## What! A Mistake!

Time required
ID: 12678
20 minutes

## Activity Overview

In this activity, students will learn about Type I and Type II errors. Then, for a given scenario, students will calculate the probabilities of errors and the power of the test.

## Topic: Statistical Inference

- Hypothesis testing
- Type I and II errors


## Teacher Preparation and Notes

- This can be used as a stand alone lesson on Type I and Type II errors. Multiple choice questions allow students to check their understanding of the concepts. The AP exam does not require students to calculate the probability of the errors, however the second problem discusses the calculation of Type I and Type II errors and the power of the test.
- Students should have previously completed work on hypothesis testing to complete the extension. Knowledge of using the calculator to find probabilities and z-intervals is important. This problem serves as excellent review of previously learned concepts.
- Students can enter their responses directly into the TI-Nspire handheld or on the accompanying worksheet. On self-check questions, students can then press menu and select Check Answer (or ctrl + $\boldsymbol{\Delta}$ ). If desired, by using the TI-Nspire Teacher Edition software, teachers can change these self-check questions to exam mode so students cannot check their answer. On any question click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.
- To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12678" in the keyword search box.


## Associated Materials

- WhatAMistake_Student.doc
- WhatAMistake.tns


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Type I and Type II Errors (TI-84 Plus family with TI-Navigator) - 1977
- Confidence Levels (TI-Nspire technology) - 13482
- Estimating a Population Proportion (TI-Nspire technology) - 9986


## Problem 1 - Introducing Type I and Type II errors

Students are introduced to type I and type II errors. They are led through a practical example and then given questions to check their ability to differentiate between the types of errors.

The graph on page 1.13 shows a representation of Type I and Type II errors and Power (discussed later). The light gray area represents the power. The blue or darkly shaded area represents Type II errors and the red or medium shaded area represents Type I errors.

| 1.11 .2 | 1.3 *WhatAMistake $\nabla$ |  | 们 x |
| :---: | :---: | :---: | :---: |
| Drag the labels into the appropriate area in the chart. |  |  |  |
|  | Ho true | $H_{2}$ t |  |
| Reject $\mathrm{Ho}_{0}$ | Type Ierror | Correct |  |
| Fail to reject $H_{0}$ | Correct.decision | Type II. |  |



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\begin{tabular}{|c|c|c|c|}
\hline 1.15 & 2.1 & 2.2 & WhatAMistake \(\nabla\) \\
\hline
\end{tabular}
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| 1.15 | 2.1 | 2.2 | WhatAMistake $\nabla$ |
| :---: | :---: | :---: | :---: |

Probability of Errors
Probability of Errors
$\alpha=$ probability of type I error
$\alpha=$ probability of type I error
$=$ the significance level of the test
$=$ the significance level of the test
$\beta=$ probability of type II error
$\beta=$ probability of type II error
power $=$ probability of a false null hypothesis
power $=$ probability of a false null hypothesis
$=1-\beta$

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    \(=1-\beta\)
```

Probability of Errors
$\alpha=$ probability of type I error
$=$ the significance level of the test
$\beta=$ probability of type II error
power $=$ probability of a false null hypothesis

$$
=1-\beta
$$

## Problem 2 - Calculating the probability of errors

Students learn to calculate type I and type II errors. They are introduced to the concept of power. Explain to students that the significance level of any test is the probability of rejecting the null hypothesis when it is in fact true. Thus, the probability of a type I error occurring is the significance level. Type II errors are more difficult to calculate because it requires estimating the distribution of the alternative hypothesis, which is usually unknown.

## Additional Properties:

- Type I $(\alpha)$ and Type II ( $\beta$ ) errors are inversely related: as one increases the other decreases.
- Type I errors are generally set by the statistician since they are the significance level of the test.
- High power is desirable. Increasing the sample size always increases the power.

In Step 1, students will find the interval that contains the values of sample proportions ( $\bar{x}$ ) that do not lead to a rejection of $H_{0}$. They can use the $\mathbf{z}$ Interval command (MENU > Statistics > Confidence Intervals > z Interval), selecting Stats for the Data Input Method.
$\sigma: 0.1$
$\bar{x}: 12$
n: 50
C Level: 0.95

These values can also be found with the formula $\mu \pm z_{\alpha / 2} \frac{\sigma}{\sqrt{n}}$.

In Step 2, students will standardize the endpoints of the interval with the formula given on the worksheet.

In Step 3, students use the normCdf command (MENU > Statistics > Distributions > Normal Cdf) to find the probability that the $z$-value will lie in the interval, of the probability of a type II error.

From the type II error, students can calculate the power and from the given situation students should know that the probability of a type I error is 0.05 or $5 \%$.

Discuss with students what the implications the probabilities have on the test.

## Solutions - student worksheet

1. Type I error
2. Type II error
3. Type I error
4. No error, correct decision
5. True
6. Type II
7. 0.057398
8. 0.942602
9. 0.05


Step 1: Find the interval that contains $\overline{\mathrm{X}}$.
(Use the $\mathbf{z}$ Interval command.)

|  | $\begin{aligned} & \text { "尺F" } \\ & \text { "ME" } \\ & \text { "n" } \\ & \text { " } \sigma \text { " } \end{aligned}$ | $\begin{gathered} 12 \\ 0.027718 \\ 50 \\ 0.1 \end{gathered}$ | $\widehat{\square}$ |
| :---: | :---: | :---: | :---: |
| 1/99 |  |  |  |



The standardized values are
-5.494 and -1.577

| $2.5\|2.6\| 2.7 \mid$ WhatAMistake $\nabla$ |
| :--- |
| Step 3: Using the standardized values, find <br> the probability that the $z$-value will lie in the <br> interval. This is $\beta$. (Use the norm Cdf <br> command.) <br> normCdf $(-5.494,-1.577,0,1)$ 0.057398 <br> $1-0.057397747507478$ 0.942602 |

