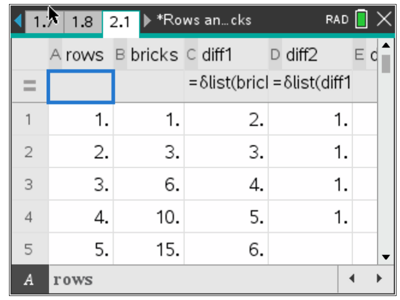


	A rows	B bricks	C diff1	D diff2
=			=δlist(bricl	=δlist(diff1
1	1.	1.	2.	1.
2	2.	3.	3.	1.
3	3.	6.	4.	1.
4	4.	10.	5.	1.
5	5.	15.	6.	

Math Objectives
- Students will use finite differences to find the degree of a polynomial that will fit data.
- Students will Use a polynomial function to model data.
- Students will graph functions, plot data in scatter plots, perform statistical calculations on a dataset, use simple formulas in a spreadsheet and apply them.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary
- common differences
- constant
- regression
- coefficient of determination
- residual

About the Lesson
- This lesson involves plotting and analyzing data.
- As a result, students will:
- Plot data points and find a polynomial regression model.
- Find and discuss the coefficient of determination (R^2).
- Make connections and predictions based on sample problems using the found regression models.

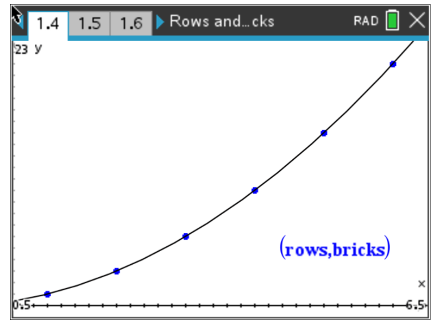
TI-Nspire™ Navigator™
- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials
Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:
- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Stacking_Bricks_Nspire_Student.pdf
Stacking_Bricks_Nspire_Student.doc

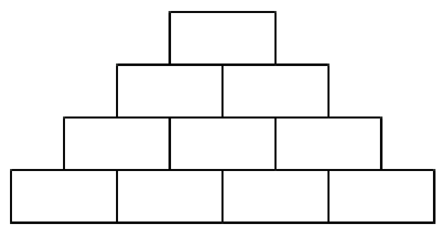
This activity presents a real-world situation—stacking bricks in a pile—that can be modeled by a polynomial function. Students create a small table to show how the number of bricks relates to the number of rows, then calculate the first, second, and third differences of the data to determine what degree of polynomial model to use. Next, they use the handheld's statistical calculation functions to perform the correct regression. Finally, they evaluate the model using a variety of methods: by graphing the model and the data together, by examining the value of R^2, analyzing Residual plots, and by discussing the model's applicability to the real-world situation.



Tech Tip: If your students are using the TI-Nspire CX II have them turn on the Lined Grid on a **Graphs** page by pressing **menu**, **2 View**, **6 Grid**, **3 Lined Grid** to add them. This activity is relying on the students being able to use both **Lists and Spreadsheets** page and regression on the handheld, therefore some time needs to be spent learning or refreshing those skills.

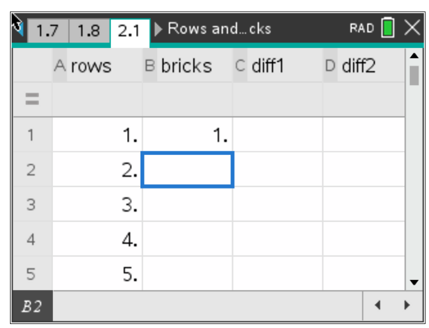
Problem 1 – A Flat Triangular Stack

In this problem, you are stacking bricks according to the pattern shown at the right. Each row contains one more brick than the row above it.



How many bricks will be in the stack when it is 50 rows high?

You can solve the problem easily by creating a polynomial model to describe the number of bricks in the stack, *f*(*x*), given a number of rows *x*.



Look for a pattern using a small table. Go to a List and Spreadsheet page, name column A rows, and enter the numbers as shown. Name column B bricks.

rows = the number of rows in the stack
bricks = the number of bricks in the stack

Complete column B (**bricks**).

Which polynomial model should you use—linear, quadratic, cubic, or quartic?

To decide, calculate the successive differences.

Enter the first differences in column C and name it **diff1** (by hand or by using the Difference List command on the handheld by moving your cursor to the second row of **diff1** and pressing **menu**, **3 Data**, **7 List Operations**, **2 Difference List(bricks)**, then **enter**), the second differences in column D and name it **diff2**, and the third differences in column E and name it **diff3**. Record your lists at the right.

Col A	Col B	Col C	Col D	Col E
rows	**bricks**	**Diff1**	**Diff2**	**Diff3**
1	1	**2**	**1**	**0**
2	**3**	**3**	**1**	**0**
3	**6**	**4**	**1**	**0**
4	**10**	**5**	**1**	
5	**15**	**6**		
6	**21**			

If the first differences are constant or close to constant, a first degree (linear) model is a good fit for the data. If the second differences are constant or close to constant, a second degree (quadratic) model is a good choice, and so on.

1. Which set of differences is constant?

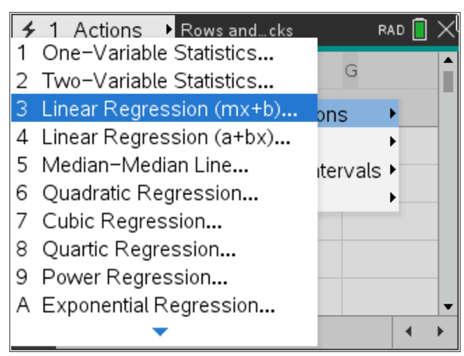
<u>Solution:</u> Second Differences

2. What degree polynomial is the best fit for this data?

<u>Solution:</u> A second degree or quadratic polynomial

Move your cursor to column F. Press **menu**, **4 Statistics**, **1 Stat Calculations**, and use the regression command to create the model for the data.

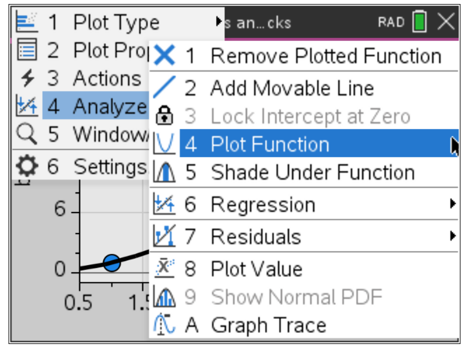
Press the arrow to choose the appropriate **Regression** command. Enter **rows, bricks,** for your X List and Y List respectively and store the equation into $f1$, press **enter**.



3. Record the equation of the model here:

<u>Solution:</u> $f(x) = 0.5x^2 + 0.5x$

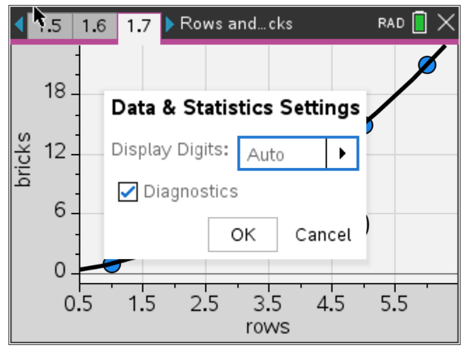
Check your model graphically by plotting the points with the model. Add a Data and Statistics page by pressing **ctrl doc 5**. Add **rows** as the horizontal variable along the bottom and **bricks** as the vertical variable along the left side. Press **menu**, **4 Analyze**, **4 Plot Function**, then type in $f1(x)$ **enter**.



If the model is correct, its graph will pass through all the data points.

Now check your model by calculating the coefficient of determination, R^2. The closer the R^2 value is to 1, the better the model fits the data.

Press **menu**, **6 Settings** and check the box that says **Diagnostics**.



If the R^2 value was not seen originally, run the regression again.

4. What is the R^2 value for your model? What does that mean?
Solution: $R^2 = 1$; This means that the model fits the data perfectly.

5. When you checked your model, did the function go through all the points?

Solution: Yes, the model seemed to pass through each and every point.

6. If your model is correct, use it to calculate the number of bricks in a stack 50 rows high. (Remember that $f1(x)$ is the number of bricks and x is the number of rows.)

Solution: 1275 bricks

7. Discuss the shortcomings of the model for this situation. For what numbers of rows is it valid? For what numbers of rows does it not make sense?

<u>Solution:</u> It is valid for all whole numbered rows. The model does not make sense for negative numbers of rows.

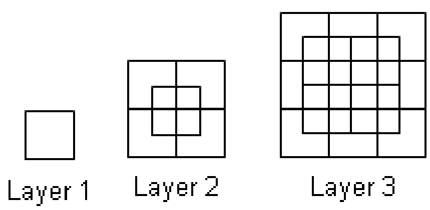
8. Write a domain for this model.

<u>Solution:</u> $\{x|\, x \geq 0\}$

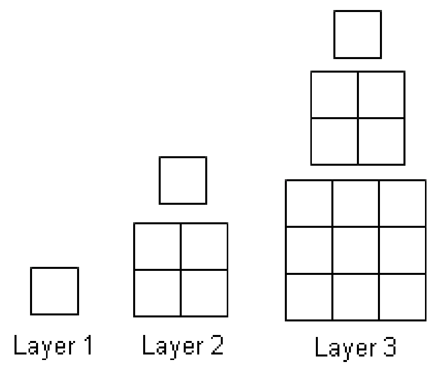
Problem 2 – A Pyramidal Stack

In this problem, you are stacking bricks in pyramids.

The diagram below shows the stacks from above.



To see the pattern more clearly, the layers of the pyramids are shown separately below.



9. Use the method from Problem 1 to find the number of bricks in a pyramid with 50 layers. Calculate the successive differences and record the values in the table. What do you notice about the common differences?

<u>Solution:</u> The second differences are linear and the third differences are constant.

Col A	Col B	Col C	Col D	Col E
rows	bricks	Diff1	Diff2	Diff3
1	1	**4**	**5**	**2**
2	**5**	**9**	**7**	**2**
3	**14**	**16**	**9**	**2**
4	**30**	**25**	**11**	
5	**55**	**36**		
6	**91**			

10. Choose and perform a polynomial regression. Record it here.

<u>Solution:</u> $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$

11. Look at the R^2 value for the regression. What is it? What does this mean?

Solution: $R^2 = 1$; This means that the model fits the data perfectly.

12. Check your model. Graph total bricks vs. number of layers as a scatter plot on a Data and Statistics page, and graph your model into $\boldsymbol{f1}(\boldsymbol{x})$. Does the model go through all the points?

Solution: Yes, the model passes through each point.

13. If your model is correct, use it to calculate the number of bricks in a pyramid 50 layers high.

Solution: 42,925 bricks

14. Discuss the shortcomings of your model for this situation. For what numbers of layers is it valid? For what numbers of layers does it not make sense?

Solution: The model is valid for all whole numbered layers of bricks. The model does not make sense for negative numbers of layers of bricks.

15. Write a domain for this model.

Solution: $\{x \mid x \geq 0\}$

Teacher Tip: In Problem 3, your class may or may not have discussed residuals. It might be a good idea to spend a little class time discussing this idea and its usefulness. Feel free to extend beyond and discuss when the values are over-estimates and under-estimates as well. Residuals are a nice connection to a Statistics class and could be used throughout any modeling activity.

Problem 3 – Extension, Beyond R^2

In this final problem, students will analyze the residuals created by the results in problem 2. A **residual** value is the difference between the actual value (given data) and the predicted value (value found by using the regression model). Students will examine the residual plot from problem 2's model and discuss what they notice.

16. Go to the **Data and Statistics** page from Problem 2 and make sure your regression curve is still passing through the data. Press **menu**, **4 Analyze**, **7 Residuals**, **2 Show Residual Plot**. Discuss with another student what you recognize about the residual plot. Explain what each of the values represent.

Possible Solution: Since the residual values represent how far each data point is from the prediction model, the residuals are indicating that they are extremely close to the cubic model. The difference between the predicted values and the actual values is almost zero.

17. Using the residual plot, how can you tell if the model found in Problem 2 is a good fit?

Possible Solution: When examining a residual plot, you are looking for a lack of a pattern. A lack of a pattern means that the selected model is a good fit. For Problem 2, there is no visible pattern, therefore this cubic model is a good fit.

Teacher Tip: If time permits, have the students go back to Problem 1 and look at the residuals for that as well and have them discuss what they see. Having them discuss and then write concise explanations is a very useful tool as they progress in mathematics.