How Far Am I Off: Estimating σ

ID: 12442

Time required
15 minutes

Activity Overview

In this activity, students will calculate a confidence interval using the chi-square distribution to estimate a population variance. The homework problems have students estimate the mean and standard deviation of the population given a sample.

Topic: Statistical Inference

- *Chi-square distribution*

Teacher Preparation and Notes

- *Graphs are included in the solution TI-Nspire document if more investigation is desired.*
- *The solution document also contains the complete data sets for the homework problems. Random samples were used to produce the mean and standard deviation. Other random samples could be found with other sample means and standard deviations.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12442" in the quick search box.***

Associated Materials

- *HowFarOff_Student.doc*
- *HowFarOff.tns*
- *HowFarOff_Soln.tns*

Suggested Related Activities

To download any TI-Nspire technology activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Population Mean: σ unknown (TI-Nspire technology) — 12388*
- *Chi-Square Distributions (TI-Nspire technology) — 9738*

Problem 1 – Assumptions

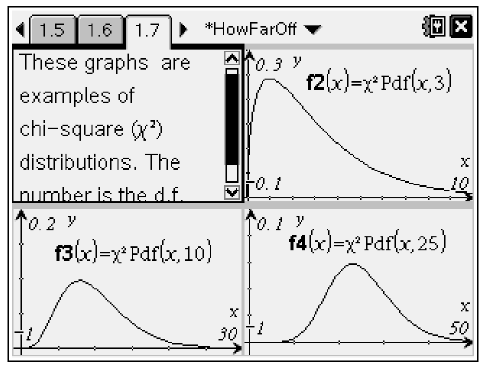
Explain to students that when they previously learned to estimate a population mean, it was possible because of the Central Limit Theorem (sample means follow a normal distribution).

Students will now estimate a population variance. However, the standard deviation follows a chi-square distribution. Review with students that the chi-square (χ^2) distribution is represented by the formula $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ where n = sample size, s = standard deviation for the sample, and σ = standard deviation for the population.

This distribution is not symmetric; it is skewed to the right. On page 1.7, students will see three graphs with degrees of freedom (n – 1) of 3, 10, and 25.

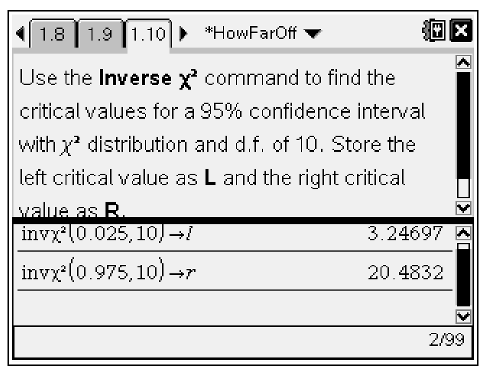
Note: The graphing windows have different scales.

Ask students: *What do you notice about the shape of the distribution as the degrees of freedom increase?* They should see that the graph becomes more symmetric.

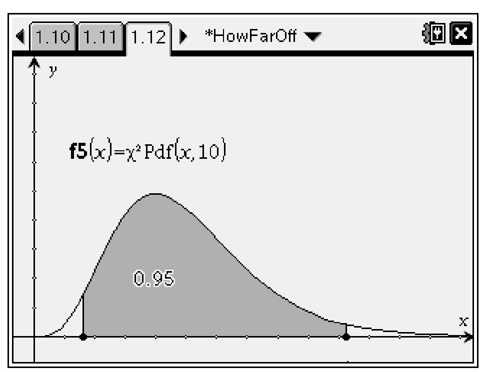


Discuss with students that confidence intervals which estimate a population variance need two different critical values as shown in the figure at the right. (This is different than with a t- or z-distribution where they are symmetric.)

Students are to find the critical values for a sample size of 11 at 95% confidence.



On page 1.12, students will verify that the area under the curve between the two critical values is 95% or 0.95. This is to be done using the **Integral** tool (**MENU > Measurement > Integral**). They will once again see that the area underneath the curve is not symmetric.



Problem 2 – Estimating the Interval

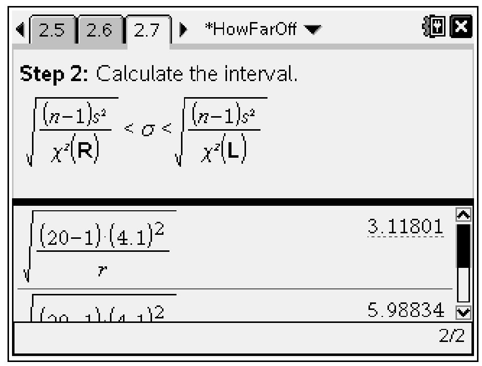
The formula for the confidence interval is given in the .tns file and on the worksheet. Discuss with students or have them discuss with a partner how to get from the formula for the χ^2 distribution to the confidence interval formula. (It is solved for σ).

Explain to students that when the square root of both sides of an equation in Algebra is taken, one side is $\pm\sqrt{\ }$. In Statistics, the positive and negative sign is the left and right critical values.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \Rightarrow \quad \sigma^2 = \frac{(n-1)s^2}{\chi^2} \quad \Rightarrow \quad \sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

Ask students: *Why are the denominators of the fractions seemingly reversed?* They should see that if not reversed, the fraction on the left would be larger than the fraction on the right.

$$\left(\frac{1}{\text{large}} = \text{small} \quad \frac{1}{\text{small}} = \text{large}\right)$$



Students are given the details of a random sample on page 2.3. A random sample of 20 cereal boxes has a standard deviation of 4.1 grams per box.

To find the 95% confidence interval students will:

- Find χ_L^2 and χ_R^2. (8.9 and 32.85)
- Calculate the endpoints of the interval using the formula above.

Sample Response: I am 95% confident that the population standard deviation of all the cereal boxes is between 3.12 and 5.99.

Homework

For both homework problems, students will need to use what they have previous learned to calculate a confidence interval to estimate the population mean and then also practice calculating a confidence interval to estimate the population standard deviation. Encourage students to write their findings in complete sentences.

The data for the entire population is given for each problem. Students can use the **One-Variable Stats** command to find the actual mean and standard deviation.

1. Mean: (Use *t*-distribution since σ is unknown.) (173343, 488119)

Standard deviation: (289931, 526915)

Actual: μ = 240,253; σ = 277,200

2. Mean: (Use *t*-distribution since σ is unknown.) (66.37, 69.83)

Standard deviation: (56.89, 84.86)

Actual: μ = 68.7; σ = 6.82

Extension (or review):

A. The data sets could be used to review normal distributions. Students could graph the data set, find means, and standard deviations. They can verify that the data set is normal using percentages and normal probability plots.

B. Students could use the data sets to find other random samples. Have each group create a different random sample and compare the standard deviations and means. How close are they to each other? Why is there a difference? This stresses the idea that different random samples can have different results.