REAL LIFE REAL WORLD Activity: Forestry
Similar Triangles and Trigonometry

Topic: Similar Triangles, Proportions & Trigonometry **Grade Level:** 7 - 12
Objective: To model and solve forestry problems with geometry software.
Time: 30-60 minutes

Introduction

Foresters, environmentalists, and other professionals in the timber industry take many measurements in their field work for purposes including forest management planning, forest health monitoring, and timber sales appraisal. Two tools commonly used are a clinometer, to measure angles of elevation, and a Biltmore Stick, to measure diameter and height of trees. These tools are based on the concepts of similar triangles and trigonometric ratios in a right triangle.

Discuss with Students

Similar triangles are triangles with corresponding angles that are congruent and corresponding sides that are proportional.

1. In Figure 1, $\Delta ABC \sim \Delta DEF$. Complete the proportion.

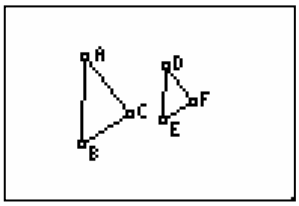

Figure 1

$$\frac{AB}{DE} = \frac{BC}{?} = \frac{?}{?}$$

2. In Figure 2, $\overline{AB} \parallel \overline{DE}$. What angles will be congruent? What triangles are similar? Complete the proportion.
If desired, have students measure angles and sides to discover this relationship.

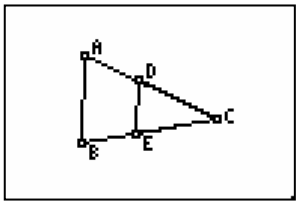

Figure 2

$$\frac{AB}{DE} = \frac{BC}{?} = \frac{?}{?}$$

3. The three trigonometric ratios *sine*, *cosine*, and *tangent* are found by taking ratios of side lengths in right triangles. *If needed, review with students how to identify the opposite, adjacent and hypotenuse for each angle in a right triangle.*

$$Sin = \frac{Opposite}{Hypotenuse} \qquad Cos = \frac{Adjacent}{Hypotenuse} \qquad Tan = \frac{Opposite}{Adjacent}$$

4. In Figure 3, what lengths are needed to find each ratio?

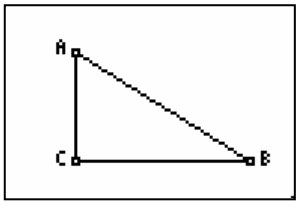

Figure 3

$sin\angle A = \frac{?}{?}$ $cos\angle A = \frac{?}{?}$ $cos\angle B = \frac{?}{?}$ $tan\angle A = \frac{?}{?}$

Karen Droga Campe, Yale University, New Haven, CT

Discuss With Student Answers

1. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

2. $\angle A \cong \angle EDC$ and $\angle B \cong \angle DEC$ because corresponding angles are congruent when parallel lines are cut by a transversal. $\angle C \cong \angle C$ by the reflexive property. The similar triangles are $\Delta ABC \sim \Delta DEC$.
   $\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$

4. $\sin\angle A = \frac{BC}{AB}$ $\cos\angle A = \frac{AC}{AB}$ $\cos\angle B = \frac{BC}{AB}$ $\tan\angle A = \frac{BC}{AC}$

Student Page Answers

Activity 1

5. One possible proportion is $\frac{OS}{OB} = \frac{TS}{AB}$

7. If $\overline{AB}$ is a different length, then the value used in the proportion is different. The units will also be different.

8. It does not matter if $\overline{SB}$ is a horizontal line. The distance measurement could still be $\overline{OB}$ and $\overline{SB}$, but it is more likely in the field to measure the horizontal distance from the feet of the observer to the foot of the tree ($\overline{DS}$ in Figure 4). This is the same as the perpendicular distance from *O* to the tree and is the altitude of the larger triangle. The proportion used in this case would be $\frac{DS}{CS} = \frac{TS}{AB}$.

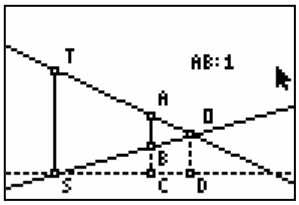

Figure 4

Activity 2

5. Use the trig ratio *tangent*. Use the values measured for $\angle O$ and the length *OS* to solve $\tan\angle O = \frac{TS}{OS}$.

9. Use the trig ratio *tangent*. Solve $\tan\angle TOX = \frac{TX}{OX}$. You also need to know the height of the observer, *OF*. Add this value to the value of *TX* to find the total height *TS* of the tree.

10. The two angles are complements and their sum is 90°. The angle of elevation = 90 – $\angle STO$.

Technology Reference

This activity uses the following Cabri Jr. functions:

F2	Point	F3	Perpendicular	F5	Hide/Show
	Line		Parallel		Alph-Num
	Segment				Measure: D. & Length
	Triangle				Measure: Angle
					Calculate

Refer to the “Introduction to Cabri Jr.” for more details.

Karen Droga Campe, Yale University, New Haven, CT

Name_____ Date_____

REAL LIFE REAL WORLD Activity: Forestry

Foresters, environmentalists, and other professionals in the timber industry take many measurements in their field work for purposes including forest management planning and forest health monitoring. Workers in these industries also need to calculate the height of trees in order to determine the "merchantable height" of the tree to find out how much lumber will be available to sell. Two tools commonly used are a clinometer, to measure angles of elevation, and a Biltmore Stick, to measure diameter and height of trees. These tools are based on similar triangles, since a tree's height usually cannot be measured directly. In this activity, you will use Cabri Jr. or Cabri II+ Geometry software to model and solve forestry problems.

Activity 1

1. Create a segment that represents the measuring stick. See Figure 1.
   - Construct a segment that appears vertical and label it $\overline{AB}$.
   - Measure the segment's length.
   - Drag either endpoint so that the length is 1.

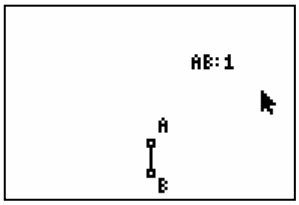

Figure 1

2. Create a parallel segment that represents the tree. See Figure 2.
   - Construct a point not on $\overline{AB}$.
   - Construct a line parallel to $\overline{AB}$ through the new point.
   - Hide the point.
   - Construct a segment whose endpoints are on the parallel line and label it $\overline{TS}$.
   - Hide the parallel line.

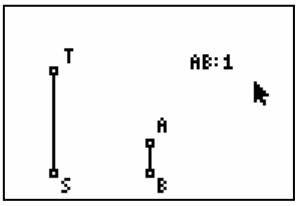

Figure 2

3. Use lines or rays to represent the observer's line of sight and the level of the ground. See Figure 3.
   - Construct a line from *T* (the top of the tree) through *A* (the top of the stick).
   - Construct line $\overleftrightarrow{SB}$.
   - Find the point of intersection of $\overleftrightarrow{TA}$ and $\overleftrightarrow{SB}$ and label it *O* (for observer).

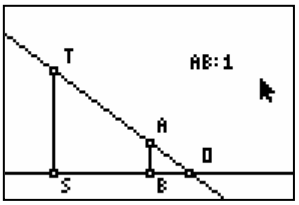

Figure 3

4. Measure the distances from the observer to the bottom of the stick and tree.
   - Measure the lengths of $\overline{OB}$ and $\overline{OS}$.

5. Calculate the height of the tree by setting up and solving a proportion involving the lengths of $\overline{OB}$, $\overline{OS}$, $\overline{AB}$ and $\overline{TS}$.

6. Confirm that your calculation is correct by measuring $\overline{TS}$ in your figure.

7. How will your work change if the original segment $\overline{AB}$ is known to be 3 feet long? Or 25 inches long?

Karen Droga Campe, Yale University, New Haven, CT

8. When a Biltmore Stick is used, it is held in front of the worker above the ground so that the bottom of the tree trunk is sighted in a line with the bottom of the stick. This can be modeled by the diagram in Figure 4. Does it matter if $\overleftrightarrow{SB}$ is a horizontal line? Drag $\overline{AB}$ as shown. Which distance measurement should now be used? (Hint: What measurement is easily made in the field?)

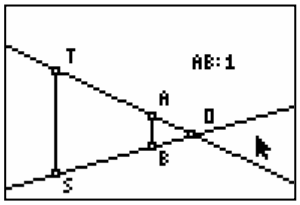

Figure 4

Activity 2

Another way to calculate heights of objects is to measure angles and use trigonometric ratios to solve for the needed height.

1. Construct a segment to represent the tree and label it $\overline{TS}$.

2. Construct a point that is the observer and create a right triangle. See Figure 5.
   - Construct a line perpendicular to $\overline{TS}$ through *S*.
   - Place a point on the perpendicular line and label it *O*.
   - Construct a segment connecting *T* (the top of the tree) with *O* (the observer).

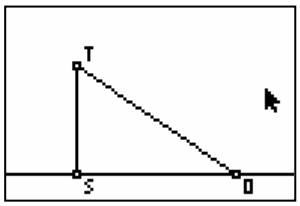

Figure 5

3. Measure the angle of elevation from the observer to the top of the tree. See Figure 6.

4. Measure the length of $\overline{OS}$, the distance from the observer to the bottom of the tree.

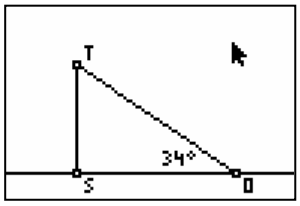

Figure 6

5. Use one of the three trigonometric ratios *sine, cosine,* or *tangent* to set up an equation and solve for the height of the tree.

6. Instead of representing the observer with a point, use a line segment. See Figure 7.
   - Construct line segment $\overline{TS}$ to represent the tree.
   - Construct a line perpendicular to $\overline{TS}$ to represent the ground.
   - Construct a point *F* on the perpendicular line to represent the observer's foot.
   - Construct a line perpendicular to $\overline{SF}$.
   - Place a point on the perpendicular line and label it *O*.
   - Hide the perpendicular line and construct segment $\overline{OF}$

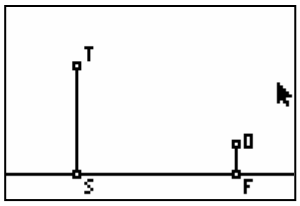

Figure 7

7. Create a new right triangle. See Figure 8.

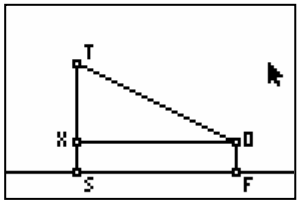

Figure 8

Karen Droga Campe, Yale University, New Haven, CT

8. Measure the angle of elevation and the distance from *O* to *X*.

9. Use a trigonometric ratio to set up an equation and solve for the length of $\overline{OX}$. What other distance do you need to find in order to calculate the height of the tree?

10. A clinometer actually measures the angle made by the tree and the line of sight ($\angle STO$). How is this angle related to the angle of elevation?

Extensions & Resources

Extension 1
Use a yardstick, meterstick or other stick of known length to measure a tree, flagpole, or other tall object near your home or school. Use lengths of shadows on a sunny day.

Extension 2
Create a clinometer and use it to measure heights of tall objects.

Websites to explore:

How a Biltmore Stick is used:
http://www.tree-tech.com/reports/biltmore_stick.xhtml

An interactive lesson on the Biltmore Stick is "Let's Cruise" found at
http://www.ext.vt.edu/resources/4h/virtualforest/

Using a clinometer to measure tree height:
http://www.offwell.free-online.co.uk/newpage2.htm

Making a clinometer:
http://hilaroad.com/camp/projects/clinometer/clinometer.html

Karen Droga Campe, Yale University, New Haven, CT