



Activity Overview

In this activity, students will explore the integral of a periodic function both numerically and graphically in order to gain an understanding of area accumulation. Students will also explore the relationship between the integral and the derivative by studying the kinematic relationship between the acceleration and the velocity of a moving object.

Topic: Derivatives

- Second Fundamental Theorem of Calculus
- Curve sketching and the graphical relationship between the integral and the derivative
- Relationship between displacement, change in velocity, change in acceleration

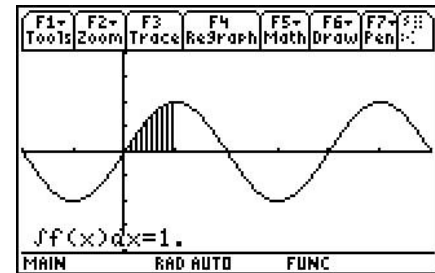
Teacher Preparation and Notes

- Part 1 of this activity takes about 15–20 minutes. Part 2 can be used as an extension or homework. In Part 2, students should predict the graph and discuss their prediction with a group or partner.
- Before beginning the activity, review with students the second Fundamental Theorem of Calculus. Also review what happens to an integral when the limits of integration are reversed, and review the graphical relationship between an integral and derivative.
- **To download the student worksheet, go to education.ti.com/exchange and enter “12240” in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Introduction to the Fundamental Theorem (TI-89 Titanium) — 9778
- Accumulation Functions (TI-84 Plus family) — 4372
- Fundamental Theorem of Calculus (TI-84 Plus family) — 4373
- Exploring the Fundamental Theorem of Calculus (TI-Nspire™ CAS Technology) — 9205



This activity includes screen captures taken from the TI-89 Titanium.

Compatible Devices:

- TI-89 Titanium

Lesson Files:

- AreaAccumulation_Student.pdf
- AreaAccumulation_Student.doc

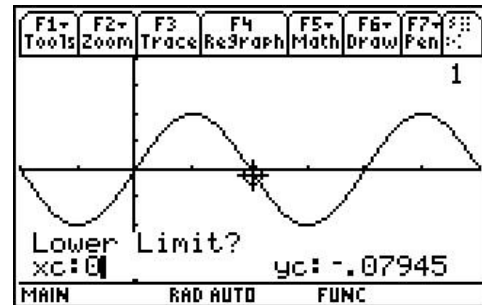
Click [HERE](#) for Graphing Calculator Tutorials.



Part 1 – Explore Area Accumulation

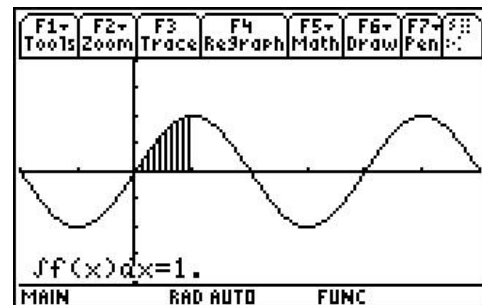
Students begin the activity by entering $y1(x) = \sin x$ in the handheld, adjusting the viewing window as instructed, and evaluating $F(x) = \int_a^x \sin(x)dx$ for different limits of integration.

When entering the limits of integration, have students enter the values using the handheld and press **ENTER**. Tracing along the graph to determine the limits will not result in exact integrals.

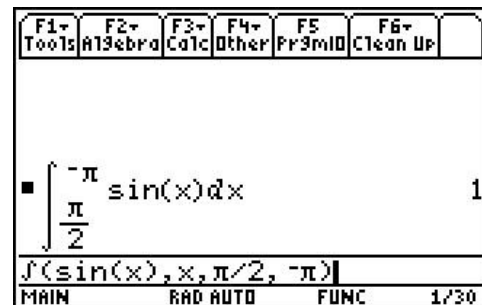


Student Solutions

1. 1
2. 0
3. -2
4. 0



Students then navigate to the **HOME** application and complete a table of integral values for $a = \frac{\pi}{2}$. In this section, the value of x (the upper limit of integration) is sometimes less than a . Remind students of the fact that $\int_a^b f(x)dx = -\int_b^a f(x)dx$.



Student Solutions

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$F(x)$	1	0	-1	0	1	0	-1

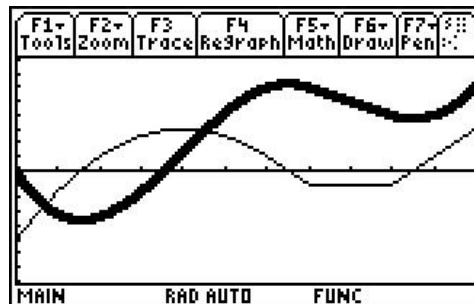
5. $F(x) = -\cos(x)$
6. When $f(x)$ is positive, $F(x)$ is increasing. When $f(x)$ is negative, $F(x)$ is decreasing.
7. The integral of $f(x)$ is *sometimes* negative when $x < a$. If there is more area above the x -axis, the integral is negative since $\int_a^b f(x) dx = -\int_b^a f(x) dx$. If there is more area beneath the x -axis, the integral is positive. The integral could also be 0.



8. For $a < x$, if there is more area beneath the x -axis, the integral is negative. For $x < a$, if there is more area above the x -axis, the integral is also negative.
9. For $a < x$, if there is more area above the x -axis, the integral is positive. For $x < a$, if there is more area beneath the x -axis, the integral is also positive.
10. -1

Part 2 – Curve Sketching and Kinematic Calculus

In this section, students compare the graph of the integrand to the graph of the integral in the context of distance, velocity, and acceleration. Students should recognize that the integral of the acceleration of an object with respect to time is the change in velocity of the object. Similarly, the integral of the velocity of an object with respect to time is the change in position of the object.



Student Solutions

1. change in velocity
2. change in position
3. Check students' graphs. The correct velocity graph is shown above as a bold line.
4. From a resting position, the object first has negative velocity and moves backward. Then the velocity of the object increases and remains positive, so the object moves forward at varying rates.