Triangle Inequalities

ID: 11756

Time Required
45 minutes

Activity Overview

In this activity, students will discover the Triangle Inequality Theorem and explore the possible lengths of the third side of a triangle given the other two side lengths. Students will also classify the type of triangle as acute, obtuse, or right using the Pythagorean inequalities.

Topic: Right Triangles & Trigonometric Ratios

- *Triangle Inequality*
- *Derive the Triangle Inequality as a corollary of the Pythagorean Theorem and apply it*

Teacher Preparation and Notes

- *To complete this activity, students will need to know how to change between pages, grab and move points.*
- *Students can write their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students enter their response and select **MENU** > **Check Answer** (or press ctrl +▲).*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "11756" in the quick search box.***

Associated Materials

- *TriangleInequalities_Student.doc*
- *triangleInequalities.tns*

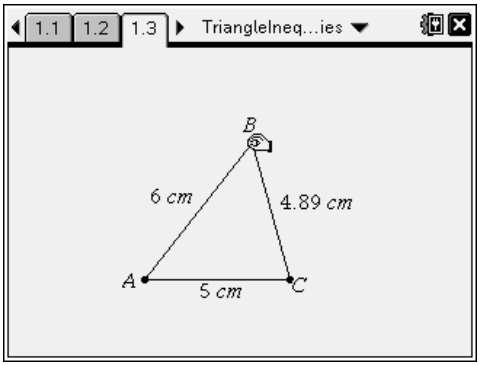
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Triangle Inequality (TI-84 Plus family) — 7299*
- *Discovering the Triangle Inequality Theorem (TI-Nspire technology) — 9767*
- *Triangle Inequalities (TI-Nspire technology) — 7857*

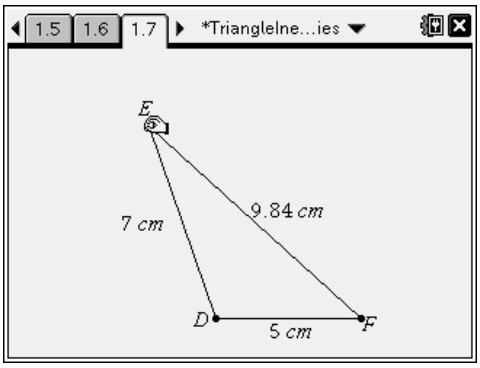
Problem 1 – Triangle Inequality Theorem

Students will begin this activity by looking at a triangle and investigating the possible lengths of the third side given the other two sides. On page 1.3, students are given triangle *ABC*. Segments *AB* and *AC* have fixed lengths 6 cm and 5 cm, respectively. By moving point *B*, students will discover the possible lengths of *BC*.



Note that when *A*, *B*, and *C* lie on the same line, there is not a triangle.

Students are asked several questions about how the minimum and maximum lengths of the third side are determined by the lengths of the other two sides. The questions that ask about minimum and maximum values are not the true maximum and minimum lengths, because when the sides are of the maximum and minimum lengths the triangles will be degenerate and form a line.

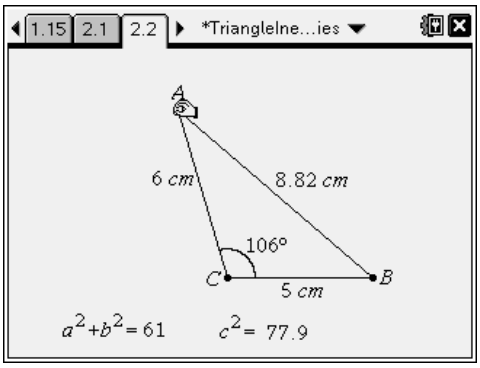


Students will repeat this for a triangle with lengths 5 and 7 to determine a pattern.

Students will be led to the discovery of the *Triangle Inequality Theorem*—the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Problem 2 – Pythagorean Inequalities

Students begin Problem 2 by exploring a triangle on page 2.2. Students are given triangle *ABC*, m∠*C*, the lengths of all sides, $a^2 + b^2$, and c^2. Make sure that students are aware that *a* refers to the length of the side opposite ∠*A*, *b* refers to the length of the side opposite ∠*B*, and *c* refers to the length of the side opposite ∠*C*. Students will be asked to move point *A* and to determine for what kind of triangle is $a^2 + b^2 = c^2$, $a^2 + b^2 > c^2$, and $a^2 + b^2 < c^2$.

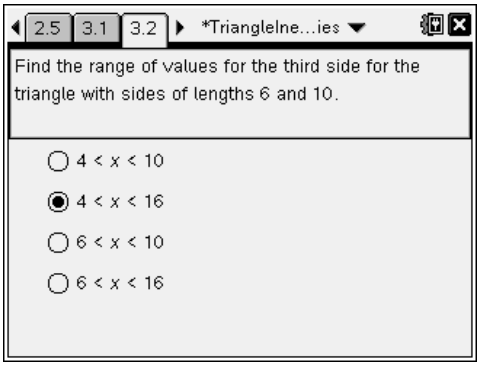


Students will discover that $a^2 + b^2 = c^2$ for a right triangle, $a^2 + b^2 > c^2$ for an acute triangle, and $a^2 + b^2 < c^2$ for an obtuse triangle. On pages 2.3–2.5, students are asked several questions confirming these relationships between $a^2 + b^2$ and c^2.

Note that $a < b < c$ for Problem 2. Discuss with students what kind of triangles are formed if $a = b$.

Problem 3 – Application of the Triangle Inequalities

In Problem 3, students are asked to apply what they have learned about the Triangle Inequality Theorem and the Pythagorean Inequalities.



Student Solutions

1. 1
2. 11
3. $2 < x < 12$
4. $7 + 5 = 2$
5. $7 – 5 = 12$
6. No
7. Yes
8. No
9. $a + b < c$, $a + c < b$, and $b + c < a$
10. Right
11. Acute
12. Obtuse
13. No
14. $4 < x < 16$
15. Obtuse
16. 8, 10, 12
17. 10, 11
18. 12, 13