Area Accumulation

ID: 12241

Time Required
15 minutes

Activity Overview

In this activity, students will numerically and graphically investigate the integral using area accumulation. This exploration uses animation and graphical observations to develop understanding. Self-check questions engage students and help deepen understanding. Geometry Trace is used to see the family of functions produced by changing the value of a in $\int_a^x f(x)dx$ *. Also the kinematic relationships for integration are used with a piecewise function and with actual data. Students are to predict and sketch the graph with the given initial conditions.*

Topic: Average

- *Fundamental Theorem of Calculus*
- *Curve sketching and the graphical relation between the integral and the derivative.*
- *Relationship between displacement, change in velocity and acceleration.*

Teacher Preparation and Notes

- *Part 1 of this activity takes about 15 minutes. Part 2 can be used as an extension or homework. In Part 2, students should predict the graph, explain their sketch with a group or partner, and then reveal the solution.*
- *Students will write their responses directly into the TI-Nspire document and/or on the accompanying handout. On self-check questions, after answering the question students can press* (menu) *and select **Check Answer** (or* (ctrl) *+ ▲).*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12241" in the quick search box.***

Associated Materials

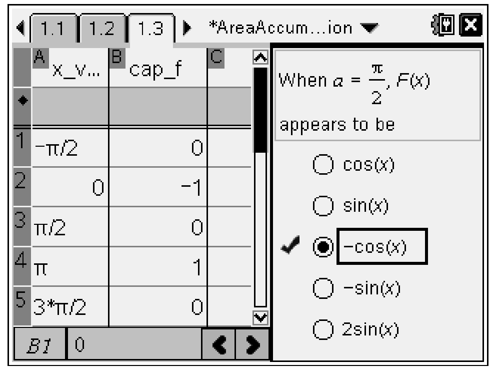
- *AreaAccumulation_Student.doc*
- *AreaAccumulation.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

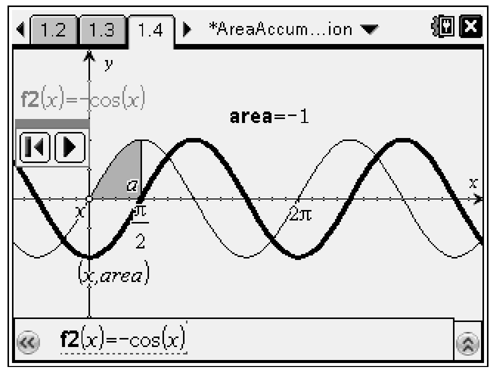
- *Fundamental Theorem of Calculus (TI-84 Plus) — 4373*
- *Exploring the Fundamental Theorem of Calculus (TI-Nspire CAS technology) — 9205*

Part 1 – Numerically and Graphically Investigate Integral

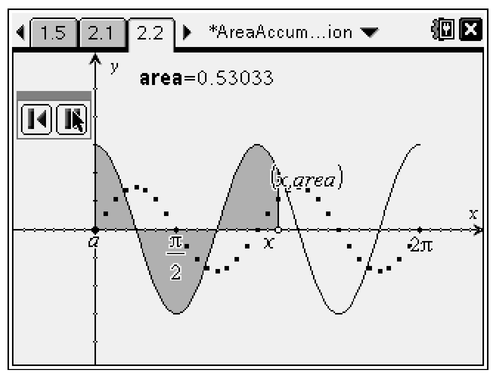


A graph of the integrand is provided and the students are to discover the function for the integral numerically and graphically. Students are also instructed to make observation on page 1.4 for what characteristics of $f(x)$ result in $F(x)$ increasing and decreasing. Instruct students and press play to observe when the point (x, *area*) is increasing and when it is decreasing.

Note: Questions on pages 1.3 and 1.5 pertain to the graph on page 1.4 so that students only need to advance forward and backward one page when exploring and pondering their answer.



Students should to graphically confirm the answer to Question 1, by graphing their answer in **f2**(x). Alternatively, students could quickly numerically confirm their answer by entering the formula **–cos(a[ ])** or **–cos(x_val)** in the gray cell of Column C. Note: The underscore symbol on the handheld can be found in the symbols menu (⌸).

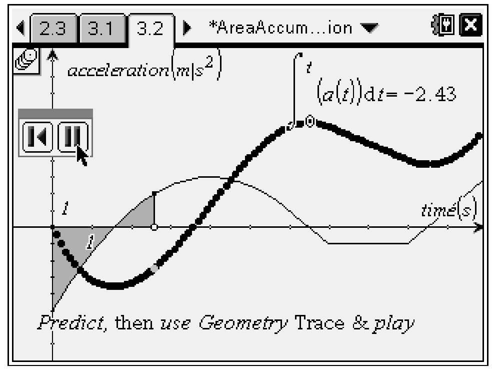


Using **Geometry Trace** (**MENU > Trace > Geometry Trace**) on page 2.2 to get a graph similar to the one on the right, requires that the students click (not grab) the point (x, *area*) only once. They should then press play. So as not to make their graph too sloppy, when the area is undefined, students should grab and move point a to other values, like $\frac{\pi}{4}$ and π.

Student Solutions

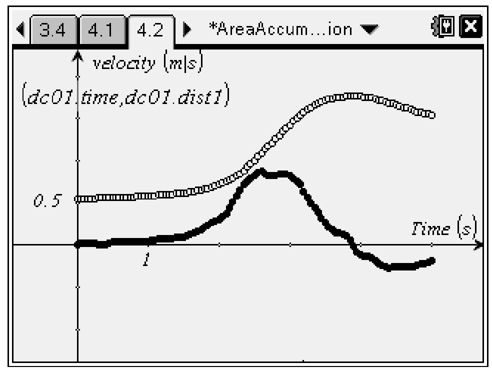
1. When $f(x)$ is positive, $F(x)$ is increasing. When $f(x)$ is negative, $F(x)$ is decreasing.
2. $F\left(-\pi/2\right) = 0$, $F(0) = -1$, $F\left(\pi/2\right) = 0$, $F(\pi) = 1$, $F\left(3\pi/2\right) = 0$, $F(2\pi) = -1$
3. $F(x) = -\cos(x)$
4. The integral of $f(x)$ is *sometimes* negative when $x < a$. (But as shown on the handheld the integral from a to b is ALWAYS the negative integral from b to a.) Students should observe that $F(x)$ is the same general function but shifted vertically.
5. –1
6. 0.75
7. Vertically shifts the graph 0.75 up and down from the origin.
8. The integral is increasing with **f1**(x) is positive and decreasing when **f1**(x) is negative.

Part 2 – Homework/Extension – Kinematic Curve Sketching

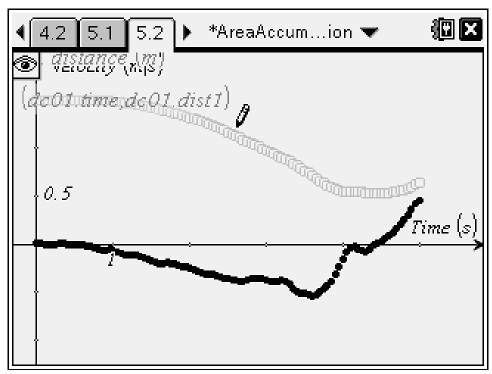


This section enables students to compare the graph of the integrand to the graph of the integral in the context of distance, velocity, and acceleration. Students should predict and sketch the curve before revealing and confirming their solution. It is also recommended to have students discuss their sketch with their neighbor and explain what they did before they reveal the answer. Communicating the relationship of a graph of a function and the graph its integral is important in calculus.

The solutions to the graphs are given on the right.



In considering the final position (or final velocity) have students think about the area. Note the initial condition is important since the integral gives the *change in* position.



Student Solutions

1. **a.** change in velocity

b. displacement = change in velocity

2. The object starts from rest, has negative velocity (moves backward), then positive velocity (forward)

3. Integral of a parabola is cubic
Integral of constant function is linear
Integral of an oblique line is quadratic

4. **a.** The object on 4.2 (left graph on worksheet) ends at about 1.5 m

b. The object on 5.2 (right graph on worksheet) ends at about 0.5 m.