

Activity 3

Objective

- Describe the relationship between relative frequency and theoretical probability to formalize the Law of Large Numbers
- Distinguish between relative frequency and cumulative frequency

Law of Large Numbers: Adding It Up

Introduction

If you toss a coin, it lands either heads or tails up. If you toss that coin a second time, it once again lands either heads or tails up. Each time you toss the coin, the chances of it landing heads up is 1 out of 2. No outcome is affected by the outcome preceding it. That is, each toss is an *independent* event.

If you look at the pattern of a series of coin tosses, you can determine the *relative frequency* of the coin landing heads up by comparing the number of times the coin lands heads up to the total number of times the coin is tossed. The Law of Large Numbers suggests that the relative frequency of heads landing up gets closer to the theoretical probability of heads landing up as the number of tosses increases.

Problem

If you tossed a fair coin a large number of times, how many times would the coin land heads up? Theoretically, the probability would be $\frac{1}{2}$ because there are two possible outcomes. Will the relative frequency of heads landing up match the theoretical probability of heads landing up?

Exploration

Getting Started

For this activity your graphing calculator must have at least 14,000 bytes of memory available in RAM. Before beginning, you should archive any lists that contain data you want saved.

Tossing the Coin

1. Press **[2nd] [MEM]**, select **4:ClrAllLists**, and then press **[ENTER]**.

```

MEM
1:About
2:Mem Mgmt/Del...
3:Clear Entries
4:ClrAllLists
5:Archive
6:UnArchive
7↓Reset...

```

```

ClrAllLists      Done

```

2. Press **[2nd] [MEM]** and then select **2:Mem Mgmt/Del**. If RAM FREE is less than 14,000 bytes, archive or delete calculator variables such as programs, matrices, and pictures.

```

RAM FREE      23967
ARC FREE      50692
1:All...
2:Real...
3:Complex...
4>List...
5:Matrix...
6↓Y-Vars...

```

3. Press **[STAT]**, select **5:SetUpEditor**, and then press **[ENTER]**.

```

STAT CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor

```

```

ClrAllLists      Done
SetUpEditor      Done

```


9. Select **ESC**.
10. Select **YES** to return to the **Simulation** menu.

Note: This only clears the data from the App's memory. The data saved in step 5 is still stored in the indicated lists.

11. Select **QUIT**, and then select **YES** to return to the Home screen.

This will clear all trials from memory. Are you sure you want to continue?			
YES			NO

Analyzing the Data

You are now ready to use the data you have gathered to investigate the *relative frequency* of the coin landing heads up. For this part of the Exploration, you will create a line graph that displays the relative frequency for the coin landing heads up for each coin was toss.

For your analysis, you only need the data saved in the lists CUMH and TOSS. Therefore, to conserve memory, clear the data from the C1 list created by the App.

1. Press **[STAT]** and then select **4:ClrList**.
2. Press **[2nd]** **[LIST]**, and select **C1** from the **NAMES** menu. Press **[ENTER]** twice.
3. Press **[STAT]** and then select **1:Edit**.

EDIT	CLC	TESTS
1:Edit...		
2:SortA<		
3:SortD<		
4:ClrList		
5:SetUpEditor		

4. Highlight **L1**, and then press **[2nd]** **[INS]** to insert a new list.
5. Press **[2nd]** **[LIST]**, and select **CUMH** from the **NAMES** menu. Press **[ENTER]** twice.

NAME	L1	L2	1
1	-----	-----	
1			
1			
1			
1			
2			
3			
CUMH = { 1, 1, 1, 1, 1, ...			

6. Highlight **L1**, press **[2nd]** **[INS]** **[2nd]** **[LIST]**, and select **TOSS** from the **NAME** menu. Press **[ENTER]** twice.

CUMH	TOSS	L1	2
1	1	-----	
1			
1			
1			
1			
2			
3			
TOSS = { 1, 2, 3, 4, 5, ...			

7. Highlight **L1**, and press $\boxed{2nd}$ $\boxed{[INS]}$ to insert a new list. Name this list **RF** by pressing **R F** \boxed{ENTER} .

Note: If the list names *CUMH* and *TOSS* already exist on the list editor screen, you do not need to create them again. However, if the list name *RF* already exists on the list editor screen, you **MUST** clear the old data from *RF*.

CUMH	TOSS	RF	3
1	1	-----	
1	2		
1	3		
1	4		
1	5		
2	6		
3	7		

RF =

8. Highlight **RF**. Press $\boxed{2nd}$ $\boxed{[LIST]}$ and select **CUMH**. Press \boxed{ENTER} $\boxed{\div}$ $\boxed{2nd}$ $\boxed{[LIST]}$ and select **TOSS**. Press \boxed{ENTER} twice.

CUMH	TOSS	RF	3
1	1	-----	
1	2		
1	3		
1	4		
1	5		
2	6		
3	7		

RF = LCUMH / LTOSS



Respond to # 4 – 6 on the Student Worksheet.

Graphing the Data

You will now graphically view the first 25 tosses of the coin versus the relative frequency.

1. Press $\boxed{2nd}$ $\boxed{[STAT PLOT]}$ and select **Plot 1**. Set up an xyLine plot as shown.

$\boxed{2nd}$ $\boxed{[STAT PLOT]}$	Plot2	Plot3
$\boxed{1}$	Off	
Type:	\boxed{L}	\boxed{S}
Xlist:	TOSS	
Ylist:	RF	
Mark:	\square	\bullet

2. Press $\boxed{Y=}$. Set **Y1 = 0.5**. This graphs a horizontal line representing the theoretical probability of a coin landing heads up.

$\boxed{2nd}$ $\boxed{[Y=]}$	Plot2	Plot3
\backslash Y1	0.5	
\backslash Y2	=	
\backslash Y3	=	
\backslash Y4	=	
\backslash Y5	=	
\backslash Y6	=	
\backslash Y7	=	

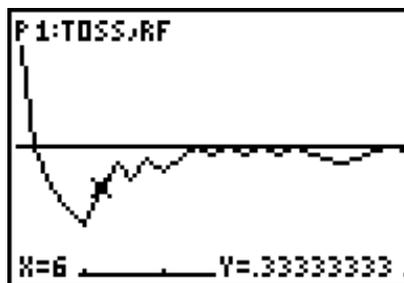
3. Press **WINDOW** to enter the values shown.

```

WINDOW
Xmin=1
Xmax=25
Xscl=5
Ymin=0
Ymax=1
Yscl=.1
Xres=1

```

4. Press **S** to see the graph of the first 25 data entries.
5. As you trace the plot, think about the story your graph tells. Press **TRACE**. Press **▶** repeatedly.



Respond to # 7 – 13 on the Student Worksheet.

6. Look at your plot as it grows. Change **Xmax** to 100 and **Xscl** to 25. Press **GRAPH** to view the plot of your first 100 tosses. The first quarter of this plot is the same graph you sketched of the first 25 tosses. What new story does the remaining three-quarters of this graph tell?
7. Change **Xmax** to 200 and **Xscl** to 100. Press **GRAPH**. Does your story of the graph now change?
8. Change **Xmax** to 300. Press **GRAPH**. What is happening in your story now?
9. Predict what the graph will look like when the last 100 tosses are added to the plot. Change **Xmax** to 400. Press **GRAPH**.



Respond to # 14 – 18 on the Student Worksheet.

After the Activity

Upon completing this activity, you should restore your calculator's memory. To do this:

1. Press $\boxed{2\text{nd}}$ $\boxed{[\text{MEM}]}$ and select **4:ClrList**.
2. Press $\boxed{2\text{nd}}$ $\boxed{[\text{MEM}]}$, select **2:Mem Mgmt/Del**, and select **4:List**. Delete all list names after L6.
3. Press $\boxed{[\text{STAT}]}$ and select **5:SetUpEditor**.

Extension

With the Probability Simulation application, you can create an unfair model as you did in Activity 2. Have another student secretly change the weights of heads and tails on your calculator. Next, examine the plot of the relative frequency of heads to guess the weights and to approximate the theoretical probability.

Student Worksheet

Name _____

Date _____

Law of Large Numbers: Adding It Up

1. Record the results of the 400 tosses.

Cumulative frequency of heads = # of heads = _____

Relative frequency of heads = $\frac{\text{number of heads}}{\text{number of tosses}}$ = _____

2. Ideally, what is the expected number of heads for 400 tosses? _____
How does this compare with the cumulative frequency of your experiment?

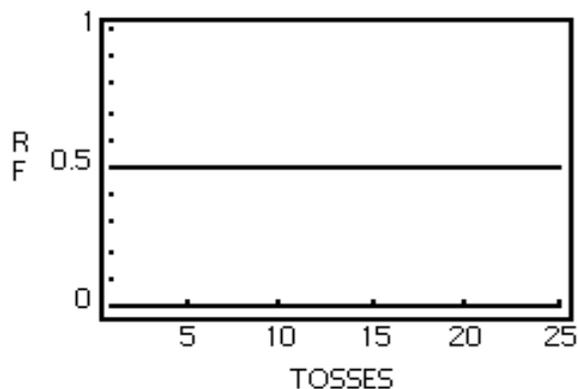
3. What is the theoretical probability of a coin landing heads up? _____
How does this compare with the relative frequency of your experiment?

4. Scroll through the first 25 data entries of your relative frequency (RF) list. What is your minimum RF? _____
What is your maximum RF? _____
Why are all the RF values between 0 and 1, inclusive?

5. Move the cursor up past the first entry to the 400th element in the RF list. Scroll through the last 25 data entries. What is your minimum RF? _____
What is your maximum RF? _____
Compare those results with your first 25. Explain any differences.

6. Compare your findings for #4 and #5 with your classmates' results. Explain any differences and similarities that you notice.

7. Sketch the graph of your first 25 tosses.



8. Explain why the window values were chosen.

9. Compare your graph with others. Notice that all graphs either start with $RF = 1$ or $RF = 0$. Why?

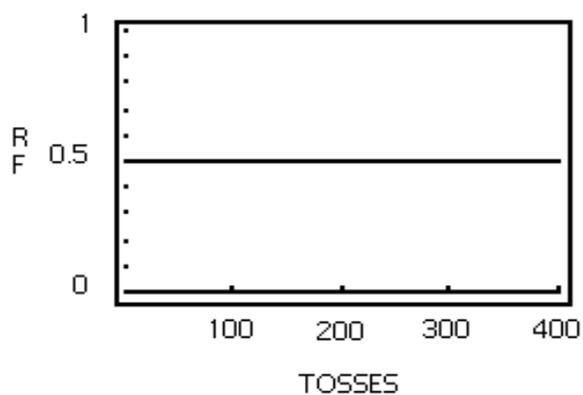
10. What does a downward slope mean?

11. What does an upward slope mean?

12. Some graphs touch or cross over the line $y = 0.5$. What does this mean?

13. What is the overall story of your graph?

14. Sketch the final graph of all 400 tosses.



15. a. What story is your graph of 400 tosses telling? How is this story different than the one told by the graph of the first 25 tosses?

b. Compare your plot with the graphs of your classmates. How are their stories similar? How are they different?

16. What would you expect to happen to the graph of 400 more tosses?

17. A die was tossed 10,000 times. The results are recorded in the table below. Complete the table and answer the true/false questions.

Tosses	Cumulative Heads (CumH)	Expected Number of Heads (ExH)	CumH - ExH	Relative Frequency (nearest hundredth)
400	212			
1,000	520			
5,000	2541			
10,000	5045			

- a. True or False As the number of tosses increases, the cumulative frequency of heads gets closer to the expected number of heads.
- b. True or False As the number of tosses increases, the relative frequency of heads gets closer to the theoretical probability.
18. Now you have explored the Law of Large Numbers in depth. Give a formal definition of the Law of Large Numbers.

Teacher Notes



Activity 3

Law of Large Numbers: Adding It Up

Objective

- Describe the relationship between relative frequency and theoretical probability to formalize the Law of Large Numbers
- Distinguish between relative frequency and cumulative frequency

Materials

- TI-84 Plus/TI-83 Plus

Teaching Time

- 60 minutes

Preparation

The goal of this lesson is for students to recognize the patterns of probability and to formalize the Law of Large Numbers. As the number of trials (tosses of the coin) increases, the relative frequency tends to get closer to the theoretical probability. It is important for students to realize that each toss is an independent event. Some people believe that after tossing heads consecutively ten times in a row, the probability becomes more or less favorable to tails for the next toss, when it is still one out of two chances, or 50%, that it will be tails.

Students compare the tables and graphs of the relative frequencies for the first and last 25 tosses. To see the pattern more clearly, the students examine a series of graphs for an increasing number of tosses.

Answers to Student Worksheet

1. Answers will vary.
2. 200. Answers will vary.
3. 0.5 or 50%. Answers will vary, likely being between 0.4 and 0.6.
4. Minimum RF: Answers will vary.
Maximum RF: Answers will vary.
The values of RF are between 0 and 1, inclusive, because the first coin either lands heads up (1) or not (0).
5. Minimum and maximum RF will vary; however, they should be closer to 0.5.
6. Answers will vary. Differences could include how the graph of the first 25 tosses will have a greater variance dependant on the outcome of the first few tosses. Similarities could be that the graph approaches 0.5 as more tosses are recorded.
7. Graphs will vary.

8. X_{\min} and X_{\max} represent the lowest and highest number of tosses. Y_{\min} and Y_{\max} represent the first coin landing heads up (1) or not (0).
9. The first toss of the coin is either heads up (1) or not (0).
10. The coin is tails up.
11. The coin is heads up.
12. When the graph touches the line, the RF has an even number of heads and tails. When the graph crosses 0.5, the number of heads and tails has switched.
13. Answers will vary, but should indicate that more tosses bring the RF closer to 0.5.
14. Graphs will vary.
15. a. Answers will vary; however, the story should indicate the RF coming closer to the theoretical probability of 0.5. The first story of the 25 tosses depends on the outcome of the first toss.
- b. Answers will vary.
16. Answers will vary. Answers should include that as the number of tosses increases, the graph will more closely approach 0.5 with less variance.

17.

Tosses	Cumulative Heads (CumH)	Expected Number of Heads (ExH)	CumH - ExH	Relative Frequency (nearest hundredth)
400	212	200	12	0.53
1,000	520	500	20	0.52
5,000	2,541	2,500	41	0.51
10,000	5,045	5,000	45	0.50

- a. False
- b. True
18. The Law of Large Numbers says that in repeated, independent trials with the same probability p of success in each trial, the relative frequency of success is increasingly likely to be close to the theoretical probability of success as the number of trials increases. In other words, the difference between the *relative frequency* of success and the probability of success approaches zero. In contrast, the difference between the cumulative frequency success and the expected number of successes tends to grow as n grows.