



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students use previous experience with proportional relationships of the form $y = kx$ to consider relationships of the form $y = mx$ and eventually $y = mx + b$.



Proportional relationships can be used to make connections to more general linear equations involving two variables.

Learning Goals

1. Understand that proportional relationships of the form $y = kx$ for positive x , y , and k extend naturally to linear relationships of the form $y = kx$ for real x , y , and k ;
2. connect the unit rate in a proportional relationship to the slope of its graph using similar triangles;
3. connect the equation for a proportional relationship $y = mx$ to the equation for a line $y = mx + b$;
4. understand that a proportional relationship between Δy and Δx holds for any line of the form $y = mx + b$;
5. identify slope and intercepts from the graph and the symbolic representation of a line.



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Prerequisite Knowledge

Moving From Proportional Relationships to Linear Equations is the fourteenth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students consider relationships of the form $y = mx$ and eventually $y = mx + b$.

Students should already be familiar with the properties of the graph of a proportional relationship of the form $y = kx$ (where x , y , and k are all positive): points (x, y) satisfying such a relationship all lie on a line in the coordinate plane, and this line always contains the origin $(0, 0)$. If a pair of horizontal and vertical line segments are drawn to connect any two such points, a right triangle is formed with hypotenuse lying along the graphed line. Students also should be familiar with the ideas of similar triangles, so that they can identify that all such triangles will be similar. This similar triangle property is one that is generalized in the move from proportional relationships to more general linear equations in two variables.

Students should understand:

- how to apply the distributive law and
- how to solve and justify the solution for a linear inequality in one variable.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Moving From Proportional Relationships to Linear Equations_Student.pdf
- Moving From Proportional Relationships to Linear Equations_Student.doc
- Moving From Proportional Relationships to Linear Equations.tns
- Moving From Proportional Relationships to Linear Equations _Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

Vocabulary

- **proportional relationship:** a relationship that describes two quantities that vary directly with one another
- **linear equation:** an algebraic equation that makes a line when it is graphed
- **intercept:** the coordinate at which lines and curves intersect a coordinate axis



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Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.



Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.



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Mathematical Background

In earlier work, students have seen proportional relationships of the form $y = kx$, where x and y are positive quantities and k is a positive proportionality constant. When the ordered pairs (x, y) that satisfy such a relationship are graphed in the coordinate plane, the resulting points are all in the first quadrant (since both x and y are positive) and all the points lie on a line that must pass through the origin. The constant k is identified with the graphical idea of *slope* of a line.

This lesson moves students first to consider relationships of the form $y = mx$ where x and y are no longer required to be positive, and the constant m also can be any real number. (Note: If m is nonzero, then such relationships are still referred to as proportional in some textbooks.) When solutions to these equations are graphed, the results are non-vertical lines that pass through the origin (the ordered pair $(x, y) = (0, 0)$ always satisfies the equation $y = mx$). When $m > 0$, the lines lie in the first and third quadrants of the coordinate plane; when $m < 0$, the lines lie in the second and fourth quadrants; when $m = 0$, the line lies along the x -axis. What all of these lines have in common is that they represent a relationship where

$$\frac{y}{x} = m \text{ for all nonzero } x \text{ and } y.$$

Another property that equations of the form $y = mx$ have is that between any two distinct solution points, the vertical change Δy (the vertical change in the y -coordinates) and the horizontal change Δx (the horizontal change in the x -coordinates) also satisfy $\Delta y = m\Delta x$.

If horizontal and vertical arrows are drawn to connect one point A on the graph of $y = mx$ to another point B on the same line, then a right triangle is formed with the arrows as legs and having its hypotenuse along the line. This picture affords a connection to geometry and the “proportional change” property can be seen as a consequence of properties of similar triangles.

This “proportional change” property is precisely the relationship that characterizes linear equations in two variables. A linear equation of the form $y = mx + b$ will have a graph that is a line that does not necessarily pass through the origin. The constant of proportional change is called the *slope*. Starting with any point (c, d) in the plane and a slope m , the lesson uses the property that $\frac{\Delta y}{\Delta x} = m$ to generate what is known as the point-slope form of a linear equation: $y - d = m(x - c)$ or $y = d + m(x - c)$.

Two other important forms of linear equations in two variables are special cases of the point slope form. The *slope-intercept* form of a linear equation identifies the y -intercept of the line (the point $(0, b)$ where the line crosses or intercepts the y -axis: $y = mx + b$).

The *x-intercept* form of a linear equation in two variables is the point-slope form using the point $(a, 0)$ where the line crosses or intercepts the x -axis: $y = m(x - a)$.



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Part 1, Page 1.3

Focus: The questions in this part connect students' previous understandings of proportional relationships to more general linear relationships of the form $y = mx$ where the variables x and y , and the slope m can be any real numbers.

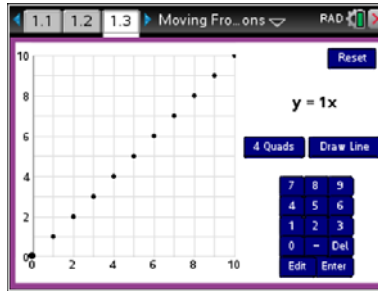
On page 1.3, the graph displays the lattice points in the first quadrant of the coordinate grid that satisfy the equation.

Draw Line displays the graph of the equation.

4 Quads will display points and/or line in four quadrants. **1 Quad** returns back to a display of only the first quadrant.

Edit or **menu > Slope** and the keypad to change the constant of proportionality.

Reset returns to the default display of $y = 1x$ in the first quadrant.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through numerator, denominator, and the buttons.

ctrl del resets the page.



Class Discussion

Have students...

Open page 1.3.

- **How are the coordinates of the points plotted related to the equation $y = 1x$ that is displayed?**
- **Remember your work with proportional relationships. Why would this equation be a proportional relationship between y and x ?**

Look for/Listen for...

Answer: The points are $(0, 0)$, $(1, 1)$, $(2, 2)$, etc. The first coordinate is an x -value and the second coordinate is a y -value. The y -value is equal to 1 times the x -value in any of these pairs, so these pairs of values satisfy the equation.

Answer: All of the ordered pairs representing solutions to the equation can be associated with equivalent ratios. The quotient of the y -value to the x -value is always the same for any x - and y -values satisfying the equation.



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Class Discussion (continued)

- **Are there other points not shown that represent solutions to this equation? Explain.**
- **Select Draw Line. Then Select 4 Quads. Jeremy says that if one imagined this line continuing infinitely far in both directions, it would have all the solutions to the equation on it. Do you agree? Why or why not?**

The multiplier m in an equation of the form $y = mx$ sometimes is called the constant of proportionality or the slope of the line that is the graph of the solution to that equation. The rest of this lesson uses the word “slope” for this multiplier m .

- **Reset and change the slope to $\frac{3}{2}$ by selecting the multiplier 1 and using the keypad to enter a denominator of 2 and a numerator of 3, so that the equation is $y = \frac{3}{2}x$ and select Enter. How are the displayed points related to the new equation? Are any of the displayed points the same as for the equation $y = 1x$? If so, which points?**

Answer: Yes, but reasons will vary. Some of the students may note that the coordinate grid is only shown up to (10, 10). If one continued moving up and to the right, there would be other points whose x - and y -coordinates are equal. Other students may point out that there are points in the coordinate plane with negative coordinates such as $(-1, -1)$, $(-2, -2)$, and so on that satisfy the equation. And some students may point out that these are just points with integer coordinates, and points with coordinates such as $(2.5, 2.5)$ or $(\frac{-9}{4}, \frac{-9}{4})$ also would satisfy the equation.

Answer: Jeremy is correct. Every point whose y -coordinate is equal to its x -coordinate would be on this line, and any point off the line would have coordinates that do not satisfy the equation.

Answer: Assuming the line extends infinitely far in both directions, it represents all of the solutions (x, y) to this equation. For example, $(-2, -3)$ is a point on this line and the x - and y -values satisfy the equation, since $-3 = \frac{3}{2}(-2)$.



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Class Discussion (continued)

- **Reset and change the slope to $\frac{2}{3}$ so that the equation is $y = \frac{2}{3}x$ and select Enter.**

Explain whether the displayed points are solutions to this equation.

- **Select 4 Quads and Draw Line. Based on the two questions above, Marlita says that whenever the slope is less than 1, the line slants up less than the line for $y = 1x$, and if the slope is greater than 1, the line slants up more than the line for $y = 1x$.**

- **Try each of these slopes. Select 4 Quads and Draw Line each time. Do you agree with Marlita? Why or why not?**

Slope: $\frac{4}{1}$ $\frac{5}{2}$ $\frac{1}{4}$ $\frac{0}{1}$ $\frac{-2}{3}$ $\frac{-3}{2}$

Note: In the question above, you may want students to experiment with more slopes than those listed in order to verify their conjectures.

Answer: Yes, the points shown are (0, 0), (3, 2), (6, 4), and (9, 6). For each of the points, the y-coordinate value is exactly $\frac{2}{3}$ times the x-coordinate value.

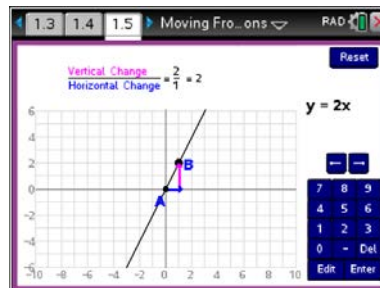
Answer: The total number of circles will change by 4 and the total number of squares by 5 because each row has 4 circles and 5 squares.

Answer: Marlita is correct provided the slopes that are less than 1 are also positive. When the slope is $\frac{0}{1}$, the line does not slant at all, since the points are all along the x-axis. When the slopes are negative, the lines slant *down* from left to right.

Part 1, Page 1.5

Focus: The questions in this part connect students' previous understandings of proportional relationships to more general linear relationships of the form $y = mx$ where the variables x and y , and the slope m can be any real numbers.

On Page 1.5, the proportionality relationship equation and its line graph are displayed in the coordinate plane. Use **tab** to highlight points **A** or **B** and use the horizontal arrows on the screen or keypad to move the selected point left or right one unit at a time.



Edit or **menu** > **Slope** and the keypad to change the constant of proportionality.

Reset returns to the default display of $y = \frac{3}{2}x$.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through numerator, denominator, and the points.

ctrl **del** resets the page.



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Class Discussion

Have students...

Look for/Listen for...

Open Page 1.5. An equation of the form $y = 2x$ is graphed.

- *What is the slope of the line whose equation is $y = 2x$? Explain how you know.*
- *Use the right horizontal arrow button once to move point B. Explain how the numbers displayed in the fraction of vertical change over horizontal change are related to the points A and B.*
- *Use the horizontal arrows to move point B both right and left of point A. How would you describe the relationship between the different fractions displayed for the vertical change over the horizontal change and the slope of the line?*

Answer: The slope is 2, the multiplier of x in the equation.

Answer: The blue arrow represents the horizontal change in the x -coordinate if you move from point A to point B, and the magenta arrow represents the vertical change in the y -coordinate from point A to point B. The fraction of the vertical change over the horizontal change is $\frac{4}{2}$. (Some students may note that this is the same as the fraction of the y -coordinate of B over the x -coordinate of B.)

Answer: All of the fractions are equivalent to the fraction representing the slope of the line.

Reset, and tab once so that point A is highlighted.

- *Move point A using the horizontal arrows. What is different about the movement of points A and B and the blue and magenta arrows?*
- *How does the fraction of vertical change over horizontal change behave as you move point A?*
- *Move point A to a point on the line in the first quadrant, and then Tab to highlight point B and move it into the third quadrant. What happens to the fraction of vertical change over the horizontal change in this case?*

Answer: Point B moves along with point A when you move point A, and the blue and magenta arrows stay exactly the same length. When you moved point B, point A did not move at all and the blue and magenta arrows changed length.

Answer: The fraction always remains the same because the lengths and directions of the blue and magenta arrows do not change.

Answer: The actual locations students move the two points to will vary, but vertical and horizontal changes both should be negative and the fraction will still be equivalent to $\frac{2}{1}$.



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Class Discussion (continued)

Edit the slope so that the equation is $y = \frac{-1}{2}x$

Enter.

- **Carla says the vertical change will have to be negative because the numerator of the slope is negative, and the horizontal change will be positive, because the denominator of the slope is positive. Do you agree with Carla? Why or why not?**
- **Jake says that no matter where you move points A and B on the line, the fraction of the vertical change over the horizontal change will be equivalent to $-\frac{1}{2}$.**
- **Make a conjecture about what will happen if both the horizontal change and the vertical change are negative. Enter a slope and try it. Check your conjecture.**
- **Torrance claims that if the slope is $\frac{0}{1}$, then the fraction formed by the vertical change divided by the horizontal change will not be defined. Do you agree? Give an example to support your reasoning.**

Answer: Carla is not correct that the vertical change would have to be negative and the horizontal change would have to be positive. Move point A to the fourth quadrant and point B to the second quadrant and the vertical change from A to B will be positive and horizontal change will be negative. The fraction formed will still be equivalent to $-\frac{1}{2}$.

Answer: As long as A and B are distinct points, Jake is correct about the fraction formed by the vertical change divided by the horizontal change being equivalent to $-\frac{1}{2}$.

Answer: The slope will have to be positive, and the point B will be above and to the left of point A.

Answer: Torrance is incorrect. The line is a horizontal line along the x-axis, and if A and B are different points, both of the y-coordinates will be 0. The vertical change is 0 and the horizontal change will be nonzero, but a fraction with 0 in the numerator and a nonzero denominator will be OK, and equivalent to the fraction $\frac{0}{1}$.

For example, if A is the point (2, 0) and B is the point (5, 0), then the vertical change is 0 and the horizontal change is 3, and $\frac{0}{3}$ is equivalent to $\frac{0}{1}$.



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Class Discussion (continued)

- *Is there a line where points A and B could be placed in different locations, but the fraction of vertical change over horizontal change does not make sense? If so, describe the line. If not, explain why it is not possible.*

Answer: If the line was on top of the vertical y-axis, then the x-coordinates of any two different points would both be 0 and the horizontal change would be 0. Since division by 0 is not permitted, the fraction of vertical change over horizontal change would not make sense.

Note: There is no value of m that makes the graph of $y = mx$ vertical.

Part 2, Page 1.7

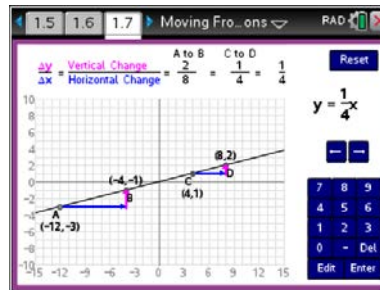
Focus: Connect the idea of the slope m as the proportionality constant in the equation $y = mx$ to the geometric idea of the slope of the line that is the graph of the solution to that equation.

On Page 1.7, the proportionality relationship equation and its line graph are again displayed in the coordinate plane along with four movable points, labeled A, B, C, and D.

Select or use **tab** to highlight any of the four points. Each point moves independently by dragging or using the arrow keys.

Edit or **menu** > **Slope** and the keypad to change the constant of proportionality.

Reset returns to the default display of $y = \frac{1}{4}x$.



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menu accesses page options.

tab cycles through numerator, denominator, and the points.

ctrl **del** resets the page.



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Class Discussion

Have students...

Look for/Listen for...

Open Page 1.7.

This page shows the symbol Δ (the Greek letter “delta”) that is often used as an abbreviation for “change in.” For example, Δy is read as “delta y ” and stands for the “change in y ”, or the change in the y -coordinate values. The Δx is “delta x ” and stands for the “change in x ,” or the change in the x -coordinate values. The points A, B, C, and D are all on the line that is the graph of $y = \frac{1}{4}x$. The ratio

of vertical change to horizontal change is shown from point A to B and from point C to D.

- *How is the fraction under “A to B” determined? Under “C to D”?*

Answer: The fraction under “A to B” is $\frac{2}{8}$. The

numerator 2 means that point B is 2 units above point A vertically: the y -coordinate of B is -1 , which is 2 units more than A’s y -coordinate -3 . The denominator 8 means that B is 8 units to the right of A horizontally: the x -coordinate of B is -4 , which is 8 units more than A’s x -coordinate -12 .

The fraction under “C to D” is $\frac{1}{4}$. The numerator

1 means that point D is 1 unit above point C vertically: the y -coordinate of D is 2, 1 unit more than C’s y -coordinate 1. The denominator 4 means that D is 4 units to the right of C horizontally: the x -coordinate of D is 8, which is 4 units more than C’s x -coordinate 4.

- *Tab so that point A is selected. Use the horizontal arrows to move A left and right. Move A to the right until the two triangles formed are congruent. What are the coordinates of point A, and how do you know that the two triangles are congruent?*

Answer: The coordinates of point A should be at $(-8, -2)$. The two triangles are congruent because they are both right triangles with legs of length 1 and 4.



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Class Discussion (continued)

- **Move A to the right until it is the same as point B. What happens to the fraction under “A to B.” Explain why this happened.**

Answer: When A and B are the same point, the fraction disappears. The vertical and horizontal changes between the two points are both 0, and the fraction $\frac{0}{0}$ does not make sense. (The denominator of a fraction can never be zero.)

Teacher Tip: A discussion of why the denominator of a fraction cannot be 0 may need to be revisited here, for the calculation of slope from the coordinates of two points is a natural example where students could encounter such a fraction (when the two points coincide). The fact that the statement “if $\frac{a}{b} = c$, then $a = bc$ ” fails to hold if $b = 0$ is the mathematical basis for prohibiting $b = 0$. Notice that in the case that a is nonzero and b is zero, then no value of c could work. And *any* value could be assigned to c if both a and b are 0.

Reset the page. Select point C. Move point C to the left until it is the same as point A.

- **Sonia claims that these two triangles must be similar. Do you agree? Why or why not?**

Answer: Sonia is correct. Both triangles are right triangles, and they share the same angle at a common vertex (points A and C). Since the angles in a triangle add up to 180° , the third angle in each triangle must also have the same degree measure, so the two triangles must be similar.

- **Armando says that he thinks that no matter where the points are placed, you will get two similar triangles. Reset and use Tab to move point C to (12, 3) and point D to (-8, -2). Are these two triangles similar? Why or why not? Do you agree with Armando?**

Answer: The two triangles are similar. The picture below shows the situation for the original points A and B and the specified locations for points C and D.

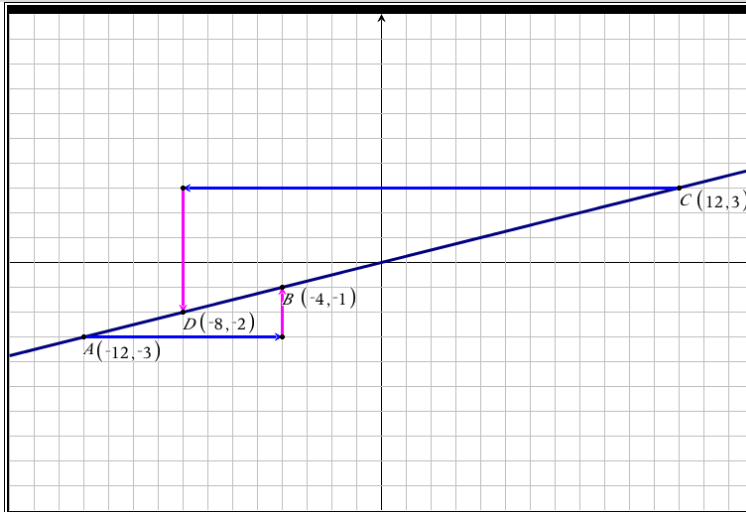


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Class Discussion (continued)



Each triangle is a right triangle with a horizontal leg and a vertical leg. The line makes congruent angles with the parallel horizontal legs of each triangle. The line also makes congruent angles with the parallel vertical legs of each triangle. So all three corresponding angles are congruent, making the two triangles similar.

Armando is correct, provided A and B are any two distinct points on the line, and C and D are any two distinct points on the line (so that triangles are actually made), for exactly the same explanation can be provided.

- ***If the locations of point C and D are interchanged, so C has coordinates $(-8, -2)$ and D has coordinates $(12, 3)$ how will the fraction under “C to D” change?***

Answer: Both the numerator and denominator would be the opposite of what they were before, changing the fraction from $\frac{-5}{-20}$ to $\frac{5}{20}$, but the result is still equivalent to $\frac{1}{4}$.

Interchanging the locations of two points switches the signs of both the numerator and denominator, so the order of the two points does not affect the fraction’s value.

- ***How would you explain why the fractions displayed at the top of the page are always equivalent to $\frac{1}{4}$ for the line $y = \frac{1}{4}x$?***

Answer: For all choices of two distinct points on the line, similar triangles are formed, so the ratio of the vertical leg length to the horizontal leg length always will be the same.



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Class Discussion (continued)

Teacher Tip: This question gives rise to an opportunity to derive the familiar formula for slope: Given any two distinct points A and B on a (nonvertical) line, we now have a way of computing the slope m of that line. If point A has coordinates (x_1, y_1) and point B has coordinates (x_2, y_2) , then in moving from point A to point B, the change in y is given by $y_2 - y_1$ and the change in x is given by $x_2 - x_1$. Therefore, the slope m of the line is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. This is the formula for the slope of a line that is encountered in most textbooks.



Student Activity Questions—Activity 1

1. Can you do it?

- a. Reset and Edit the slope so that the equation is $y = \frac{-2}{5}x$.

Can you find locations for A, B, C, and D so that:

the numerators and denominators of the A to B fraction and the C to D fraction are all different integers.

AND

the numerator of the A to B fraction has the opposite sign of the numerator of the C to D fraction and the denominator of the A to B fraction has the opposite sign of the denominator of the C to D denominator?

Answer: Answers will vary. One example is to place A at $(-15, 6)$ and B at $(15, -6)$, and to place

C at $(10, -4)$ and D at $(-10, 4)$. The fraction from A to B is $\frac{-12}{30}$ and the fraction from C to D is

$$\frac{8}{-20}.$$

- b. Can you Edit the slope so that the numerators of both the A to B and C to D fractions are always the same, no matter where you place the points? If so, what would the line look like?

Answer: Make the slope equal to 0, and the vertical changes will all be 0. The line is horizontal line $y = 0$ and lies along the x -axis.

- c. Can you Edit the slope so that the denominators of both the A to B and C to D fractions are always the same, no matter where you place the points?

Answer: Not possible. Lines with equations of the form $y = mx$ will always have points with lots of different x -coordinates, so there will always be lots of different horizontal changes possible as denominators of the fractions.



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Student Activity Questions—Activity 1 (continued)

2. Suppose you had the graph of a line that passes through the origin (0, 0) but you do not have the equation of the line.

a. If the point (8, -6) is also on your line, could you figure out what equation it must have? Explain why or why not.

Answer: Yes. The line must have an equation of the form $y = mx$ for some slope m .

We can find m by calculating $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{-6 - 0} = \frac{8}{-6} = -\frac{4}{3}$, so the equation of the line must be

$$y = -\frac{4}{3}x.$$

b. Is there a point on this line where the y -coordinate is 1,000,000? If so, then find the x -coordinate of the point, or explain why there is no such point.

Answer: Any point (x, y) that satisfies $y = -\frac{4}{3}x$ will be on the line. If $y = 1,000,000$, then

$$1,000,000 = -\frac{4}{3}x, \text{ so } x = -\frac{3}{4}(1,000,000) = -750,000.$$

Part 3, Page 2.2

Focus: Generalize the idea of slope to lines that do not go through the origin.

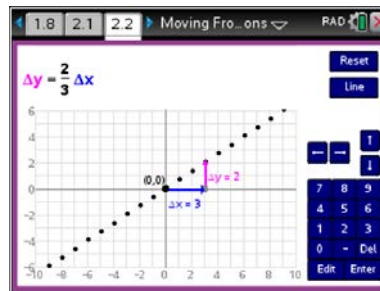
On Page 2.2, the equation displayed is for the proportionality relationship between the change in y -values (Δy) and the change in x -values (Δx).

Edit or **menu** > **Slope** and the keypad to change the constant of proportionality.

On Page 2.2, **Enter** displays the lattice points satisfying the relationship as measured from the origin (0, 0) as the point indicated by its coordinates. Select the gray point and drag or use the horizontal arrows to move.

Line displays the graph of the line through the indicated point with the specified slope.

Show Equation displays the point-slope form of the equation of this line.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles between the points and the buttons.

enter activates a button.

ctrl **del** resets the page.



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Class Discussion

Have students...

Look for/Listen for...

Open Page 2.2.

A slope equation of the form $\Delta y = \frac{2}{3}\Delta x$ is shown.

Two points satisfy this slope equation if Δy , the change in the *y*-coordinates, over Δx , the change in *x*-coordinates, from one point to the other is

equal to $\frac{\Delta y}{\Delta x} = \frac{2}{3}$.

- **One point (0, 0) is shown and one other point is indicated by the blue and magenta arrows. What are the coordinates of this other point, and do the two points satisfy the slope equation?**
- **Use the left and right horizontal arrows on the screen or keypad to change Δx . Move to other points both to the right and left of the origin (0, 0). Explain how changes.**
- **Predict the equation of the line through these points. Explain your prediction. Then select Line and Show Equation to check your answer.**

Reset. Use Tab to highlight the point at the origin (0, 0). The directional arrows on the screen or keypad will move this point to a new location. Move the point to the coordinates (-4, 2).

- **What other point is indicated by the blue and magenta arrows from (-4, 2)? Explain why the equation $\frac{\Delta y}{\Delta x} = \frac{2}{3}$ is still satisfied by (-4, 2) and this other point.**

Answer: The other point is (2, 3) and it does satisfy the slope equation since from (0, 0) to (2, 3) we have $\Delta y = 2$ and $\Delta x = 3$, so $\frac{\Delta y}{\Delta x} = \frac{2}{3}$.

Answer: Whatever Δx is, $\Delta y = \frac{2}{3}\Delta x$. The second point and the origin will always satisfy the slope equation. For example, if $\Delta x = 5$, then $\Delta y = \frac{2}{3} \cdot 5 = \frac{10}{3}$. If $\Delta x = -4$, then $\Delta y = \frac{2}{3}(-4) = -\frac{8}{3}$.

Answer: $y = \frac{2}{3}x$. A line through the origin has equation $y = mx$ where m is the slope.

Answer: The new point is (-1, 4). The change in *y*-coordinate values from (-4, 2) to (-1, 4) is 2 units. The change in *x*-coordinate values from (-4, 2) to (-1, 4) is 3 units.

So $\frac{\Delta y}{\Delta x} = \frac{2}{3}$.



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Class Discussion (continued)

- **Tab to highlight the gray point and use the horizontal arrows to change Δx to -6 . What are the coordinates of the point indicated by the blue and magenta arrows? Is the slope equation still satisfied? Explain why or why not.**
- **Select line to see the graph of the line of points through $(-4, 2)$. Sal says that for any other point on this line, the Δy and Δx satisfy the slope equation. Do you agree? Why or why not?**
- **Select Show Equation. Explain why any point (x, y) on this line satisfies this equation.**

Answer: The new point has coordinates $(-10, 2)$. The slope equation is still satisfied since Δy , the change in y -coordinate values, from $(-4, 2)$ to $(-10, 2)$ is -4 , so $\frac{\Delta y}{\Delta x} = \frac{2}{3}$ because

$$\frac{\Delta y}{\Delta x} = \frac{-4}{-6} = \frac{2}{3}.$$

Answer: Sal is correct. For all other points on the line different than $(-4, 2)$, the blue and magenta arrows form similar triangles and $\frac{\Delta y}{\Delta x} = \frac{2}{3}$.

Answer: If (x, y) is a point on this line, then the left-hand side of the equation is $y - 2$ and represents Δy . On the right-hand side of the equation $(x - (-4))$ represents Δx . So the equation is just another way of saying that (x, y) and $(-4, 2)$ satisfy the slope equation.

NOTE: This equation is called the *point-slope* form for the equation of a line through the point $(-4, 2)$ with slope $\frac{2}{3}$.

Reset. Edit the slope so that $\Delta y = \frac{5}{-4}\Delta x$. Tab to highlight the point at the origin and use the directional arrows to move it to $(3, -4)$.

- **If a second point is located $\Delta x = -8$ (8 units left) from $(3, -4)$, what is the corresponding Δy that satisfies $\Delta y = \frac{5}{-4}\Delta x$? What are the coordinates of this point? Use the TNS activity to check your answer.**
- **Select Line. What is the point-slope equation of this line? Use Show Equation to check your answer.**

Answer: $\Delta y = \frac{5}{-4}(-8) = \frac{-40}{-4} = 10$, so the point is located 10 units up from $(3, -4)$ and the $\Delta x = -8$ indicates that the point is 8 units to the left from $(3, -4)$. The point is located at coordinate $(3 - 8, 4 + 10) = (5, 6)$.

Answer: $y - 4 = \frac{5}{-4}(x - 3)$.



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Class Discussion (continued)

- Do the coordinates of the point $(3, -4)$ satisfy this equation for the line?**

Answer: Yes, since $-4 - (-4) = 0$ and $\frac{5}{-4}(3 - 3) = \frac{5}{-4} \cdot 0 = 0$
- Is the origin $(0, 0)$ on this line? Explain why or why not.**

Answer: The origin is not on this line, because $0 - (-4) = 4$, but $\frac{5}{-4}(0 - 3) = \frac{5}{-4}(-3) = \frac{-15}{-4} = \frac{15}{4} \neq 4$ (The line passes close to $(0, 0)$ but not through it!)



Student Activity Questions—Activity 2

- Find an equation for each of the following lines. If possible, find the point where each line crosses the y -axis and where each line crosses the x -axis. Use the TNS activity to check your answers.
 - The line that contains the point $(-2, 5)$ and slope $\frac{3}{2}$.

Answer: Equation in point-slope form is $y - (-5) = \frac{3}{2}(x - (-2))$ or $y + 5 = \frac{3}{2}(x + 2)$. The point where the line crosses the y -axis would have first coordinate $x = 0$. The y -coordinate of this point would have to satisfy $y + 5 = \frac{3}{2}(0 + 2)$, so solving for y gives us $y = \frac{6}{2} - 5$ or $y = -2$ and the point is $(0, -2)$.

The point where the line crosses the x -axis would have second coordinate $y = 0$. The x -coordinate of this point must satisfy $0 + 5 = \frac{3}{2}(x + 2)$. Solving for x , we have $x = \frac{4}{3}$ and the point is $(\frac{4}{3}, 0)$.



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Student Activity Questions—Activity 2 (continued)

- b. The line that that goes through $(-2, 5)$ and $(3, -6)$.

Answer: First, we can use the two points to figure out what the slope needs to be. If we move from point $(-2, 5)$ to $(3, -6)$ the change in y -coordinate values is from 5 to -6 , so $\Delta y = -11$, and the change in x -coordinate values is from -2 to 3 , so $\Delta x = 5$. The slope is given by $m = \frac{\Delta y}{\Delta x} = \frac{-11}{5}$.

We can use this slope and either point to find an equation. If we use the first point given $(-2, 5)$, the equation in point-slope form is $y - 5 = \frac{-11}{5}(x - (-2))$, or equivalently $y + 5 = \frac{-11}{5}(x + 2)$. We

can check that the other point $(3, -6)$ satisfies this equation: $-6 + 5 = \frac{-11}{5}(3 + 2)$; $-1 = \frac{-11}{5}(5)$ and $-1 = -11$.

The point where the line crosses the y -axis would have first coordinate $x = 0$. The y -coordinate of this point would have to satisfy $y + 5 = \frac{-11}{5}(0 + 2)$, so solving for y gives us $y = \frac{-22}{5}$ or $y = -4\frac{2}{5}$, and the point is $(0, -4\frac{2}{5})$.

The point where the line crosses the x -axis would have second coordinate $y = 0$. The x -coordinate of this point must satisfy $0 + 5 = \frac{-11}{5}(x + 2)$. Solving for x , we have $x = -27$, and the point is $(-27, 0)$.

- c. The horizontal line that lies two units below the x -axis.

A horizontal line has slope 0 , and points 2 units below the x -axis all have y -coordinate -2 . The equation of this line is $y = -2$. This line intersects the y -axis at $(0, -2)$. This line would not intersect the x -axis since it is parallel to it.

Part 3, Page 2.4

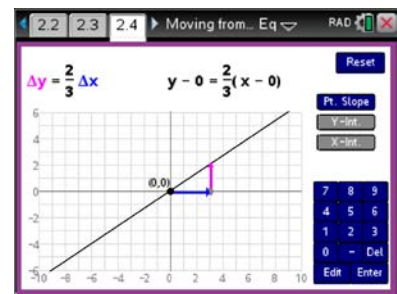
Focus: Generalize the idea of slope to lines that do not go through the origin.

On Page 2.4, the proportionality relationship between Δy and Δx is displayed along with an equation for the line. Students may move the indicated point or change Δx as they did on Page 2.2.

Pt. Slope displays the point-slope form of the equation.

Y-Int. displays the y -intercept form.

X-Int. displays the x -intercept form of the equation for the line.





Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Class Discussion

Have students...

Go to Page 2.4. The Pt. Slope button indicates that the point-slope form of the line shown is displayed.

- **Zack says that the form for this line shown could have been written as $y = \frac{2}{3}x$. Do you agree? Why or why not?**
- **The point where a line intersects the y-axis is called its y-intercept and the point where a line intersects the x-axis is called its x-intercept. For the line shown, what are the y-intercept and x-intercept? Select Y-Int and X-Int and look at the form of the equation to see the y-intercept and x-intercept forms of the equation of the line. How do they compare with each other and the point-slope form?**

A linear equation in two variables x and y is an equation whose solution can be graphed as a line in the coordinate plane. There are several forms that a linear equation could have for the same line. One of these forms is the point-slope form.

Look for/Listen for...

Answer: Yes, since $y - 0$ on the left-hand side of the equals sign could have been written as just y , and $\frac{2}{3}(x - 0)$ on the right-hand side of the equals sign could have been written as just $\frac{2}{3}x$.

Answer: The y-intercept and x-intercept of this line are the same point, namely the origin $(0, 0)$. The y-intercept form of the equation is $y = \frac{2}{3}x$ and the x-intercept form of the equation is $y = \frac{2}{3}(x - 0)$, so the left-hand side of both is simply y and the right-hand side of the x-intercept form is the same as the right-hand side of the point-slope form.



Building Concepts: Moving from Proportional Relationships to Linear Equations

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Class Discussion (continued)

- **Reset. Tab to highlight the point at the origin and use the directional arrows on the keyboard to move it to (3, 4). Use the graph to find the coordinates of the y-intercept and of the x-intercept of this new line. Is there another way to find these intercepts without using the graph? Explain your reasoning.**

Answer: The y-intercept is the point (0, 2) and the x-intercept is the point (-3, 0).

The y-intercept always has 0 as the first coordinate. If one substitutes 0 for x in the equation and solves for the corresponding value of y, that gives the second coordinate of the y-intercept.

The x-intercept, if the line is not horizontal, is a point having 0 as second coordinate. If one substitutes 0 for y in the equation and solves for the corresponding value of x, that gives the first coordinate of the x-intercept.

Here the point-slope form is $y - 4 = \frac{2}{3}(x - 3)$

Substituting 0 for x results in

$$y - 4 = \frac{2}{3}(0 - 3) = \frac{2}{3}(-3) = -2 \text{ . So } y = -2 + 4 = 2$$

and the y-intercept is (0, 2).

Substituting 0 for y results in

$$0 - 4 = \frac{2}{3}(x - 3) = \frac{2}{3}x - 2, \text{ so } -4 + 2 = \frac{2}{3}x \text{ and}$$

solving for x gives us $x = \frac{3}{2}(-2) = -3$. The

x-intercept is (-3, 0).

Another form of linear equation is called the y-intercept (or slope-intercept) form, and this form makes it easy to identify the point where the line crosses the y-axis. Another form is called the x-intercept form, and this form makes it easy to identify the point where the line crosses the x-axis.



Building Concepts: Moving from Proportional Relationships to Linear Equations

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Class Discussion (continued)

- **Use Y-Int. to see the y-intercept form of the equation for this line. Why is it easy to find the y-intercept from this equation? Use X-Int. to see the x-intercept form and explain why it is easy to find the x-intercept from this equation.**

Answer: The y-intercept form is $y = \frac{2}{3}x + 2$. If you substitute 0 for x it is easy to see that the corresponding value of y is 2, since $\frac{2}{3}(0) + 2 = 2$, and the y-intercept is (0, 2). The x-intercept form is $\frac{2}{3}(x+3)$. If you substitute 0 for y, then $\frac{2}{3}(x+3)$ must also be 0 and it is easy to see that $x = -3$ the value that works. The x-intercept is (-3, 0).



Student Activity Questions—Activity 3

1. A line passes through the point (4, -3) and has slope $m = -\frac{3}{2}$.

- a. What is the point-slope form of the equation for this line using the given point and slope? Create a line using the TNS activity that passes through this point with this slope to check your answer. What are the coordinates of the x-intercept and y-intercept of this line?

Answer: The point-slope form of the equation is $y - (-3) = -\frac{3}{2}(x - 4)$. The x-intercept is (2, 0) and the y-intercept is (0, 3).

- b. Tayeen notices that if you use the x-intercept of this line as the point in the point-slope form, then you will just get the x-intercept form. Use the TNS activity to verify her claim. She wonders if that would always be true for any line. What do you think? Explain your reasoning.

Answer: As long as the line is not horizontal, the slope m will not be 0, and it will have an x-intercept with coordinates (a, 0) for some number a. The point-slope form would be $y - 0 = m(x - a)$, and if you write $y - 0$ as just y, then that is x-intercept form.

- c. Tayeen wonders if you used the y-intercept as your point in the point-slope form, would the equation be the same as the y-intercept form. Use the TNS activity to investigate. What do you think? Explain your reasoning.

Answer: The equations are not of the same form. The point-slope form using (0, 3) and slope $-\frac{3}{2}$ is $y - (-3) = -\frac{3}{2}(x - 0)$. The y-intercept form is $y = -\frac{3}{2}x + 3$. These forms would be the same only when the y-intercept happens to be the origin.



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Student Activity Questions—Activity 3 (continued)

- d. Monique says that in all three of these forms of the equation for a line, it is easy to see what the slope of the line is. Do you agree?

Answer: Yes. In every case, the slope is the multiplier of the factor involving x .



Deeper Dive

- You also have seen that lines can have equations of the form $ax + by = c$, where a , b , and c are numbers. How would you find the slope m of a line whose equation is written this way?**
- Enterra says that the following could also be another “point-slope” form for a linear equation: Given the slope m and one point on the line (x_1, y_1) , every point (x, y) on the line would satisfy the equation**

$$m = \frac{y - y_1}{x - x_1}.$$

Would every point on the line really satisfy this equation? Why or why not?

Answer: Answers will vary. One strategy would be to simply find two specific ordered pairs (x, y) that satisfy the equation, say (x_1, y_1) and (x_2, y_2) , and then use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Another strategy would be to rewrite the equation algebraically into slope-intercept form by solving for y so that the slope could be easily read off of this equation: $y = -\frac{a}{b}x + \frac{c}{b}$, so the slope must be $m = -\frac{a}{b}$.

(Note: in any case, if $b = 0$, the equation describes a vertical line with undefined slope.)

Answer: Not quite! There is exactly one point on the line that will not satisfy this equation, namely the given point (x_1, y_1) , since that would result in the denominator being equal to 0. (Note that the usual point-slope form does not involve such a denominator.)



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES



Deeper Dive (continued)

- **Josefina says that an equation like $(x - 5)(x - 3) = 0$ is easy to solve. Do you agree? Do you think she would also say that $(x - 5)(x - 3) = 10$ is as easy to solve? Why or why not?**

Answer: When a product of factors is equal to 0, then at least one of the factors must be 0, so it is easy to see that either $x = 5$ or $x = 3$ are solutions to Josefina's original equation $(x - 5)(x - 3) = 0$. However, if the product of two factors is 10, one cannot conclude that either factor must be equal to a particular number. Finding a value of x that makes the product $(x - 5)(x - 3) = 10$ will not be easy to see from this form of the equation. (Note: the strategy of rewriting an equation to involve factors whose product is 0 is one that will be important later in students' study of quadratic and polynomial equations.)



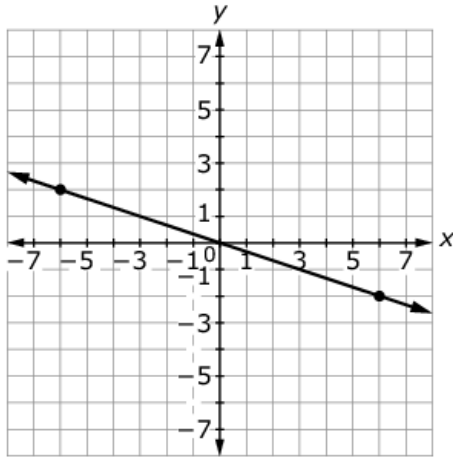
Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Consider the graph of a line. What is the equation of the line?



Adapted from SBAC Practice Test Scoring Guide Grade 8 Mathematics, 1863

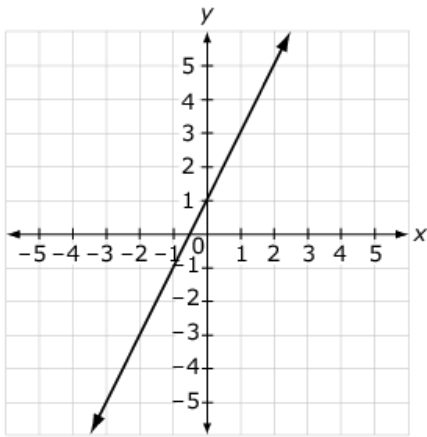
Answer: $y = -\frac{1}{3}x$



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

2. Consider the graph of a line.



Which equation has a slope **greater than** the slope for the line shown?

- a. $y = 3x - 1$
- b. $y = \frac{x}{2} + 4$
- c. $y = 2x + 2$
- d. $y = \frac{x}{3} - 3$

Adapted from PARCC Spring Operational 2015 Grade 8 Released Items

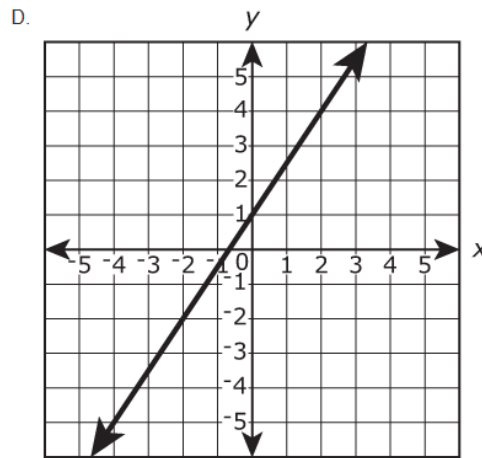
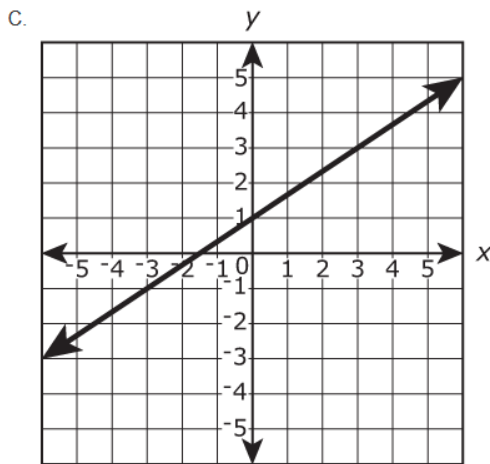
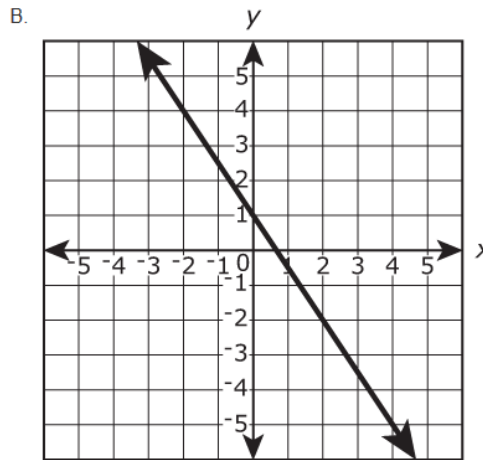
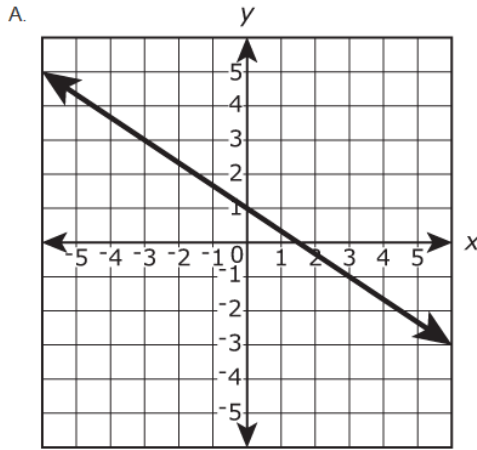
Answer: A



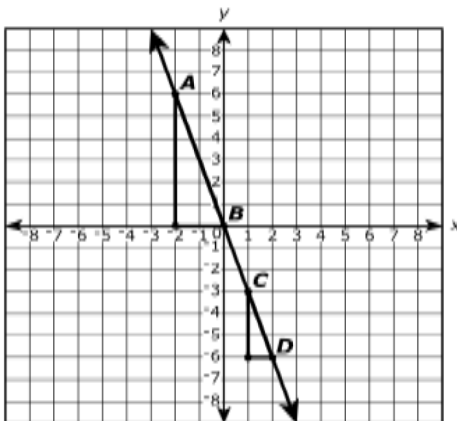
Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

3. A line is defined by the equation $y = -\frac{2}{3}x + 1$. Which of the following is the graph of the equation?



Answer: A





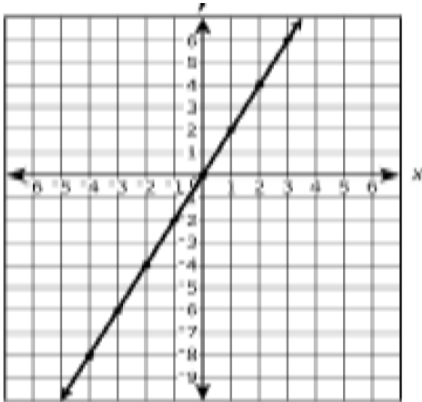
Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

4. On the coordinate plane above, points A , B , C , D lie on the same line. Fill in the blank with wording that correctly complete the sentences.

The slope of \overline{AB} is _____ the slope of \overline{CD} because the fraction of the vertical change over the horizontal change between points A and B is _____ to the fraction of the vertical change to the horizontal change between points C and D .

Answers: "equal to" and "equivalent to"



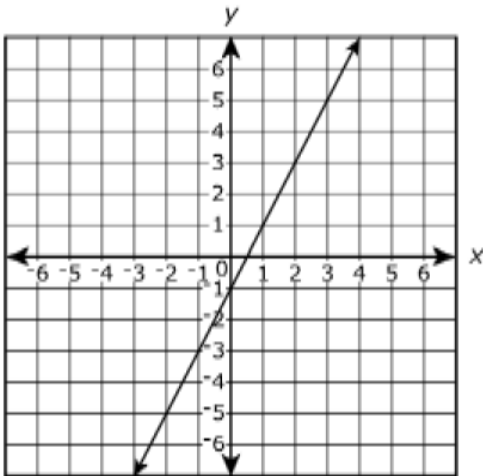
5. Line j contains the points $(0, 1)$ and $(4, 2)$. Line k is shown above. Which of the following describes the y -intercepts of the lines.
- The y -intercept of j is equal to the y -intercept of k .
 - The y -intercept of j is one unit less than the y -intercept of k .
 - The y -intercept of j is one unit greater than the y -intercept of k .
 - The y -intercept of j is two units greater than the y -intercept of k .

Answer: B



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6. The graph of a line $l1$ is shown above. Below are equations of three other lines.

Line $l2$

Line $l3$

Line $l4$

$$y = 3 + 2x$$

$$y = 2$$

$$y = \frac{3}{2}x + 6$$

Which of these lines has the same slope as line $l1$?

- a. Line $l2$ only
- b. Line $l4$ only
- c. Lines $l2$ and $l3$ only
- d. Lines $l2$ and $l4$ only

Answer: A



Building Concepts: Moving from Proportional Relationships to Linear Equations

TEACHER NOTES

Student Activity Solutions

In these activities, you will explore relationships of the form $y = mx$ and eventually $y = mx + b$. After completing the activities, discuss and/or present your findings to the rest of the class.



Activity 1 [Page 1.3]

1. Can you do it?

- a. **Reset** and **Edit** the slope so that the equation is $y = \frac{-2}{5}x$.

Can you find locations for A, B, C, and D so that:

the numerators and denominators of the A to B fraction and the C to D fraction are all different integers.

AND

the numerator of the A to B fraction has the opposite sign of the numerator of the C to D fraction and the denominator of the A to B fraction has the opposite sign of the denominator of the C to D denominator?

Answer: Answers will vary. One example is to place A at $(-15, 6)$ and B at $(15, -6)$, and to place

C at $(10, -4)$ and D at $(-10, 4)$. The fraction from A to B is $\frac{-12}{30}$ and the fraction from C to D is

$$\frac{8}{-20}.$$

- b. Can you **Edit** the slope so that the numerators of both the A to B and C to D fractions are always the same, no matter where you place the points? If so, what would the line look like?

Answer: Make the slope equal to 0, and the vertical changes will all be 0. The line is horizontal line $y = 0$ and lies along the x-axis.

- c. Can you **Edit** the slope so that the denominators of both the A to B and C to D fractions are always the same, no matter where you place the points?

Answer: Not possible. Lines with equations of the form $y = mx$ will always have points with lots of different x-coordinates, so there will always be lots of different horizontal changes possible as denominators of the fractions.



Building Concepts: Moving from Proportional Relationships to Linear Equations

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2. Suppose you had the graph of a line that passes through the origin (0, 0) but you do not have the equation of the line.
- a. If the point (8, -6) is also on your line, could you figure out what equation it must have? Explain why or why not.

Answer: Yes. The line must have an equation of the form $y = mx$ for some slope m .

We can find m by calculating $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{-6 - 0} = \frac{8}{-6} = -\frac{4}{3}$, so the equation of the line must be

$$y = -\frac{4}{3}x.$$

- b. Is there a point on this line where the y -coordinate is 1,000,000? If so, then find the x -coordinate of the point, or explain why there is no such point.

Answer: Any point (x, y) that satisfies $y = -\frac{4}{3}x$ will be on the line. If $y = 1,000,000$, then

$$1,000,000 = -\frac{4}{3}x, \text{ so } x = -\frac{3}{4}(1,000,000) = -750,000.$$



Activity 2 [Page 2.2]

1. Find an equation for each of the following lines. If possible, find the point where each line crosses the y -axis and where each line crosses the x -axis. Use the TNS activity to check your answers.
- a. The line that contains the point (-2, 5) and slope $\frac{3}{2}$.

Answer: Equation in point-slope form is $y - (-5) = \frac{3}{2}(x - (-2))$ or $y + 5 = \frac{3}{2}(x + 2)$. The point where the line crosses the y -axis would have first coordinate $x = 0$. The y -coordinate of this point would have to satisfy $y + 5 = \frac{3}{2}(0 + 2)$, so solving for y gives us $y = \frac{6}{2} - 5$ or $y = -2$ and the point is $(0, -2)$.

The point where the line crosses the x -axis would have second coordinate $y = 0$. The x -coordinate of this point must satisfy $0 + 5 = \frac{3}{2}(x + 2)$. Solving for x , we have $x = \frac{4}{3}$ and the point is $(\frac{4}{3}, 0)$.



Building Concepts: Moving from Proportional Relationships to Linear Equations

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- b. The line that that goes through $(-2, 5)$ and $(3, -6)$.

Answer: First, we can use the two points to figure out what the slope needs to be. If we move from point $(-2, -5)$ to $(3, -6)$ the change in y -coordinate values is from -5 to -6 , so $\Delta y = -1$, and the change in x -coordinate values is from -2 to 3 , so $\Delta x = 5$. The slope is given by $m = \frac{\Delta y}{\Delta x} = \frac{-1}{5}$. We can use this slope and either point to find an equation. If we use the first point given $(-2, -5)$, the equation in point-slope form is $y - (-5) = \frac{-1}{5}(x - (-2))$, or equivalently $y + 5 = -\frac{1}{5}(x + 2)$. We can check that the other point $(3, -6)$ satisfies this equation: $-6 + 5 = -\frac{1}{5}(3 + 2)$; $-6 + 5 = -1$ and $-\frac{1}{5}(3 + 2) = -1$.

The point where the line crosses the y -axis would have first coordinate $x = 0$. The y -coordinate of this point would have to satisfy $y + 5 = -\frac{1}{5}(0 + 2)$, so solving for y gives us

$$y = -\frac{-27}{5} \text{ or } y = -5\frac{2}{5}, \text{ and the point is } \left(0, -4\frac{3}{5}\right).$$

The point where the line crosses the x -axis would have second coordinate $y = 0$. The x -coordinate of this point must satisfy $0 + 5 = -\frac{1}{5}(x + 2)$. Solving for x , we have $x = -27$, and the point is $(0, -27)$.

- c. The horizontal line that lies two units below the x -axis.

A horizontal line has slope 0, and points 2 units below the x -axis all have y -coordinate -2 . The equation of this line is $y = -2$. This line intersects the y -axis at $(0, -2)$. This line would not intersect the x -axis since it is parallel to it.



Building Concepts: Moving from Proportional Relationships to Linear Equations

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Activity 3 [Page 2.4]

1. A line passes through the point $(4, -3)$ and has slope $m = -\frac{3}{2}$.
 - a. What is the point-slope form of the equation for this line using the given point and slope? Create a line using the TNS activity that passes through this point with this slope to check your answer. What are the coordinates of the x -intercept and y -intercept of this line?

Answer: The point-slope form of the equation is $y - (-3) = -\frac{3}{2}(x - 4)$. The x -intercept is $(2, 0)$ and the y -intercept is $(0, 3)$.

- b. Tayeen notices that if you use the x -intercept of this line as the point in the point-slope form, then you will just get the x -intercept form. Use the TNS activity to verify her claim. She wonders if that would always be true for any line. What do you think? Explain your reasoning.

Answer: As long as the line is not horizontal, the slope m will not be 0, and it will have an x -intercept with coordinates $(a, 0)$ for some number a . The point-slope form would be $y - 0 = m(x - a)$, and if you write $y - 0$ as just y , then that is x -intercept form.

- c. Tayeen wonders if you used the y -intercept as your point in the point-slope form, would the equation be the same as the y -intercept form. Use the TNS activity to investigate. What do you think? Explain your reasoning.

Answer: The equations are not of the same form. The point-slope form using $(0, 3)$ and slope $-\frac{3}{2}$ is $y - (-3) = -\frac{3}{2}(x - 0)$. The y -intercept form is $y = -\frac{3}{2}x + 3$. These forms would be the same only when the y -intercept happens to be the origin.

- d. Monique says that in all three of these forms of the equation for a line, it is easy to see what the slope of the line is. Do you agree?

Answer: Yes. In every case, the slope is the multiplier of the factor involving x .