

Name _____

Date _____

EXPLORATIONS

ACTIVITY 11

Modeling Damped Motion

Periodic oscillations, such as those experienced by a pendulum as it swings back and forth, occur in very predictable patterns. The motion of such objects can be described mathematically in terms of a simple sinusoidal equation. In the real world, however, resistive forces such as friction are always present and cause *damping*.

A *damped oscillator* moves in a modified periodic fashion so that its amplitude gets smaller and smaller with each cycle. Common examples of damped oscillators include an empty rocking chair as it comes to rest after being pushed; a diving board after the swimmer leaves it; and, to some extent, a bungee jumper as he or she slowly bounces to rest at the end of a cord. At first, the problem of modeling this type of motion with a mathematical equation may seem extraordinarily complex, but surprisingly, it can be analyzed rather thoroughly using basic math concepts with which you are already familiar.

In this activity, you will use the Calculator-Based Laboratory 2™ System (CBL 2™) to collect motion data as a paper plate attached to a loose spring oscillates up and down above a motion detector. Then you'll use the Voyage™ 200 PLT with DataMate software to find an appropriate mathematical model for the resulting data set.

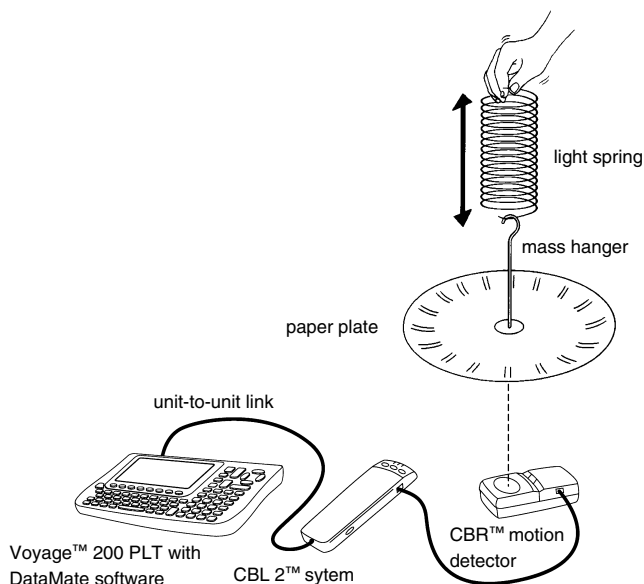
Instructions

Part A—Setup and Data Collection

Setup

You need:

- 1 CBL 2 unit with unit-to-unit linking cord
- 1 Voyage 200 PLT with DataMate software
- 1 CBR™ motion detector
- 1 light spring
- 1 standard slotted mass hanger (about 50 grams, found in most science laboratories)
- 1 paper plate



Modeling Damped Motion *(Continued)*

Data Collection

In this activity, you will record variations in position and velocity as a paper plate attached to a loose spring oscillates up and down above a motion detector.

1. Connect the Voyage™ 200 PLT and CBL 2™ system using the unit-to-unit linking cord. Push firmly on each end of the cord to be sure the connection to each unit is good.
2. Connect the motion detector to the DIG/SONIC port of the CBL 2 system.
3. Attach a paper plate to the end of a standard mass hanger and then attach the hanger to the spring, as shown in the illustration at the beginning of this activity. Place the motion detector face-up on the floor, as shown.
4. Start DataMate by pressing **[APPS]**, selecting DataMate, and pressing **[ENTER]**.
5. Press **[CLEAR]** to reset DataMate.
6. The spring-plate assembly typically takes about 10 seconds for the amplitude of motion to decay significantly. To collect data for 10 seconds, you need to adjust the data collection parameters.

Press **[1]** **[←]** **[ENTER]** to edit the MODE settings. Press **[2]** **[2]** to access the time settings. Enter 0.1 as the time between samples and 100 as the number of samples. Press **[1]** to return to the setup screen.

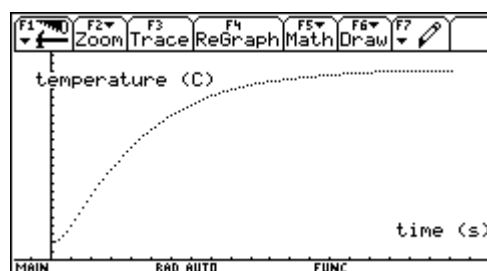
7. In order for the zero reference position to be the equilibrium position of the plate, you must zero the motion detector.

Hold the spring-plate assembly at least one meter above the detector. Press **[3]** to select ZERO from the setup screen, and then press **[1]** to select DIG-DISTANCE from the SELECT CHANNEL menu. Confirm that the plate is still stationary. Press **[ENTER]** to zero the motion detector.

8. When you are ready to start collecting data, pull the plate downward about 10 cm, and allow it to oscillate up and down. Press **[2]** on your Voyage 200 PLT to start collecting data. A graph of the motion data you are collecting will be displayed.

Modeling Damped Motion (Continued)

9. When data collection is complete, view the motion data by selecting the DIG-DISTANCE graph and pressing **ENTER**.
10. Your data should show a modified period motion, centered on the x -axis, with amplitudes that decrease noticeably as time passes.
 - If your data set is not satisfactory, press **ESC** **1** **1** and repeat steps 7–9 to do another trial. *Once you zero the motion detector, be sure to keep your hand supporting the spring-plate assembly at a constant height.*
 - If your data set is satisfactory, sketch a plot of your distance versus time data in the screenshot provided to the right. Be sure to include axes on your sketch.



Part B—Data Analysis

1. To start, you will attempt to model the motion data you collected using a simple sinusoidal function of the form

$$y = a \cos(b(x - c)),$$

where

x = time (in seconds),

y = distance (in meters), and

a = the *amplitude* of the motion, which is the largest distance above or below the center position.

Press **⬇** and **⬅** to trace along the plot.

Identify the first maximum distance value (y -value), and record it as a in the space below.

$a =$ _____ meters

2. Identify the time (x -value) at which the first distance maximum occurs, and record it below. Then trace to the second distance maximum, and record its time (x -value) below.

time for first maximum = _____ seconds

time for second maximum = _____ seconds

You can now exit DataMate. To do this, press **ESC** **1** to return to the main screen, and then press **6** to quit. Press **⬇** [GRAPH] to view the graph again. If necessary, press **F2** **9** to rescale the graph based on your data.

Modeling Damped Motion (Continued)

6. (continued)

How does the modeling equation fit the data vertically? What variable in the equation $y = a \cos(b(x-c))$ should be changed, and how should it be changed to make the curve fit the data?

7. To account properly for amplitude variations, you need to create a second mathematical model that describes how the amplitude changes as a function of time. To do this, identify the set of maximum distance values and their corresponding time coordinates as follows:

To start, press \blacklozenge [CALC HOME] to return to the Home screen. Type **DELVAR SYSDATA**, and then press [ENTER]. Press \blacklozenge [Y=] to access the Y= Editor. Turn off the functions by pressing [F5] and selecting 3:Functions Off. Return to the plot of motion versus time data by pressing \blacklozenge [GRAPH].

Press [F3], trace to the first maximum value, and press \blacklozenge **D**. This operation selects the x - and y -coordinates of the cursor location and automatically stores them in the data variable *sysdata*. Trace to the next maximum, and press \blacklozenge **D** again.

Repeat this procedure for all maximums in the data set.

To see a listing of the maximum distance data, press [APPS], highlight the Data/Matrix Editor icon, press [ENTER], and select 1:Current. Move the cursor to any cell in column c2 and press [F4]. Type **SYSDATA[1]** and press [ENTER]. Move the cursor to any cell in column c3 and press [F4]. Type **SYSDATA[2]** and press [ENTER]. The maximum distance values are now listed in columns c2 and c3.

Press [F2] \odot [F1] to define Plot 2. Press \odot **1** to set the Plot Type to Scatter; press \odot \blacktriangleright **1** to set the Mark to Box; press \odot and type **C2** to define the x -list; press \odot and type **C3** to define the y -list.

Press [ENTER] [ENTER] to store the plot definition, and then press \blacklozenge [GRAPH] to display the original data and highlighted maximums together.

Modeling Damped Motion (Continued)

8. The distance maximums start at some fixed value and continually decrease as time passes. Accordingly, their variations are appropriately modeled using an exponential curve. Use the statistical tools of the Voyage™ 200 PLT to find the exponential equation that best fits the data you found as follows.

To reopen the current Data Editor, press $\boxed{\text{APPS}}$, highlight the Data/Matrix icon, press $\boxed{\text{ENTER}}$, and select 1:Current. To perform an exponential regression on the maximum distance data, press $\boxed{\text{F5}}$ \blacktriangleright and select 4:ExpReg. Press \blacktriangleleft and type **C2** for the x -values; press \blacktriangleleft and type **C3** for the y -values.

Press \blacktriangleleft \blacktriangleright \blacktriangleleft \blacktriangleleft $\boxed{\text{ENTER}}$ to store the regression equation to y_2 . Press $\boxed{\text{ENTER}}$ again to perform the calculation.

Copy the complete ExpReg equation that appears in the dialog box in the space below.

Press \blacklozenge $\boxed{\text{GRAPH}}$ to see the data and exponential regression curve together. How well does this equation fit the distance maximums?

9. The reason the original modeling equation, $y = a \cos(b(x - c))$, provided a poor fit is that the amplitude value, a , was fixed. As you have just seen, the appropriate amplitude expression, y_2 , changes continually with time. So, to fit the original motion data completely, you'll need to replace the constant value, a , with the dynamic expression, y_2 .

To do this, press \blacklozenge $\boxed{\text{Y=}}$. Turn off the functions by pressing $\boxed{\text{F5}}$ and selecting 3:Functions. Move the cursor to highlight equation y_1 . Press $\boxed{\text{ENTER}}$ to copy y_1 to the entry line, and press \blacktriangleleft to position the cursor at the beginning of the expression.

Press \blacklozenge $\boxed{\text{DEL}}$ to remove the constant a from the equation. Press **Y2** $\boxed{\text{X}}$ $\boxed{\text{ENTER}}$ to make y_2 the new amplitude expression in equation y_1 .

Press \blacklozenge $\boxed{\text{GRAPH}}$ to see the original motion data together with the new modeling curve. How well does this curve fit the data?

Modeling Damped Motion *(Continued)*

Extension for Calculus

1. What would a plot of velocity versus time look like for the damped oscillations you studied in this activity? Create a statistical plot of velocity versus time. Note that the time and velocity values are found in columns c1 and c7, respectively, of the data variable named *cbldata*.

2. Use your Voyage™ 200 Personal Learning Tool's computational tools to find a mathematical equation that models velocity as a function of time. It is possible to create this model in just one step with the Voyage 200 PLT.

Hint: Think of velocity as the time rate of change of the distance.

How well does your modeling curve fit the data?

Modeling Damped Motion (Continued)

Questions

1. Would it be possible to completely fit the motion data using a sine model rather than a cosine model? Specifically, which variable(s) in the equation $y_1 = y_2 \cos(b(x - c))$ would have to be adjusted? Record the modified equation in the space below.

Check your prediction by storing new values to b and/or c . Then press \blacklozenge [Y=], and modify the y_1 equation. Press \blacklozenge [GRAPH] to view the modified equation. How well does it fit?

2. Suppose that your friend is resting on a playground swing. You start your friend moving with a small push and continue to push once each cycle. If the resulting amplitudes get larger and larger with each cycle, the result is called antidamping.

Describe how a short-term model for this situation might differ from the equation used in this activity if the oscillation amplitudes increased by approximately the same percentage during each swing.

Hint: Think about how the exponential equation modeling amplitudes would be modified.

3. What would a plot of velocity versus distance look like for the damped oscillator studied in this activity? Record your prediction on a separate sheet of paper.

While the CBL 2™ system was recording distance values during the damping activity, velocity values were being computed simultaneously and are stored in your Voyage™ 200 PLT in the Data/Matrix Editor. To view a plot of velocity versus distance, press \blacklozenge [Y=] [F5] and select 1:All Off to turn off all plots and functions. Press \odot to scroll above the $y(x)$ functions, highlight a plot that is not defined, and press [ENTER].

Press \blacktriangleright 2 to set Plot Type to xyline; press \odot \blacktriangleright 5 to set Mark to Dot; press \odot and type C6 to define the x -list; press \odot and type C7 to define the y -list. Press [ENTER] [ENTER] to store the plot definition, and then press [F2] and select 9:ZoomData to display the plot.

How does the velocity-versus-time plot compare with your prediction? Use your own words to explain exactly why this plot has such a peculiar shape.

Hint: First, consider what a plot of velocity versus position looks like if the motion is not damped.