# Teachers Explanatory Notes
## TI-15 Explorer™: Finding Patterns

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The introduction of linear algebra often starts with simple numerical patterns that increase by a set amount. For example 5,9,13 ,………. etc. This pattern starts at 5 and increases by 4. Most students can readily describe the pattern as "starting at 5 and increasing by 4" but have difficulty predicting say the 10th or 22nd number. In other words they can continue building the pattern from the previous term (recursion) but cannot leap ahead and make a prediction for a distant term.

Generally a trial and error process is employed by students, with a lot of prompting until a "Rule" is developed which involves multiplying by a constant and adding another number, in this case multiply by 4 and add 1 so the 10th term would be $10 \times 4 + 1$ or 41 and the 22nd term would be $22 \times 4 + 1$ or 89. This "Rule" eventually becomes known as the equation of a line or the implicit function of a line and is written in various symbolic forms but it is still the same rule that students initially articulate in words. ***The issue for students, and teachers, is arriving at this rule in the first place.***

Enter the "Division Algorithm" which basically says that any integer can be made by multiplying two other integers and adding a third, for example 57 can be described as 11 multiplied by 5 plus 2 (this is just one way) so it follows that 57 divided by 5 is 11 with 2 left over[1] (note: zero is an integer). ***This is the same as the rule we were talking about for linear patterns, ie multiply and add.***

If we go back to our example and look at each term we see that:

The 1st term, $5 \div 4 = 1$ r 1 which can be reversed to $4 \times 1 + 1 = 5$
The 2nd term, $9 \div 4 = 2$ r 1 which can be reversed to $4 \times 2 + 1 = 9$
The 3rd term, $13 \div 4 = 3$ r 1 which can be reversed to $4 \times 3 + 1 = 13$
The 10th term, $? \div 4 = 10$ r 1 which can be reversed to $4 \times 10 + 1 = ? = 41$
The 22nd term, $? \div 4 = 22$ r 1 which can be reversed to $4 \times 22 + 1 = ? = 89$

**The divisor is the same, the remainder is the same and the quotient is the same as the term number.** If students can discover this they will have developed an understanding of a linear function and the reversibility of the Division Algorithm.

This unit of work aims to help students discover this relationship and use it to develop a way of describing linear patterns and predicting outcomes based on those patterns.

The unit uses a unique property of the TI-15 Explorer™ Calculator which can be used to perform a division and display the answer as a quotient and remainder.

Also there is some terminology that should be mastered, students should learn the meaning of "Divisor", "Dividend", "Quotient" and "Remainder" and then use these terms to describe and discuss what they are doing. This will be addressed early in the unit.

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1 Division Algorithm: For integers a and b, with $b > 0$, there exists unique integers q and r such that $a = bq + r$ where $0 \leq r < b$.
In the division of a by b the number q is the **quotient** and r is the **remainder** (Concise Oxford Dictionary of Mathematics).

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### Session 1

**Definitions**

**Dividend:** **The number that is being divided**
**Divisor:** **The number that you are dividing by**
**Quotient:** **The whole number part of the answer**
**Remainder:** **What is left of the answer, in other words, what remains**

*Note: Some dictionaries vary on these definitions. Where the answer is in decimal form the entire answer is generally referred to as the quotient.*

NB:

Quotient → 6 r 4 ← Remainder
Divisor → 7) 46 ← Dividend

or

Dividend → 46 ÷ 7 = 6 r 4 ← Remainder
Divisor → ÷ 7 ; 6 ← Quotient

**Part A**

Show students how to use the Int÷ function on the TI-15 Explorer™ (see the accompanying power point presentation).

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Using the TI-15 Explorer™ key in the following:

$\boxed{4}\boxed{6}\boxed{\text{Int}\div}\boxed{7}\boxed{\text{Enter}}$

The screen should look like this:

| |
|---|
| 46 ÷ 7 = 6 r 4 |

Ask students to find numbers that when divided by 7 have a remainder of say 4. See who can get the most in a given time (1-2 minutes).

*Significant Questions:*

- Tell me some numbers that have a remainder of 4 when you divide by 7?
- Can you work out which numbers will give a remainder of 4 when you divide by 7 before you start?
- Write down a few and check using the Int÷ key on the TI-15 Explorer™
- What is the smallest non negative integer that will give a remainder of 4 when divided by 7?
- If you go from smallest to biggest can you see a pattern?

**Part B**

1. Now let's do some investigations where we always use Int ÷ 3 (i.e. the divisor will always be 3). Ask students: Can you find a starting number (i.e. a dividend) that will give us a remainder of 0 after we perform Int÷ 3? Can you find another? How many are there?

2. Write a list of all suitable "starting numbers" (dividends) on the board. Ask "Does anyone notice something special about all these numbers?" (They are all multiples of 3). [At this stage it may be suitable with some classes to briefly discuss the relationship between division and multiplication]

3. Have students record their results in Worksheet 1. W1

4. Repeat steps 1 & 2 for a remainder of 1 and record results on Worksheet 2. W2

5. Repeat steps 1 & 2 for a remainder of 2 and record results on Worksheet 3. W3

6. Have students write their results into the tables in Worksheets 1, 2 & 3.

*Significant Questions:*
- Is it possible to get a remainder of 3 when using Int÷ 3? Why not?
- Is it possible to have a quotient of zero?

### Session 2

**1. Find the missing numbers;**

**Review using Int÷**
Put the following numbers on the board and ask the students to find a divisor that gives a common remainder for all of them.
56, 83, 200, 335, 51284.

*In each case when divided by 9 there is a common remainder of 2. Ask students how many numbers they "tested" before they were confident that they had the correct "Divisor"*

Nominate a simple divisor and remainder, divisor 4 and remainder 3 are good, get students to test every number from zero to 30 and have them record the quotient and dividend in a table of those numbers between 0 and 30 that show a remainder of 3 when divided by 4.

This column often helps students recognise the multiplier/divisor but can be left out

| Divisor is 4. Remainder is 3 | | |
|---|---|---|
| Quotient | Dividend | Dividend minus remainder |
| 0 | 3 | 0 |
| 1 | 7 | 4 |
| 2 | 11 | 8 |
| 3 | 15 | 12 |
| 4 | 19 | 16 |
| 5 | 23 | 20 |
| 6 | 27 | 24 |

÷ these numbers by 3

To get these answers with a remainder of 2

*Significant Questions:*
- If you wanted to divide these numbers (dividends) by 4 and get a remainder of zero, ie divide evenly, what would you have to do to the dividends?
- What happens if you go in reverse, that is multiply the quotient by the divisor and add the remainder?
- Will this work for any divisor and remainder?
- What about something big like a divisor of 32 and a remainder of 17?

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Have students plot the dividend against the quotient on a coordinate axis (see BL Master # 1) with the quotient on the horizontal axis and the dividend on the vertical axis.

*Significant Questions:*
- What do you notice about the points you have plotted?
- What happens if you join the points with a line and keep the line going in both directions?
- Can you find any more numbers that have the same remainder when divided by your chosen divisor?

W4

*Student Review:*
In your books write down the mathematics you have learnt today.
- What did you learn from the activities?
- What did you learn about quotients and remainders?
- What sort of patterns did you see?

**2. Finding Patterns using Int÷**

Students find the missing number in linear number patterns by using the Int÷ function to find a divisor that gives the same remainder for all the numbers in the linear pattern.

| Divisor is 7. Remainder is 3 | | |
|---|---|---|
| Quotient | Dividend | Dividend minus remainder |
| 0 | | |
| 1 | | |
| 2 | 17 | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | 45 | |
| 7 | | |
| 8 | 59 | 56 |
| 9 | 66 | |

Example 1: ____ , 29, 38, 47, ____ , ____ Find the missing numbers in the pattern.

Students use the Int÷ button and discover that Int÷ 9 gives a remainder of 2 in all 3 cases. They can then find the missing numbers by adding or subtracting 9.

Example 2: ___ , ___ , 17, ___ , ___ , ___ , 45, ___ , ___ , 66

Again students use the Int÷ button to discover that 7 will divide into each of these numbers with a common remainder of 3.

Students will find it easier if they set up a table and list the quotients from 0 to 9, the quotient when 66 is divided by 7.

By completing the dividend minus remainder column students will get a clue as to how to develop a rule in the next section.

Have students practice finding patterns using the Int÷ key,

1. *To make up sets of numbers for a linear pattern choose a divisor and a remainder then multiply different integers by the divisor and add the remainder.*
2. *Don't make the numbers too hard too soon.*
3. *Encourage students to make up their own patterns and check them using the Int÷ key.*
4. *If you have an interactive white board in your classroom use the IWB package that comes with this unit to allow students to post their points and build up a graph of the pattern.*
5. *Once a few points are on the board show the line then see if any subsequent points fit the line.*

It is quite possible to get more than one linear pattern from a linear sequence of numbers,

- For example 11, 20, 29, …………………
- Gives a remainder of 2 when divided by both 3 and 9
- The pattern rule could be multiply by 3 and add 2 or multiply by 9 and add 2,
- Which one is it?
- Answer; Both. *(it is useful to let students get as many linear patterns as possible from a sequence of numbers)*
- If 11 was the 3rd term then the rule is x 3 + 2 but if 11 was the first term then the rule is x 9 + 2.

This allows discussion of the co ordinate system, if the terms were described as (1,11) or (3,11) then there could be no ambiguity

Making the transition from a group of numbers with a common divisor and remainder to a pattern with a rule in the form of $y = mx + b$ is a very important series of steps. Students should be encouraged as much as possible to discover this by themselves guided by critical/probing questions from the teacher.

Steps to discovery:

1. Numbers with a common remainder when divided by the same divisor form a linear pattern.
2. These numbers are not necessarily in sequence
3. Missing numbers in the sequence can be found by multiplying a number by the divisor and adding the remainder.
4. Choosing this number is important
5. A rule can be established for each pattern that enables you to find any number in it.

**3. Working Backwards**

It is to be hoped that at this stage students begin to realize that the quotient multiplied by the divisor plus the remainder will also give the number

From example 2 in the previous section: $2 \times 7 + 3 = 17$ next number will be $3 \times 7 + 3 = 24$ etc and don't forget $0 \times 7 + 3 = 3$.

At this point encourage students to articulate the "Rule", in this case one would hope that some student will come up with something like:
"well you have to multiply the quotient by 7 and then add 2 to get the dividend.

*Significant Questions:*
- Can anyone see a way to work out the dividend if the quotient is 2?
- How about if the quotient is 5, 9, 20, 100 (and any others that seem worthwhile)?
- Is there a common "Rule" here, if so what is it?
- If you know the "Rule is there a way you can put it on the TI-15 Explorer™?

If students have discovered the rule they could use the Operations function on the TI-15 Explorer™. *For more detailed instructions on how to do this see the accompanying Powerpoint presentation.*

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Students can discover a lot using the Int ÷ key and the IWB program, some examples:

1. Some numbers are in more that 1 pattern, for example 13 gives a remainder of 1 when divided by 2, 3, 4, 6 & 12.
2. What do patterns with the same divisor but different remainders look like (parallel lines)
3. What do patterns with the same remainder but different divisors look like (all cross the vertical axis at the same spot, ie the same y intercept)
4. Does the divisor have to be an integer.

### Using a RULE on the TI-15 Explorer™

***Note: Before commencing the calculator memory should be cleared by simultaneously pressing the "on" and "clear" keys***


**To define an operation [Op1] (formula) first press the (or [Op2] ) key then type the steps of the operation and then press [Op1] (or [Op2] ) to set the operation**

*Example:*

First set up the calculator by keying [Op1][×][7][+][3][Op1]
You have entered the "Rule Multiply by 7 and add 3".

Then enter any number say 8 followed by [Op1] .
The answer in this case is 59.

As students develop numbers they could plot the output against the input on a graph. In the previous example 8 was the input and 59 was the output.

Eg plot the point (8,59) as more students develop points these are written on the board and plotted on a graph.

If the classroom has an IWB this could be done on that using the accompanying graphing program with each student entering their coordinates.

W4

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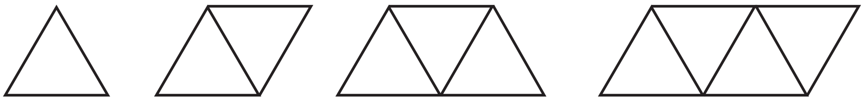
## Session 3

**Matchstick Activity**
Students use matchsticks to make triangles in the following patterns:



........................etc

Students investigate how many matches are needed to make each pattern and predict how many will be needed to make 5 triangles, 6 triangles etc.

Students use the Int÷ feature of the TI-15 Explorer™ to find a divisor that gives a common remainder and they use this information to predict the number of matches needed to make 20 triangles, 100 triangles, 257 triangles etc.

*Significant Questions:*
- Can you use your matchsticks to make the 5th and 6th triangle?
- Can you find a quicker way to work out how many matchsticks you will need to make a particular number of triangles?
- Can you complete this table? –

| Number of triangles | Number of matches needed |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| | |
| | |
| | |

Students create a formula to predict the number of matches for any number of triangles and use the operations feature to calculate the number of matches for any number of triangles *(# of matches )* = 2 × *(# of triangles)* +1

***Note: Before commencing, the calculator memory should be cleared by simultaneously pressing the "on" and "clear" keys.***

Students then use the Op feature to program their TI-15 Explorer™.
[Op1] [×] [2] [+] [1] [Op1]
Then try 27 [Op1] for 27 triangles, the answer should be 55 matches.

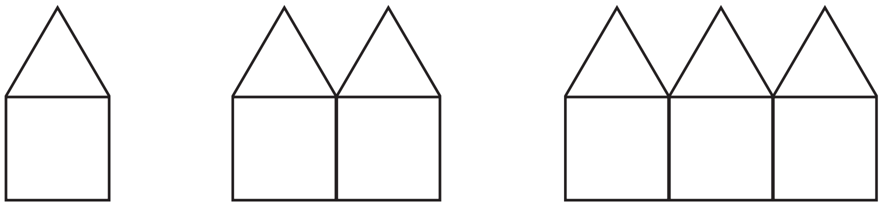
Have the students plot the number of matches (vertical axis) against the number of triangle (horizontal axis).

*Significant Questions:*

- Can you use [Int÷] to help you find a pattern?
- Can you find a "quick method" to find out how many matchsticks are needed to make 20 triangles, 100 triangles, 257 triangles?
- Can you explain your "quick method to the rest of the class?

Repeat this activity using matchstick houses:

Eg.



**More advanced work – optional**

Not all linear patterns can be found by using the [Int÷] key as this will only give a minimum remainder. In some cases a larger remainder is needed.

For example 20 ÷ 3 = 6 r 2 but how about 20 ÷ 3 = 4 r 8 is this correct; strictly speaking yes it is.

Now we come to a linear relationship such as 20, 23, 26, …….

When divided by 3 this gives answers of 6 r 2, 7 r 2 and 8 r 2, so the linear pattern would be 2 more than a multiple of 3 starting with 3 × 1 + 2 = 5 something like: 5, 8, 11, 14, 17, 20, 23, 26, 29, 32 ………… but what if we were told that 20 was the 4th term, 23 was the 5th term and 26 was the 6th term. A good problem for thinking students.

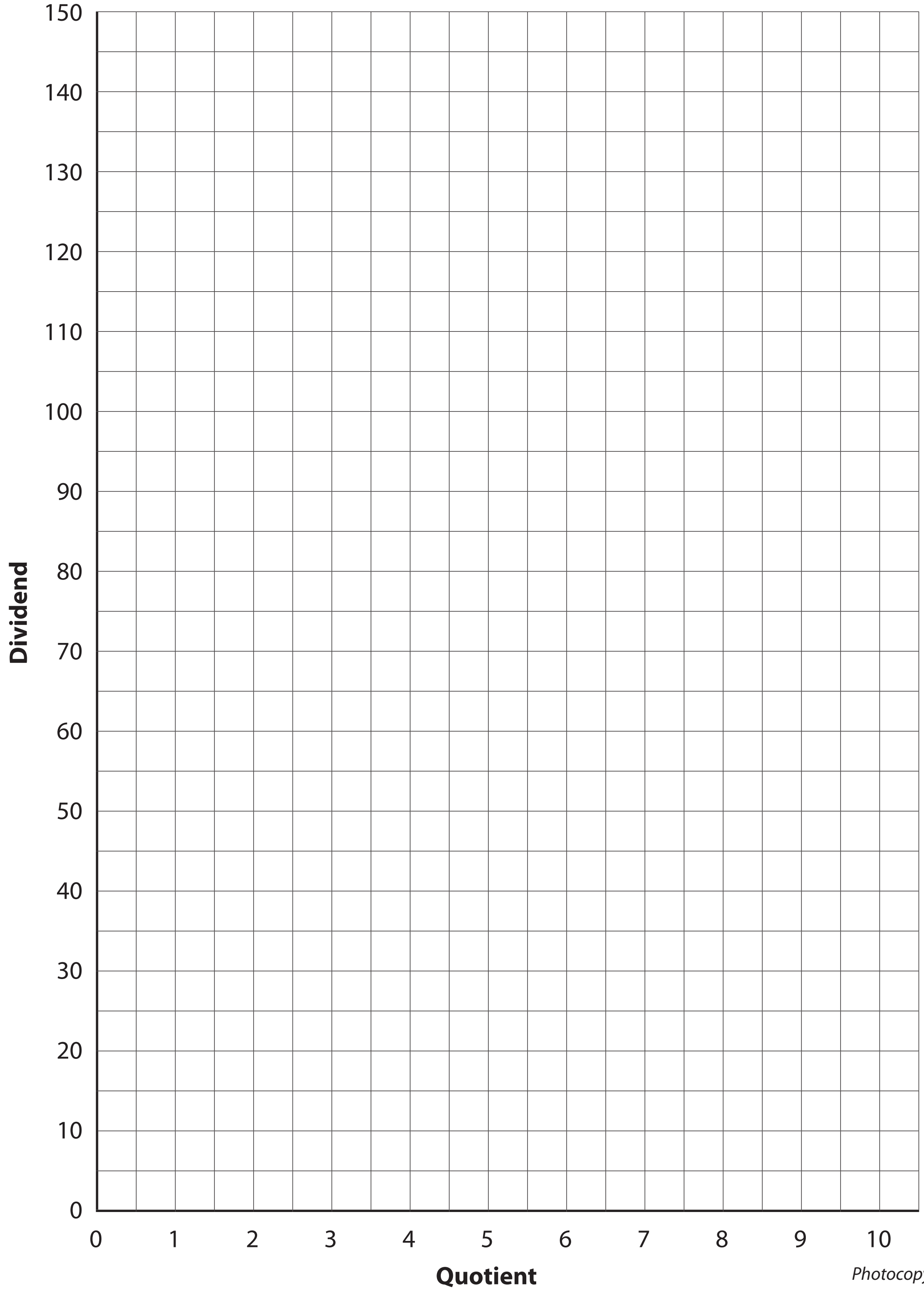
Now when we divide 20 by 3 we need to get a divisor of 4.
Eg. 20 ÷ 3 = 4 r 8 23 ÷ 3 = 5 r 8 26 ÷ 3 = 6 r 8; the 4th, 5th, and 6th terms of the linear pattern.

We can now derive the correct pattern but we needed the position or ordinal number of at least 2 of the terms.

This is similar to question 5 of the assessment.

**BLM 1**

Title: ______________________________

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