not balance.

Activity 2 [Page 2.4]

1. Generate a new mobile.

   a. Write down an equation for your mobile.

   Answers vary due to random generation.

   b. Assign a weight to one of the shapes. Can you find the weight of the other shapes in your mobile? Write down the solution if you can.

   Answers vary due to random generation.

   c. Generate a new mobile. Repeat steps a and b. Do this until you have at least two mobiles for which you can find the weight of the other shape(s).

   Answers vary due to random generation.

   d. Exchange your equations for your two mobiles with a partner. Then go to page 2.4 and build the mobiles. Find the missing weight(s). Check with your partner to see if your answers agree.

   Answers will vary.

2. Go to page 2.5. Work with a partner to create a mobile that models each situation using *at least* one triangle, one circle and one square. Assign weights to give the desired outcome. Be sure you check your work using your mobile.

   a. Determine weights for the square and circle that makes the triangle have to have a weight of 16.

   Answers will vary. An example: a mobile that has a triangle and a circle with weight 8 on one arm and a square that has weight 24 on the other, that is, $T+8=24$*. Another example: a mobile that has a triangle on one arm and a circle of weight 4 and a square of weight 12 on the other, that is* $T=12+4$*.*

   b. Assign weights to the triangle and the circle so that no whole number weight for the square will make your mobile balance.

   Answers will vary. An example: $2T+C=2S$ *where* $T=5$*,* $C=9$*. 2S would have to be 19 and S would be 9.5, which is not a whole number.*

   c. Assign weights to the square and to the triangle so than any weight for the circle will make the mobile balance.

   Answers will vary. A square, two triangles and a circle on the left arm and two squares and a circle on the right arm will produce an identity where any weight for the circle can be used if the square has weight 10 and the triangle has weight 5.

   d. Assign weights to the square and the triangle so that the only weight for the circle is 0.

   Answers will vary. An example would be $2T=S+C$*, where* $T=2$ *and* $S=4$*, which would map to the equation* $4=4+C$*.*

3. Suppose you have a balanced mobile. Decide which move will always keep the mobile balanced. Explain your reasoning in each case.

   a. removing the same shape from both arms

   b. removing the same number of shapes from each arm, regardless of what type they are

   c. adding an additional shape to both arms

   d. interchanging the shapes on the two arms

   e. doubling the numbers of shapes of each kind on each arm

   f. adding the same shape to both arms

   Answer: a, d, e, and f will keep the mobile balanced as those moves are solution preserving moves and will produce equations that have the same solutions. The other moves are not solution preserving because they do not specify that the same shape must be added or removed from both arms of the mobile to preserve the balance.

Activity 3 [Page 2.5]

1. Al and Bert lived in a large urban area and had to use some kind of transportation to get to their workplace.

   a. Bert found that it took the same amount of time for two shuttles on the weekend as for the total time it took to make the trip by train five times. If the shuttle trip takes 70 minutes, how much time would it take Bert to make the commute by train?

   Answer: $2S = 5T$ *, and if* $S = 70$. $140 = 5T$ *, so* $T = 28$*. It would take Bert 28 minutes to commute by train.*

   b. One week they both worked seven days and spent the same total amount of time commuting. Al went by car every day, and Bert took the train five days and the shuttle the other two (the train did not work on the weekend). How long did it take Al to make the trip by car?

   Answer: $7C = 5T + 2S$ *would model the situation. From a), the trip by train was 28 and S was 70, so* $7C = 5(28) + 2(70) = 280$*. So C would be 40; it took Al 40 minutes to make the trip by car.*