Binomial Experiments – ID: 9999

By Michele Patrick

Time required
30 minutes

Activity Overview

In this activity, students use the multiplication rule for independent events to find the probability of the first success in the nth trial. Students use their results to derive and test a general formula. Then, students expand on this foundation to derive and test a rule for the probability of x successes in n trials.

*An optional extension asks students to use the **binomPdf** command to find binomial probabilities.*

Concepts

- *Probability*
- *The Multiplication Rule for Independent Events*
- *Combinations of n items taken x at a time*
- *Binomial Experiments*

Teacher Preparation

- *This activity is designed to be used for students studying Statistics and Probability.*
- *Students should already understand the difference between independent and dependent events, replacement and nonreplacement when selecting items, and the multiplication rule for independent events.*
- *Students should also be familiar with arrangements of items and combinations of n items taken x at a time.*
- *The screenshots on pages 2–5 (top) demonstrate expected student results. Refer to the screenshots on pages 5 (bottom) and 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "9999" in the quick search box.***

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet StatAct13_BinomExp_worksheet_EN helps guide students through the activity and provides a place for students to record their answers.*
- *Information for an optional extension is provided at the end of this activity, both on the student worksheet and in the student .tns file. Should you not wish students to complete the extension, you may delete the extension from the student .tns file and have students disregard that portion of the student worksheet.*

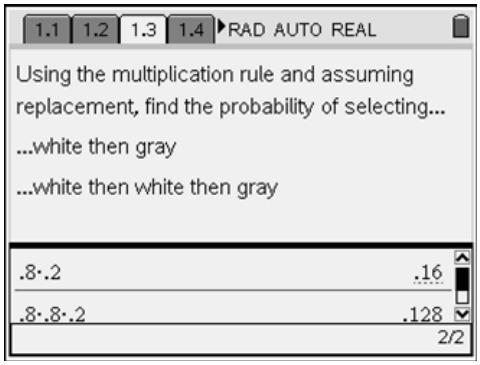
TI-Nspire™ Applications

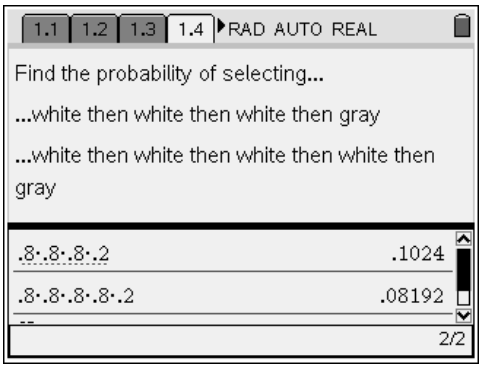
Calculator, Graphs & Geometry, Notes

Problem 1 – The First Success

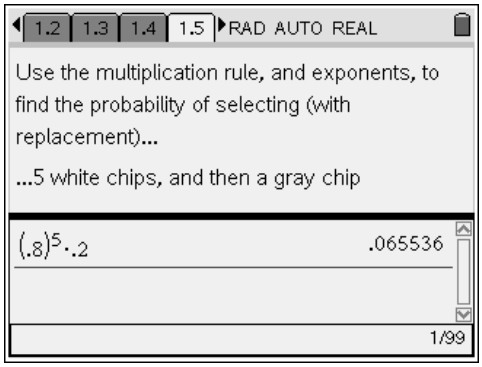
Read through page 1.2 with students. Make sure students calculate the correct probabilities: *p*(gray chip) = 0.2, *p*(white chip) =0.8. If needed, review the meaning of *replacement* and be sure students understand that the events are independent, so the probabilities of 0.2 and 0.8 remain constant.

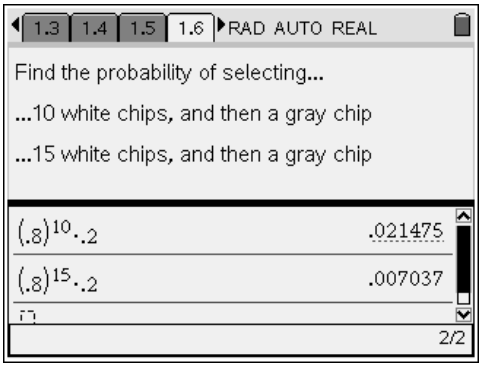
Have students find the indicated probabilities on pages 1.3 and 1.4.





Tell students to read page 1.5 and to use the patterns shown on page 1.3 and 1.4 to find the indicated probabilities. Instruct students to use exponents instead of repeated multiplication.

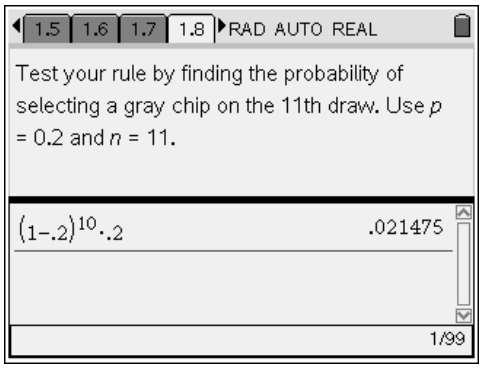




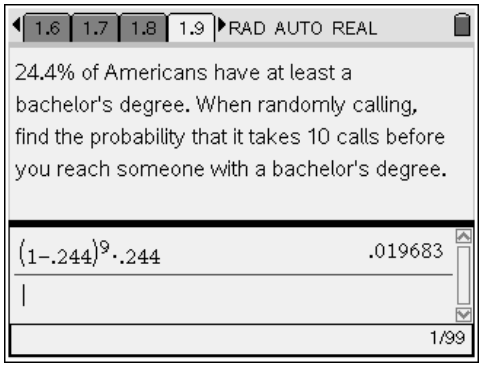
On page 1.7, students are asked to generalize a rule for finding the probability of getting the first success on the *n*th trial. Let students volunteer suggestions and determine the rule as a class. Students should end up with an expression along the lines of $(1-p)^{n-1} \cdot p$.

You may wish to explain that some books will use *q* to represent 1 – *p*, the probability of a failure.

Once the class has determined the general rule, the students will test the rule on page 1.8.



When students have created, tested, and understand the rule on page 1.7, have them advance to page 1.9 and answer the question by using the *Calculator* application at the bottom of the page. ($P \approx 2\%$)

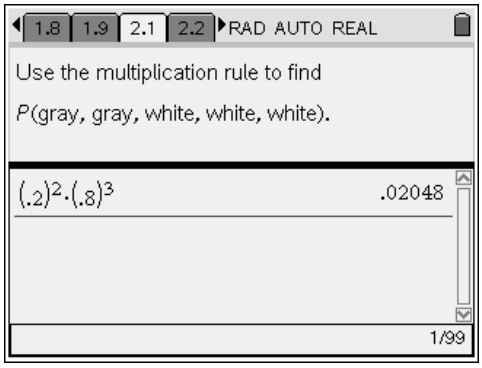


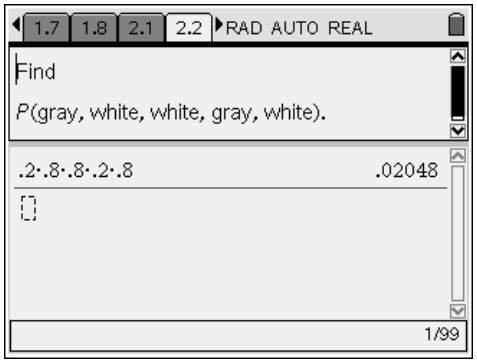
Problem 2 – The Binomial Probability Formula

Have students find the indicated probabilities on pages 2.1 and 2.2.

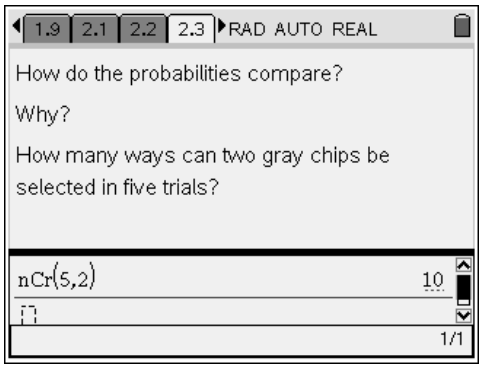
The scenario is the same as in Problem 1: *P*(gray chip) = 0.2 and *P*(white chip) = 0.8 and the selections are made with replacement.

It does not matter if students use exponents or repeated multiplication.





Students will notice that the probabilities are the same. Discuss why. Ask students how many ways two gray chips can be chosen in five selections.



Some students may wish to make a list, which is useful for confirmation, but ask if there is another way to find this number. Based on what has already been covered in your class, either show this number as $\frac{n!}{(n-x)!x!}$ or ${}_nC_x$ (found in **MENU > Probability > Combinations**).

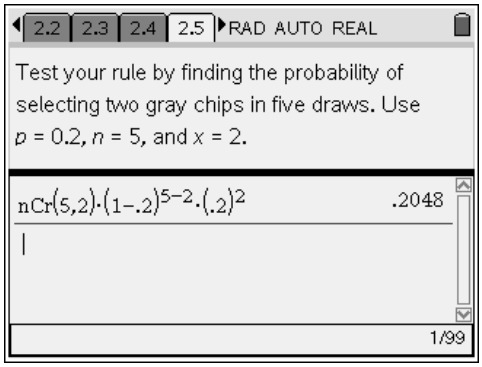
Have students move to page 2.4 to write a general rule for the probability of *x* successes in *n* trials. It may help to write the following on the board:

# of ways to arrange *x* successes • probability of *x* successes (in one arrangement)

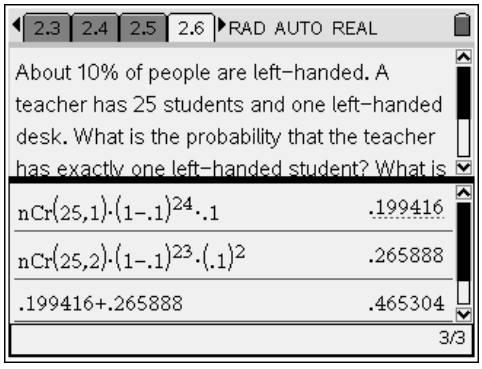
Students should agree on one of the following expressions:

$$\frac{n!}{(n-x)!x!}\cdot(1-p)^{n-x}\cdot p^x \quad \text{or} \quad {}_nC_x\cdot(1-p)^{n-x}\cdot p^x$$

When students perform the calculation for the given values of *p*, *n*, and *x*, they will see that it is 10 times the value found on pages 2.1 and 2.2. This is the same as adding 0.02048 ten times for each of the 10 possible arrangements.



Students should now be able to use the general rule to answer the questions on page 2.6. To answer the second question, they will need to add the two individual probabilities.
($P(1) \approx 20\%, P(1 \text{ or } 2) \approx 46.5\%$)

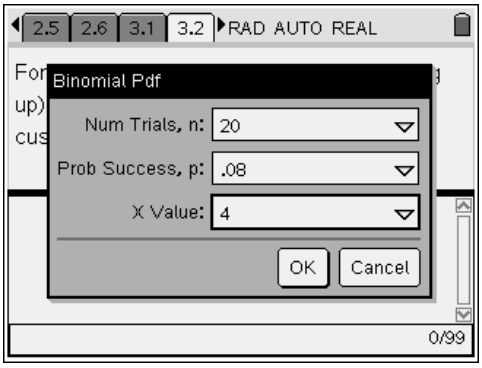


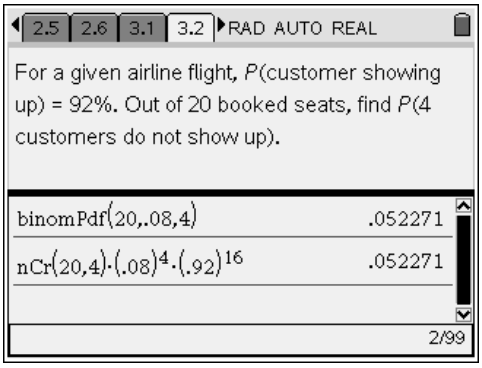
Conclude the activity by asking students what is true about all the experiments/scenarios in Problem 2. Lead them to answer that the events are always independent, so the probabilities remain constant, there are always two categories (success and failure), and there are a fixed number of trials. These are the requirements for a binomial experiment.

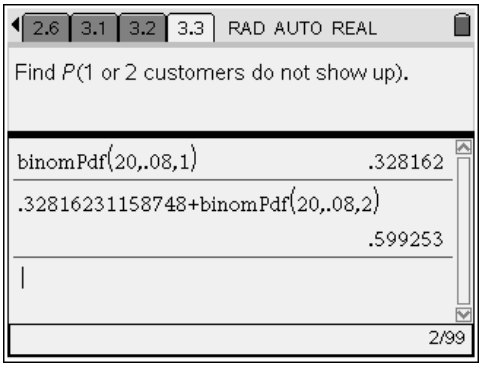
Problem 3 – Extension

In the Extension, students are asked to use the **binomPdf** command to answer questions. To use the **binomPdf** command (**MENU > Probability > Distributions > Binomial PDF**) students enter *n*, *p*, and *x* as requested. They could also just type **binompdf(20, 0.08, 4)**. Note that the **binomPdf** command is also listed in the catalog.

Encourage students to check their work by using the formula they found in Problem 2.







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(Student)TI-Nspire File: *StatAct13_BinomExp_EN.tns*