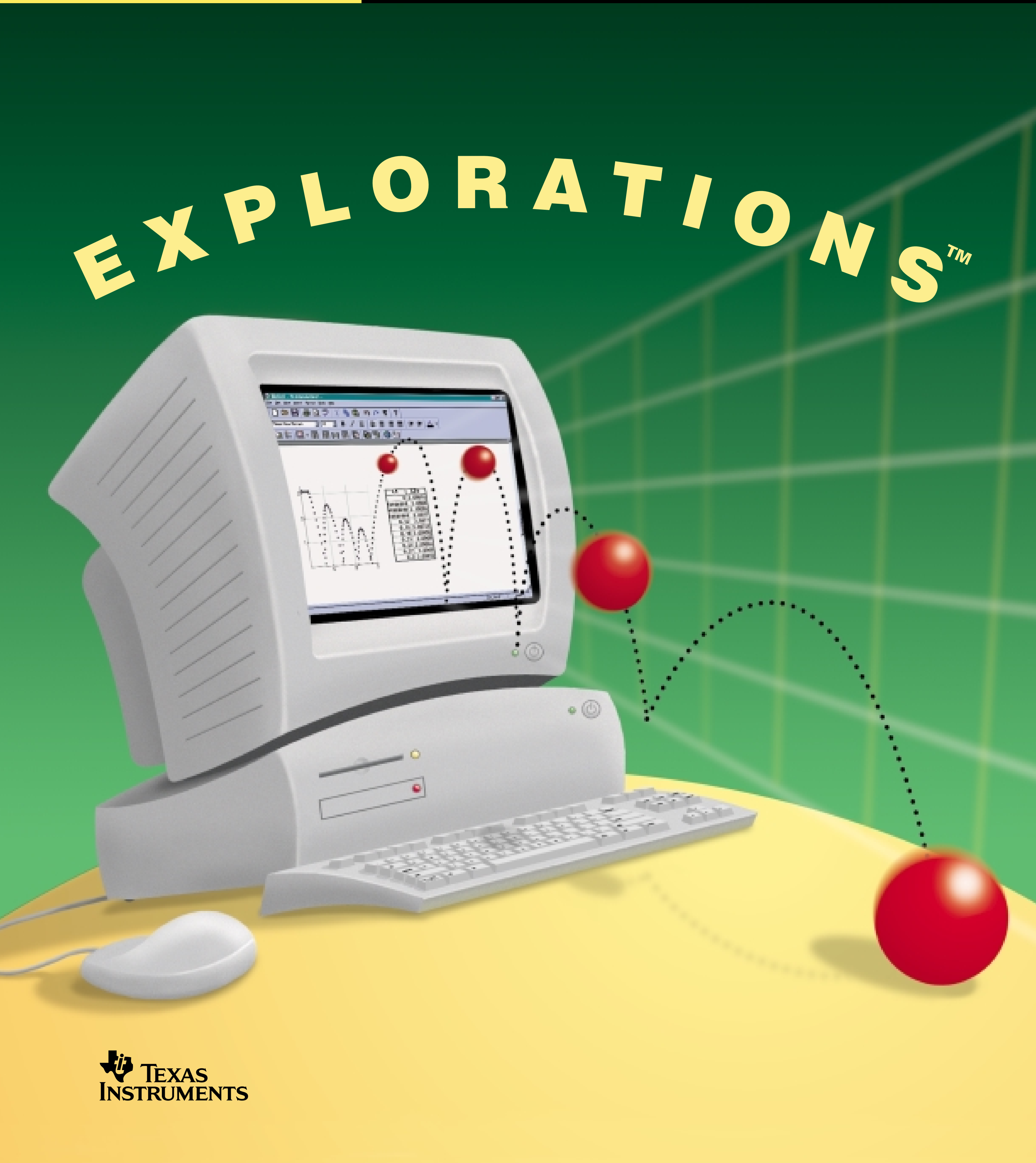
TI InterActive!™

TI InterActive! Data Collection and Analysis

Chris Brueningsen Bill Bower Ron Armontrout Liz Sumner



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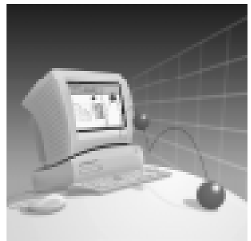

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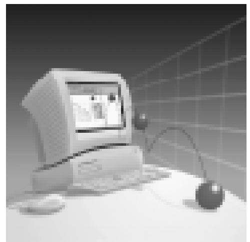


Contents

TI InterActive! Data Collection and Analysis Content Checklist

Activity	Concept											
	Internet Data Collection	CBL/CBL 2	Spreadsheets	List Manipulation	Tables	Ratios	Direct Variation	Box Plots	Linear Function	Quadratic Function	Exponential Function	Logistic Function
1: Step by Step		X		X					X			
2: Tight Rope		X							X			
3: Take the Train	X			X			X		X			
4: Pencil Me In		X					X		X			
5: Two Hot, Two Cold	X				X				X			
6: It’s a Small World	X								X			
7: On the Rebound		X	X	X		X					X	
8: Cool Off		X									X	
9: Used Cars	X		X			X			X		X	
10: Coffee Break	X				X						X	X
11: Curve Ball		X		X						X		

Preface

TI InterActive!™ is a new product that enables high school and college teachers and students to easily investigate ideas in mathematics and science. The purpose of this workbook is to introduce algebra teachers and their students to this powerful software by means of hands-on activities, each presented in a worksheet format. Background information, set-up diagrams, and instructions are provided for each activity. In some activities, space is given for students to record their answers as they work. Students can also record their answers directly into the TI InterActive! document. "Extensions" and further applications are included in most activities to allow for independent investigations. Teacher information sheets supply helpful notes for completing the activities, as well as sample data plots and answers to student questions.

The activities feature the TI InterActive! software's capacity to explore datasets collected with the Calculator-Based Laboratory™ (CBL™) or CBL 2™, and data retrieved from websites. The clear, step-by-step instructions allow even those users relatively unfamiliar with the CBL/CBL 2 or Internet to utilize these tools.

Many people helped us create this workbook. Our liaisons at Texas Instruments were instrumental in organizing this project. A special thanks to Vince Watchorn of West Nottingham Academy, whose keen mathematical insights assisted us greatly.

— *Chris Brueningsen*

— *Bill Bower*

— *Ron Armontrout*

— *Liz Sumner*

About the Authors

CHRIS BRUENINGSEN is Head of Upper School at Nichols School in Buffalo, New York. In 1996, he received the Presidential Award for Excellence in Mathematics Teaching and was a Member of the NCTM Task Force on Integrated Mathematics. He has taught math and science to all levels of middle and high school students and has co-authored several Texas Instruments technology workbooks. He has also written a number of articles for math and science journals and speaks frequently at educational conferences.

BILL BOWER is a teacher and Chairman of the Mathematics Department at The Kiski School in Saltsburg, Pennsylvania. Throughout his 19-year career he has taught all levels of high school math. In recent years, graphing calculator technology has become an integral part of his teaching. He is a co-author of *Math and Science in Motion: Activities for Middle School*.

RON ARMONTROUT is a teacher and chair of the mathematics department at The Peddie School in Hightstown, New Jersey. In 1986, he was a Woodrow Wilson fellow at Princeton University and was a site leader in the NSF funded Boston College Discrete Mathematics for Secondary School Teachers project. He co-authored the *How Should Algebra Be Taught?* section of the ALGEBRA for the 21st CENTURY report published by NCTM. He has taught math to all levels of middle and high school students and speaks frequently at mathematics conferences.

LIZ SUMNER is an Academic Technology Specialist at Nichols School in Buffalo, New York. She works closely with teachers, helping them prepare lesson plans with strong integration of technology. She also creates materials for and conducts inservice workshops on a variety of software products, such as Microsoft Office 2000. She is the school's webmaster and has authored numerous web sites.

Getting Started

About TI InterActive!

TI InterActive!™ is a user-friendly, interactive computer software program that enables high school and college teachers and students to easily investigate ideas in mathematics and science. Teachers can enhance students' learning through interactive lessons that encourage exploration, visualization, data analysis, and writing. TI InterActive! can help students master math and science concepts and improve problem-solving skills.

Features

- Word processor with integrated math system
- TI graphing calculator functionality
- Symbolic Computer Algebra System
- Integrated Web browser
- Data editor with spreadsheet
- Calculator connectivity

Things to Know

- **Math Box**

The Math Box enables you to perform mathematical calculations and integrate those with other features of TI InterActive!, such as lists and graphs.

The Math Box must be positioned above the list or graph you are working with in the TI InterActive! document. To see how changes in a Math Box affect a graph, position the Math Box above the graph, make your changes, then examine how those changes instantly update the graph.

- **List Editor/Data Editor**

The List Editor serves as a Data Editor, Matrix, and Spreadsheet. You can use a CBL™, CBL 2™, or CBR™ directly with TI InterActive! to collect data into the List Editor, and you can manipulate that data once it is collected.

- **Import data**

To import data into TI InterActive! from the Internet, you must first select the data. Press Select to select all the data on the Internet site. Then press Extract to extract it into a List Editor.

In some cases, you may not want all of the data displayed on the web site. To select a portion of the data, position your cursor at the beginning of the data segment you want to select. Click and hold the left mouse button, and drag your mouse to highlight the data you want. Then press Extract to extract the data into a List Editor.

Appendix

The blank grids in the Appendix may be duplicated and used with the activities in this book if your students do not have access to printers to print their TI InterActive! documents, or if you prefer to have students draw the graphs on paper after they have completed the activity using TI InterActive!

EXPLORATIONS

Activity 1

Step by Step

Many different situations in the real world exhibit linear behavior. Linear behavior can be defined as a situation in which equal changes in the independent variable produce approximately equal changes in the dependent variable. For example, if you collected data for pressure versus your depth underwater, each meter you descended would produce an approximately equal change in the pressure.

Introduction

In this experiment, you will create a situation that produces linear behavior by stepping heel to toe and taking distance readings as you step. You will then apply the properties of a linear function to develop a model for your motion. Finally, you will interpret the values used in your model.

Equipment Required

- Computer
- TI InterActive!™ software
- CBL™ unit or CBL 2™ unit with a compatible TI calculator
- Motion detector
- TI-GRAPH LINK™ cable

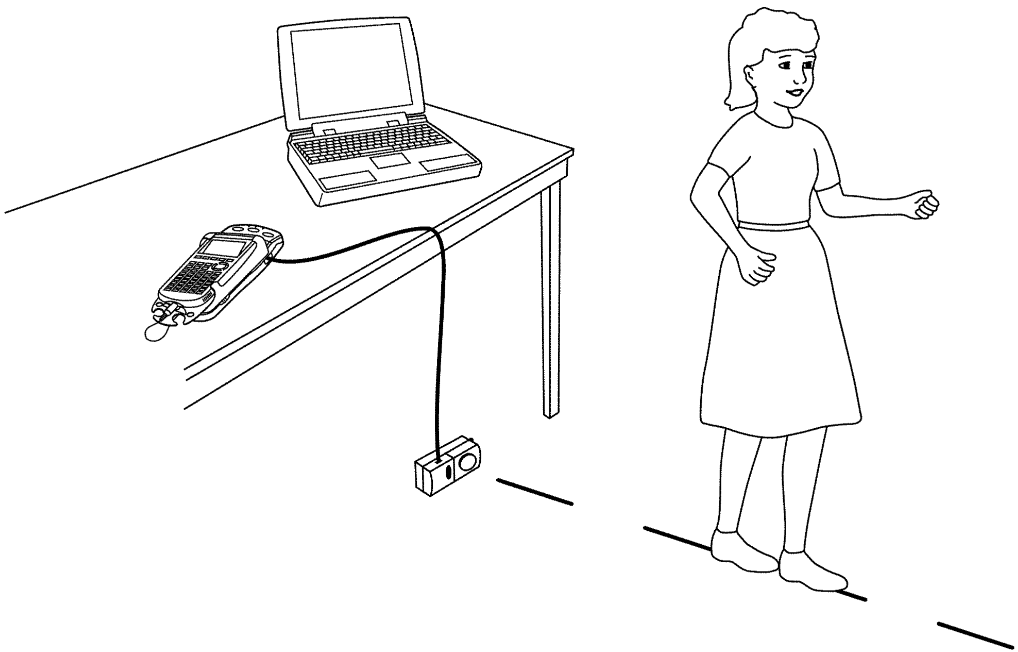
Setup

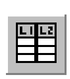
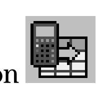
CBL	CBL 2
1. Plug the motion detector into the **SONIC** port of the CBL, and connect the CBL to a compatible TI calculator.	**1.** Plug the motion detector into the **DIG/SONIC** port of the CBL 2.
2. Press the ON/HALT button, then press the MODE key to put the CBL in multimeter mode.	**2.** Connect the CBL 2 to a compatible TI calculator. Clear the lists so your data will automatically load to **L1** and **L2**.
3. Press the CH VIEW button repeatedly until the **SONIC** channel, with **Ft** appears. The detector will begin to click and distance readings will appear on the screen of the CBL.	**3.** Run the **DATAMATE** program on the calculator. Choose **1:SETUP**. Press (down arrow) to move to **DIG:** and press Enter. Press **2** to select **MOVEMENT(FT).** Finally, choose **1:OK** to return to the main screen and begin collecting data.

4. Place the motion detector on the floor with the front of the unit facing forward. Be sure that you have a clear path in front of the detector.

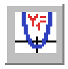
Collecting the Data

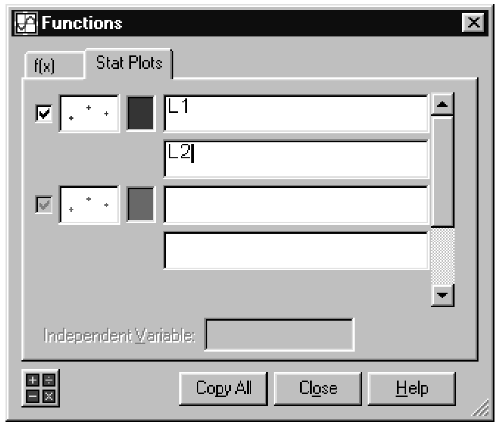
1. Have a student stand approximately two feet in front of the motion detector with the heel of one foot directly in line with the detector and the heel of the other against the toe of the first foot, as shown in the illustration below.



2. Have the student who is walking place the heel of the back foot in front of the toe of the front foot. Continue stepping away from the motion detector until you have a good sample collected. The data is stored into **L1** and **L2** of your calculator. **L1** is the step number, and **L2** is the distance reading.
3. Connect the calculator to your computer, using the TI-GRAPH LINK™ cable.
4. Start TI InterActive!™ The software opens to a new, blank document.
5. Title your new document *Step by Step* and add your name and the date. Press the Save button  to save and name your document.
6. Click the List button  to open the Data Editor.
7. Press the Import Calculator Data button  in the Data Editor.
8. Choose the appropriate calculator from the Import Calculator Data dialog box.
9. Highlight the lists containing your data (**L1** and **L2**), and click **Import**. TI InterActive! automatically inputs your data into **L1** and **L2** in the Data Editor.

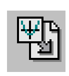
Recording the Data

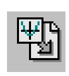
1. Press the Graph button  and then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key and move to the second text box. Type **L2** to specify the list containing the *y* coordinates.



2. Press Enter, then click the Zoom Statistics button. The viewing boundaries are adjusted automatically to show all the plotted data.

3. Your segment length versus step plot should appear to be linear. If you are not satisfied with your data, return to the *Collecting the Data* section and collect a new set of data. If you are satisfied with your plot, move to the box below the graph and enter **0**, then press Enter. This will change the minimum value of *y* to 0 and allow you to see the *x* axis.

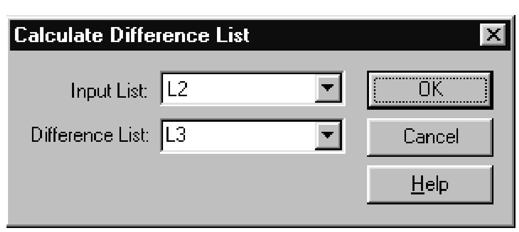
4. Click the Save to Document button  to place the graph in your TI InterActive!™ document.

5. Click the Save to Document button  on the Data Editor to place the lists in your document.

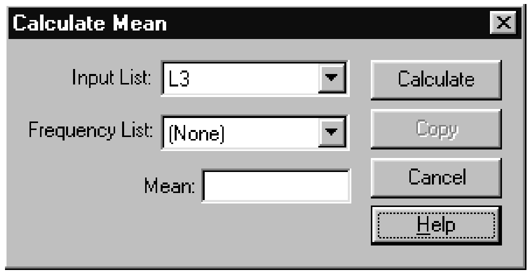
Analysis and Questions

1. To find a formula for the distance of the walker from the motion detector with respect to the number of steps taken, you need to know two things: the starting position, and the distance traveled per step. Refer to Data Editor to find the initial position of the walker and record it in your TI InterActive! document.

2. Linear data is characterized by the property that equal changes in the *x* values result in approximately equal changes in the *y* values. The graph you created already has equal changes of one for the x values. To find the changes in the *y* values, return to the Data Editor by double-clicking on the lists in your document. Choose **List**, then **Operations,** and finally **Calculate Difference List** to bring up the Calculate Difference List dialog box. Enter **L2** for the Input List and **L3** for the Difference List. Click **OK** to calculate the difference list.



3. The differences you stored in **L3** represent changes in distance per step. For the formula, you will need one value that is representative of all these changes in distance. One measure that does this is the mean of the differences. To find the mean of your difference list, from the Data Editor choose **List**, **Calculate**, and **Calculate Mean**. Enter **L3** as your Input List and click **Calculate**. Click **Copy** to copy this value onto the clipboard.



4. Your formula can be written in the form **f(x) = IP + Sx**, where *IP* is the initial position of the walker, *S* is the average distance per step, and *x* is the number of steps taken. Open a Math Box , and using the values you found above, write out your formula for distance from the motion detector with respect to the number of steps taken. Use **Edit**, **Paste** to paste the value of the mean from the clipboard into the math box. Make sure you place the math box above the graph in the document.

5. Close the Math Palette and double-click on the graph in the TI InterActive!™ document to refresh the Graph window. How well does your formula fit the data?

6. What is the physical interpretation of the value of *IP* that you found in your model?

7. What is the physical interpretation of the slope *S* in the model you found?

8. Place a yardstick on the floor and have the walker stand on it. How does the length of the walker's foot compare to the value of *S*?

9. Use the model you found to predict where the walker would be if he took 100 steps. Record your prediction in your TI InterActive! document.

10. If you were standing at the back of a 20-foot diving board, how many steps would it take before you fell in the water? Be careful with respect to the initial value. Record your answer in your TI InterActive! document.

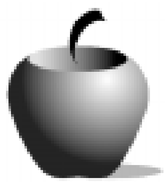
11. Save and print your TI InterActive! document.

Extensions

- Find a formula to model this data by the traditional method of selecting two points and finding a linear model containing those points. Was this model better or worse than the one you found in the activity?
- Have TI InterActive!™ find a model for this activity by performing a linear regression on the data. How does this model compare to the one you found in this activity?
- Reverse the order of this experiment. Measure your foot and the distance to the starting point of the data collection. Use these values to predict the model for the data before you collect it.
- Connect the TI-GRAPH LINK™ cable to your calculator and use TI InterActive! to send the data you collected from the computer to the calculator. Analyze the data on your calculator using the same methods applied in this activity.

Teacher Notes

Activity 1: Step by Step



Math Concepts

- CBL™/CBL 2™
- List Manipulation
- Linear Function

Activity Notes

- Make sure the student doing the stepping has his/her foot lined up directly in front of the motion detector, a minimum of two feet away.
- For some students, maintaining balance may be a problem. Take appropriate precautions to prevent injuries in case of a fall.

Sample Data

STEP#	DISTANCE (feet)
0	2.015
1	3.160
2	4.308
3	5.457
4	6.601
5	7.747
6	8.897

Analysis and Questions - Answer Key

1. The initial position was 2.015 feet.
2. The difference list table is shown below.

L3
1.145
1.148
1.149
1.144
1.146
1.150

3. The mean is 1.147 feet.
4. $f(x) = 2.015 + 1.47x$
5. The formula is an almost perfect model for the data.

6. The value *IP* represents how far the person walking was from the motion detector when data collection began.

7. The value *S* represents the length of the shoe of the person stepping.

8. The values should be very close.

9. The person walking would be 116.715 feet from the motion detector.

10. You would fall into the water on the 18^{th} step. (Remember, the initial value is zero in this problem while the length of the step remains the same.)

EXPLORATIONS

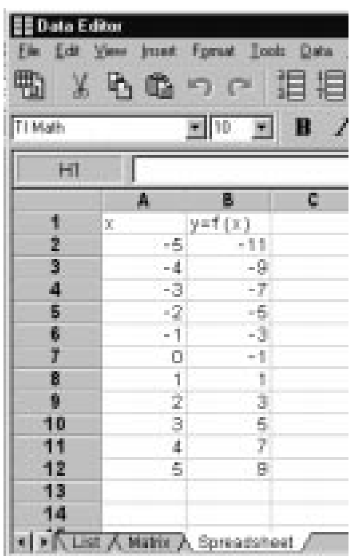
Activity 2

Tight Rope

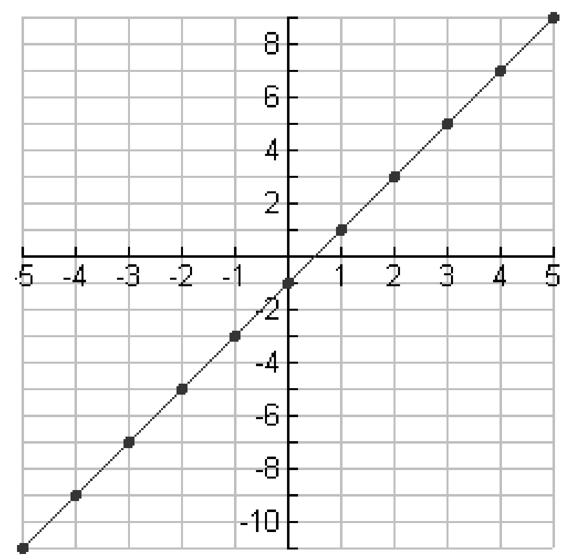
When one quantity changes at a constant rate with respect to another, we say they are related linearly. Mathematically, we describe this relationship by defining a linear function, f(x), which can be represented with a straight-line graph. In many real-world applications, quantities are linearly related and can be visually represented using a straight-line graph.

Introduction

Compare this TI InterActive!™ spreadsheet and the graph of the data points:



	A	B
1	x	y=f(x)
2	-5	-11
3	-4	-9
4	-3	-7
5	-2	-5
6	-1	-3
7	0	-1
8	1	1
9	2	3
10	3	5
11	4	7
12	5	9



As the independent variable x values increase by one, the dependent variable $y = f(x)$, or function of x, values increase by the same constant value of 2 at each step.

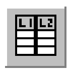
In this activity, you will create constant-speed distance versus time plots and develop linear equations to describe these plots mathematically.

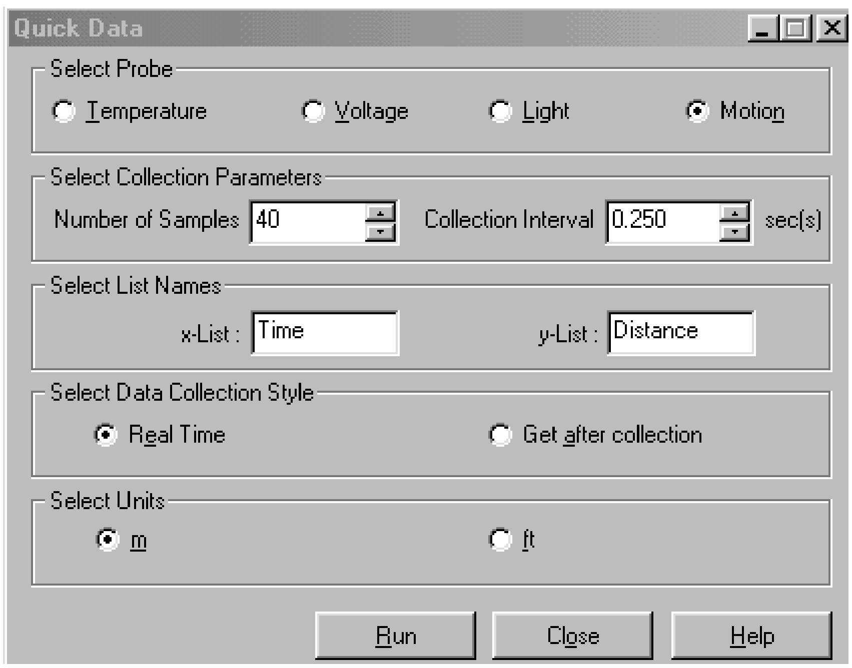
Equipment Required

- Computer
- TI InterActive! software
- CBL™ or CBL 2™ unit
- Motion detector
- TI-GRAPH LINK™ cable

Setup

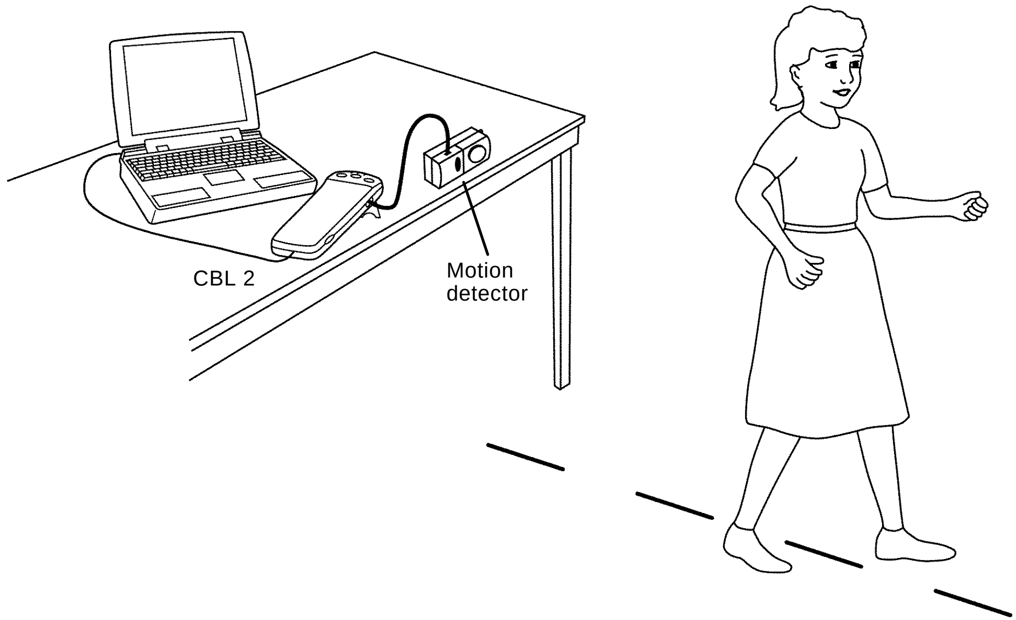
1. Plug the TI-GRAPH LINK™ cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL™/CBL 2™.

   Note: *Alternatively, you can use a CBR™ for this activity.*
3. If you are using a CBL, plug the motion detector into the **Sonic** port. If you are using a CBL 2, plug the motion detector into the **DIG/SONIC** port.
4. Place the motion detector on a table with the front of the unit facing forward. Be sure that you have a clear path in front of the detector.
5. Start TI InterActive!™ The software opens to a new, blank document.
6. Title your new document *Tightrope* and add your name and the date. Press the Save button  to save and name your document.
7. Click the List button  to open the Data Editor.
8. From the **Data** menu, choose **Quick Data**.
9. Adjust the Quick Data dialog box so that the settings match the ones shown below.

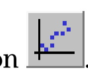


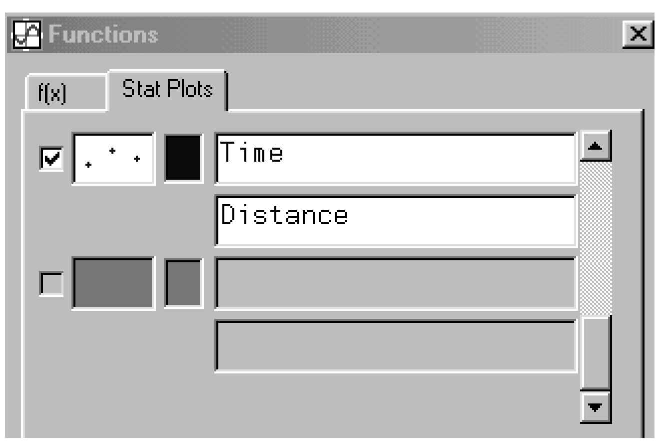
Collecting the Data

1. Place the motion detector on a table or desk and stand at least 50 cm from it. Aim the motion detector at the walker as shown in the illustration below.



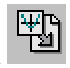
2. When you are ready to start collecting data, click **Run** in the Quick Data dialog box and start walking away from the motion detector at a slow, steady pace. You will have 10 seconds to collect the data.

3. When data collection is done, click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data. You should also see the Functions dialog box as shown below.



4. The plot of straight-line distance versus time should be linear. If you are not satisfied with your data, click **Run** in the Quick Data dialog box to perform another trial. If you are satisfied with your plot, move to the box below the graph and enter a **0**, then press Enter. This will change the minimum value of *y* to 0 and allow you to see the *x*-axis.

5. You can make a sketch of your time versus distance data plot on one of the blank grids in the Appendix. Label the horizontal and vertical axes on your sketch.

6. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

1. The slope-intercept form of a linear equation is:

$$Y = MX + B$$

where *M* is the slope or steepness of the line and *B* is the *y*-intercept or the starting value. In this activity, the control variable, *X*, represents time, and *Y* represents distance. Press 

and use the right and left arrow keys in the Trace Value dialog box or use your keyboard to move the cursor along your distance versus time plot. Identify the starting value (the *Y*-value when $X = 0$) and record this below as the intercept, *B*.

$B =$

2. Since you have a value for *B* to find the equation of the line that fits your data, you will use a guess-and-check method to determine the value of *M*. With your mouse, highlight the *B* value in the Trace window and press Ctrl+C to copy it.

Close the Trace dialog box, and click the f(x) tab in the upper left corner of the Functions dialog box. Start with an initial guess of $M = 1$. Type **f(x):= 1*x + B** (your value of *B*) in the uppermost text box of the f(x) tab. Press Ctrl+V to paste your *B* value. Record your guess for *M* in the box below. For example, **f(x):= 1*x + .599289.** Press Enter to superimpose the graph on the plotted data.

It is unlikely that your first guess for the value of *M* produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the linear function, replacing the old value, $M = 1$, with your new guess for *M*. Press Enter to update the graph. Repeat the guess-and-check procedure until you find an *M*-value that models the data well and record your guesses in the spaces below:

Guess #1	Guess #2	Guess #3	Guess #4	Final *M*-value
$M =$	$M =$	$M =$	$M =$	$M =$

Using this value of *M* and the *B*-value determined in questions **1** and **2**, complete the slope-intercept form of the equation and record it below.

3. Another way to get a linear function model for the data is to press 

again. Move along the plot with the arrow keys and identify two points ($x1$, $y1$) and ($x2$, $y2$) and record them below. Try to pick the points that are not too close together.

x1	**y1**	**x2**	**y2**

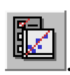
When the coordinates of two points on the same line are known, the slope of the line can be computed by finding the difference in *y*-values divided by the difference in *x*-values:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

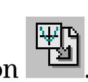
Use this formula to compute the slope of the linear plot and record the result below.

slope =

How does this value compare with the value of *M* you found experimentally in question **2**?

4. TI InterActive!™ lets you check the values of *M* and *B* you just found by calculating the line of best fit. Close the Graph window to return to the Data Editor. Click Statistical Regressions . Click the down arrow  next to Calculation Type, scroll down, and click on **Linear Regression (ax + b).** In the text box labeled **X List**, type **TIME**; in the box labeled **Y List**, type **DISTANCE**. Click **Calculate** to find the regression equation, $y = ax + b$ and its variables. Record the regression equation values of a and b in the table below.

y* = *ax* + *b
a =
b =

5. Click the Save To Document button . TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document. How does the value of a in the linear regression equation compare with the M-value you found by guess-and-check?

How does the value of *b* compare with the y-intercept value, *B*, you identified earlier? Explain.

6. Double-click on the graph in the TI InterActive!™ document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regEQ(x)** and press Enter. TI InterActive! graphs the equation that was created as the Stat Regression result. Which equation seems to fit the data better? Which equation is a better linear function model? Why?

7. Remember, slope is defined as change in *y*-values divided by change in *x*-values. Complete the following statement about slope for the linear data set you collected.

In this activity, slope represents a change in ______________________________

divided by a change in ______________________________.

Based on this statement, what are the units of measurement for the slope in this activity?

8. As mentioned earlier, the intercept value, *B*, can be interpreted as the starting position or the starting distance from the motion detector. What does the value of *M* represent physically? (Hint: Think about the units of measurement for the slope you described in question **6**).

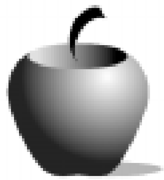
9. Save and print your TI InterActive! document.

Extensions

- Change your rate of walking speed and redo the activity with this new data. Describe any differences in the linear models.
- Start about 3.5 meters away from the motion detector and walk towards it. Describe any differences in the linear models.
- Stand in front of the motion detector and do not move. Describe any differences in the linear models.

Teacher Notes

Activity 2: Tight Rope



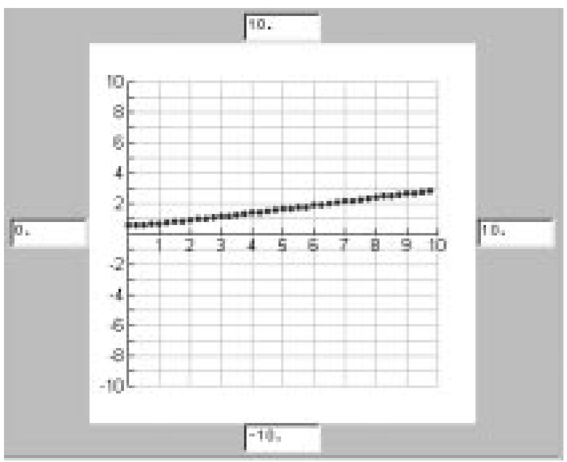
Math Concepts

- CBL™/CBL 2™
- Linear Function

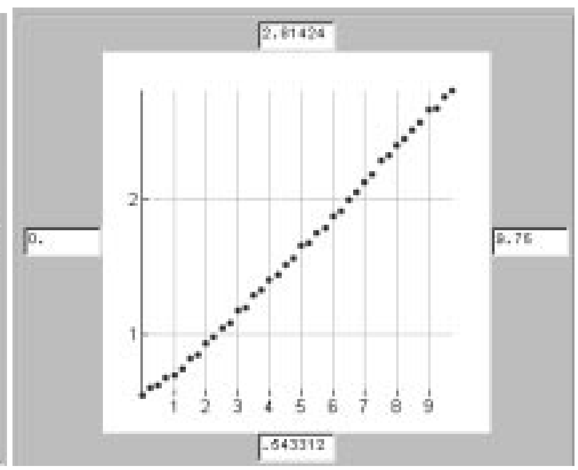
Activity Notes

- This activity investigates linear functions as models with a constant rate of change. Students will create constant-speed distance versus time plots.

Sample Data



Original StatPlot



After ZoomStat

Analysis and Questions - Answer Key

1. Answers will vary but $B \approx .5$ m (50cm). B is the distance from the motion detector at time 0.
2. Answers will vary but M represents the rate the distance from the detector is increasing with respect to time.
3. The calculated slope should be approximately equal to the slope determined by guess-and-check.
4. The regression equation for the sample data is: $regEQ(x) = .240356x + .454713$.
5. Although all the guess-and-check linear functions should ultimately be similar, one could argue that the linear regression equation is a better fit because we would all get the same linear function for the data set.
6. Change in distance divided by the change in time.
7. Slope = $\dfrac{\Delta \text{distance}}{\Delta \text{time}}$; meters per second (m/s).
8. M is the rate the person is walking away from the motion detector. In the sample data this rate is $\approx$ .24 m/s.

EXPLORATIONS

Activity 3

Take the Train

Have you ever ridden on a train? In 1869, the first railroad that crossed the continent was built. Now, railroad tracks cover the United States – you probably know of some near where you live. Railway companies such as Amtrak® can take you to almost any city or town in the country. In this activity, you will travel on one of Amtrak's most famous routes and follow its path through towns like Kankakee, Illinois and Yazoo City, Mississippi.

Introduction

Using the TI InterActive!™ web browser, you will visit Amtrak's web site and access the City of New Orleans timetable. After collecting a list of stops, times and distances, you will determine which mathematical model can best track the train.

Equipment Required

- TI InterActive! software
- A working Internet connection
- Adobe® Acrobat® Reader software (http://www.adobe.com)

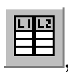
Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Take the Train* and add your name and the date. Click the Save button to save and name your document.
3. Click the Web Browser button to open the TI InterActive! browser. Click on the Data Sites button. Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 3: Take the Train.**
4. Once the page has been loaded in the browser, scroll down to the "City of New Orleans" and click this link. Adobe Acrobat Reader automatically launches, and the timetable for this route appears. Scroll through the timetable and note that the train starts in Chicago, and ends in New Orleans. The arrival times for the train are shown in the leftmost column, and the distances in the column just beside it. Note that the distance traveled is cumulative.
5. Record the data for the train's first ten stops in the table provided.

   Note*: You can print the web page to use in the* Working With the Data *section, rather than writing it in the table and transferring it to the Data Editor.*

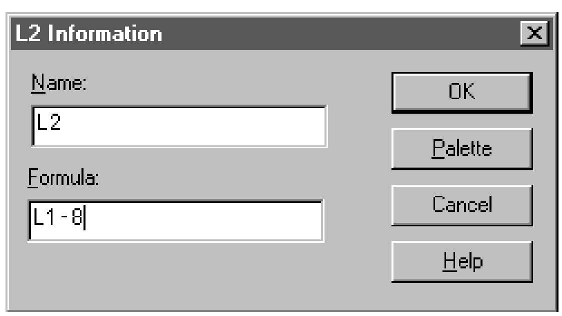
City	Time	Total Distance
Chicago, IL	8:00	0
Homewood, IL	8:51	25
Kankakee, IL	9:24	
Champaign-Urbana, IL		
Matoon, IL		
Effingham, IL		
Centralia, IL		
Carbondale, IL		
Fulton, KY		
Newbern-Dyersburg, TN		

Working With the Data

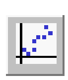
1. Click the List button , and then click the empty cell at the top of list **L1**. The times you wrote on paper will be changed slightly, and be written as decimals. Type the starting time value, 8:00, as **8**, and then press the down arrow key to move to the next cell. Enter the next time, 8:51, as a fraction, for instance **8+51/60**. Notice that TI InterActive!™ automatically enters the time in decimal form, 8.85.

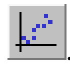
   Also, since you are interested in calculating total elapsed time, write the stops that occur after midnight as 12, plus the hour. For example, write 1:30 a.m. as **13+30/60**, and 4:25 a.m. as **16+25/60**. Continue entering the elapsed times until you have entered all of the time values into **L1**.

2. You will now use **L2** to calculate total elapsed time. Double-click on the gray box that reads **L2**. In the dialog box that appears, click in the Formula box and type **L1-8**. Click **OK**.

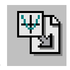


3. Click the empty cell at the top of list **L3**. Type the starting distance, **0**, and press the down arrow key to move to the next cell. Continue entering the distances until you are finished.

4. Click the Scatter Plot button  and then click the Stat Plots tab. In the uppermost text box, type **L2** to specify it as the list containing the *x*-coordinates. Press the Tab key and move to the second text box, and type **L3** to specify the list containing the *y*-coordinates.

5. Press Enter, then click the Zoom Statistics button . The viewing boundaries will be adjusted automatically to show all the plotted data.

6. The plot of elapsed time versus total distance traveled should go up and to the right. You can make a sketch on one of the blank grids in the Appendix of the data that you collected for elapsed time versus the total distance traveled. Label the horizontal and vertical axes on your sketch.

7. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

1. When two quantities are related so that when one changes, the other one changes by a constant multiple of the first, we say that the quantities are *directly proportional*. Mathematically, this type of relationship can be expressed as:

   $$y = kx$$

   where, for this activity, y represents the total distance traveled, x represents the total elapsed time, and k is called the *constant of variation*.

   In order to find a direct variation model for elapsed time and distance, you will need to find the value of k, the constant of variation. We will use the guess-and-check method. Click the f(x) tab in the upper left corner of the Functions dialog box. Start with an initial guess of $k = 10$. Type **f(x):= 10*x** in the uppermost text box of the f(x) tab. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of k produced a model that matches the data closely. Click in the text box of the f(x) tab again and edit the direct variation equation, replacing the old value, $k = 10$, with your new guess for k. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a k-value that models the data well and record it in the space below.

   $k =$

2. In science class, you have probably used the equation: $distance = rate \times time$. This is really the same as the equation $y = kx$, because the y-values represent total *distance* traveled and the x-values represent total elapsed *time*.

   For this activity, what is the real-world meaning of the k value you computed earlier using the guess-and-check method? What are the units of measure for the k value?

3. You probably noticed that increasing and decreasing the constant of variation, k, changes the steepness of the line you are graphing. For this reason, the constant k in the linear equation $y = kx$ is called the *slope* of the line. Using guess-and-check, you determined an overall slope for the train data. You also determined that this number approximates the average speed of the train over *all* of its ten stops.

We know, however, that the train was not traveling that *exact* speed between every city. You may be able to get a pretty good idea of when the train was moving fastest just by looking at the graph and finding the steepest sections. You can use another feature of TI InterActive! to figure out where the train was going fastest and slowest. Mathematically, slope is defined as the change in *y*-values divided by the change in *x*-values, $\Delta y/\Delta x$.

Click the Graph close box . Choose **Yes** when asked if you want save changes. Click the Save to Document button to save the graph in your TI InterActive! document.

To find the slope of the segment between each pair of points (the speed of the train between each of its stops), double-click on the gray box that reads **L4**. In the dialog box that appears, click in the Formula box and type **deltaList(L3)/deltaList(L2)**, and click **OK**.

The numbers that appear in list **L4** represent the average speeds for the train between stops. Between which two cities was the train moving slowest? What was the average speed between these cities?

4. TI InterActive!™ lets you check the value of *k* you found by calculating the line of best fit.

a. Click the Graph close box to return to the Data Editor.

b. Click Statistical Regressions .

c. Click the down arrow next to Calculation Type, scroll down the list and click on **Linear Regression (ax + b).**

d. In the text box labeled **X List**, type **L2**; the the box labeled **Y List**, type **L3**.

e. Click **Calculate** to find the regression equation, $y = ax + b$, and its variables. Record the regression equation values of *a* and *b* below:

$y = ax + b$
$a =$
$b =$

f. Click **Save Results**. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document. How does the value of *a* in the linear regression equation compare with the *k*-value you found by guess-and-check?

In theory, what should the *b*-value from the regression equation be? Explain.

5. Do you think that the linear modeling equation you developed in this activity could be used to accurately predict when the train will arrive in New Orleans? (Hint: What happens to the train in Memphis?) What factors would you have to take into account if you wanted to use your equation to predict arrival times?

6. Save and print your TI InterActive!™ document.

Extensions

- Edmund and Ted are going skiing for the weekend. After work, they pack the car and set off for their trip. Leaving town, they drive on a small road for 30 minutes at a constant speed of 40 mph. When they reach the highway, they speed up to 65 mph and drive for two hours. What is the average speed (total distance divided by total time)? Why isn't the answer the same as the average of the speeds: 52 ½ mph? (Hint: Think about how long they were traveling at each speed).

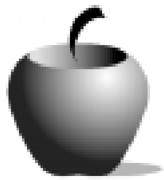
- Suppose that, in the previous question, Edmund and Ted had to drive an additional 30 minutes at 30 mph on a windy road from the highway to the ski mountain. What is the average speed for the entire trip now?

- Tonya is going to Barbara's house for a slumber party, but has to stop and pick up Mary first. Tonya walks ¼ mile to Mary's house in 5 minutes, but has to wait there while she packs her sleeping bag. Mary's dad offers to drive them, but only if they sit down and try some of his experimental grapefruit and marshmallow cookies. By the time they explain to Mary's dad that they would prefer a good walk to a stomach ache, 15 minutes have passed. They leave the house, and walk ¾ mile to Barbara's house in 10 minutes. What is the total distance for the trip? What is the total time (including the time they were at Mary's house)? What is the average speed for the whole trip? What is the average speed for just the walking portion of the trip?

- Mr. Bryan is taking his dog Tater for a walk. They leave home and walk two blocks down Amherst Street in 3.6 minutes, then stop for 1 minute while Tater watches a neighbor's cat. Next, they walk four blocks on Parkside Avenue in 7.5 minutes, three blocks on Tillinghast Place in 5.1 minutes, stop for 45 seconds so Tater can sniff a tree, and finally walk five blocks back home in 8.5 minutes. Assume that a block is 1/10 of a mile. What is the total distance for the trip? What is the total time (including the time when Tater was busy)? What is the average speed for the whole trip? What is the average speed for just the walking part of the trip?

Teacher Notes

Activity 3: Take the Train



Math Concepts

- Internet Data Collection
- List Manipulation
- Direct Variation
- Linear Function

Activity Notes

- If you do not have Internet access, you can still do this activity by obtaining a bus schedule from a local station.
- Be sure to take some time to carefully explain the idea of converting standard time values to decimal form. This concept is confusing for many students.

Sample Data

City	Time (hours)	L1=Times (decimal hours)	L2=Elapsed Time (decimal hours)	L3=Total Distance (miles)
Chicago, IL	8:00	8	0	0
Homewood, IL	8:51	8.85	0.85	25
Kankakee, IL	9:24	9.4	1.40	57
Champaign-Urbana, IL	10:37	10.61667	2.62	129
Mattoon, IL	11:18	11.3	3.30	174
Effingham, IL	11:43	11.71667	3.72	201
Centralia, IL	12:35	12.58333	4.58	254
Carbondale, IL	1:30	13.5	5.50	310
Fulton, KY	3:40	15.66667	7.67	406
Newbern-Dyersburg, TN	4:25	16.41667	8.42	442

Analysis and Questions - Key

1. $k = 54$.
2. k represents the speed of the train in miles per hour.
3. Slowest = Chicago to Homewood, 29.4 mph; fastest = Champagne-Urbana to Matoon, 66.2 mph.
4. $a = 54.860$, $b = -8.998$. The a-value is very close to the value found by guess-and-check. The value of b in the regression equation should equal zero.
5. No, since the train stops for a period of time when it arrives in Memphis. You need to consider layovers, stopping times, and variations in speed if you wish to accurately predict arrival times.

EXPLORATIONS

Activity 4

Think about a simple electrical circuit consisting of a battery and a light bulb. The battery, or voltage source, is like a pump, pushing electrons around a closed path and causing the bulb to glow. The flow of electrons is called *current*. The bulb provides resistance to electron flow; the bigger the bulb, the larger the voltage source required to maintain a steady current.

Introduction

In this experiment, the light bulb will be replaced by a segment of graphite. Instead of using bigger or smaller bulbs, you will vary the length of the graphite segment to change the resistance. A constant current will be maintained while the resistance is changed. Using a CBL™ or CBL 2™, you will measure voltages across increasing lengths of graphite, then use this data to build a mathematical model relating resistor length and voltage.

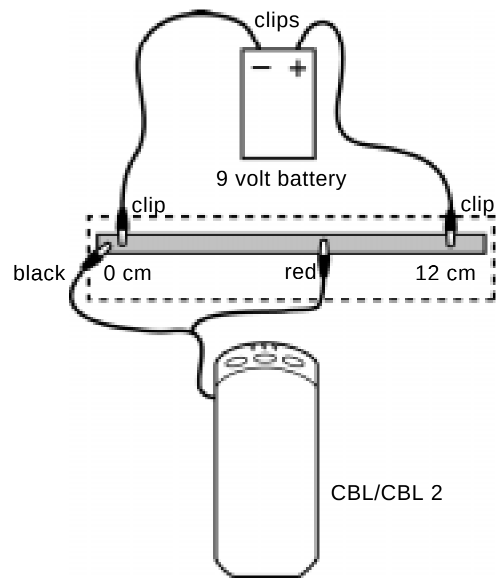
Equipment Required

- TI InterActive!™ software
- CBL or CBL 2 with a compatible TI calculator
- TI voltage probe
- Two alligator clip jumper leads (RadioShack® #278-1156C)
- A 9-volt battery
- Pair of scissors
- Graphite pencil (No. 2 pencil)

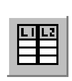
Setup

CBL	CBL 2
1. Use a pair of scissors to cut out the box on the last page of this activity. With a graphite pencil, completely darken the rectangle within this box.	**1.** Use a pair of scissors to cut out the box on the last page of this activity. With a graphite pencil, completely darken the rectangle within this box.
2. Use the alligator clip jumper leads to connect the positive lead of the battery to the **0 cm** (centimeter) mark on the rectangle, and the negative lead of the battery to the **12 cm** mark.	**2.** Use the alligator clip jumper leads to connect the positive lead of the battery to the **0 cm** (centimeter) mark on the rectangle, and the negative lead of the battery to the **12 cm** mark.

CBL	CBL 2
3. Plug the voltage probe into the **CH 1** port of the CBL™. In this experiment, you will use the CBL as a stand-alone device to measure voltage.	**3.** Plug the voltage probe into the **CH 1** port of the CBL 2™.
4. Press the ON/HALT button, then press the MODE key to put the CBL in multimeter mode.	**4.** Connect the CBL 2 to a compatible TI calculator.
5. Press the CH VIEW button repeatedly until the **CH 1** channel, with **V** appears.	**5.** Run the **DATAMATE** program. For details on how to load and use the **DATAMATE** program, see the CBL 2 guidebook.
6. Connect the black CBL/CBL 2 voltage lead to the **0 cm** mark.	**6.** Connect the black CBL/CBL 2 voltage lead to the **0 cm** mark.



Collecting and Recording the Data

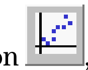
1. Start TI InterActive!™ The software opens to a new, blank document.
2. Title your new document *Pencil Me In* and add your name and the date. Click the Save button  to save and name your document.
3. Click the List button  to open the Data Editor.
4. Click the empty cell at the top of list **L1**. Type the first segment length, **0**, and then press the down-arrow key to move to the next cell. Continue until you have entered all of the segment lengths into **L1**.
5. With the experiment set up as shown above and the CBL/CBL 2 taking voltage readings, position the red CBL/CBL 2 voltage lead on the **2 cm** mark of the graphite segment.

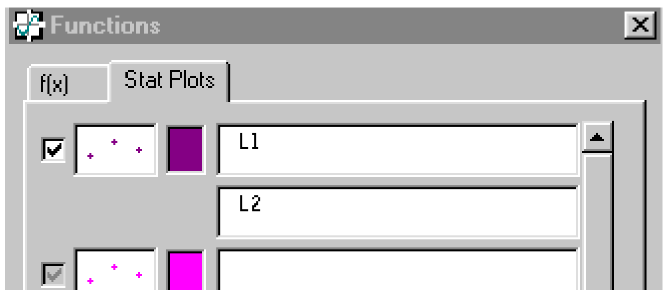
6. Click the empty cell at the top of list **L2**. Type the first recorded voltage in the top cell and then press the down-arrow key to move to the next cell.

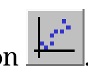
7. Repeat this procedure until you have entered all of the voltage readings into **L2**.

   Note: *Be sure to disconnect the leads from the battery when you are finished collecting the data so that the battery does not overheat.*

Working With the Data

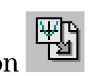
1. Click the Scatter Plot button , then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x*-coordinates. Press the Tab key to move to the second text box, and type **L2** to specify the list containing the *y*-coordinates.



2. Press Enter, then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data. Your segment length versus voltage plot should appear as a straight line passing through the origin.

   If you are not satisfied with your results, start again and collect a new set of data.

   If you are satisfied with your data, make a sketch of the segment length versus voltage data that you collected on one of the blank grids in the Appendix. Label the horizontal and vertical axes on your sketch.

3. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

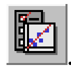
1. When the relationship can be expressed in the form $y = kx$, the variables x and y are said to *be directly proportional* or *vary directly*, and k is called the *constant of variation*. In order to find a direct variation model for segment length and voltage, you will need to find the value of k, the constant of variation. You will use the guess-and-check method. Click the f(x) tab in the upper left corner of the Functions window. Start with an initial guess of $k = 1$. Type **f(x):= 1*x** in the uppermost text box of the f(x) tab. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of k produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the direct variation equation, replacing the old value, $k = 1$, with your new guess for k. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a k-value that models the data well and record it in the space below:

   $k =$

2. TI InterActive!™ lets you check the value of *k* you just found by calculating the line of best fit.

a. Click the Save to Document  button and return to the Data Editor.

b. Click Statistical Regressions .

c. Click the down arrow  next to Calculation Type, scroll down the list and click on **Linear Regression (ax + b)**. In the text box labeled **X List**, type **L1**; in the box labeled **Y List**, type **L2**.

d. Click **Calculate** to find the regression equation, $y = ax + b$, and its variables. Record the regression equation values of a and b below:

$y = ax + b$
$a =$
$b =$

e. Click **Save Results**. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

f. Close the Data Editor.

How does the value of a in the linear regression equation compare with the k-value you found by guess-and-check?

What should the b-value from the regression equation be? Explain.

3. Drag the graph to position it below the Statistical Regressions result if necessary, and then double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq(x)** and press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions result.

Which equation seems to fit the data better? Which equation is a better direct variation model? Why?

4. Use the direct variation modeling equation you found in this activity to predict the voltage reading corresponding to a 5 cm long graphite segment. Record this value in the table below as the predicted voltage reading.

Voltage Readings	
Predicted	**Actual**

Test your prediction by positioning the red CBL™ voltage lead at the **5 cm** mark on the graphite segment you used in this experiment. Be sure the CBL is in multimeter mode (or, if you are using the CBL 2™, be sure that you have executed the **DATAMATE** program). Record this value in the table above as the actual voltage reading. How do these results compare?

5. Save and print your TI InterActive! document.

Extensions

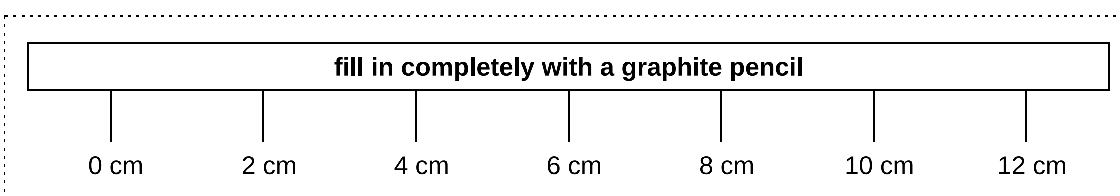
- Repeat the experiment, but this time start with an 8 cm segment instead of a 12 cm segment of shaded with graphite paper. Take voltage readings every 2 cm. Note the value of the constant of variation, *k*, between voltage and segment length. Repeat for a 20 cm segment of graphite paper (take readings every 5 cm). Record the *k*-value. Does there appear to be a relationship between the total length of the shaded segment and the voltage of the battery?

  ***Note**: To control experimental conditions, try to keep the width and thickness of the graphite shading constant in both cases.*

- Use the bottom of a coffee can and a graphite pencil to make a circle on a blank sheet of paper, leaving a 1 cm gap between the starting and ending points of the circle. Color over the circle's outline with the pencil (but maintain the gap), making the width of the circle's outline about ½ cm. Use a ruler or measuring tape to mark off 2 cm, 4 cm, and 8 cm lengths on the circle measured from the gap. Attach the jumper cables from the battery to starting and ending points of the circle. Be sure these two cables are not in contact with each other.

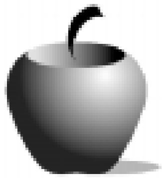
  Set up the experiment as before and record voltage readings for 2 cm, 4cm, and 8 cm lengths. Develop a direct variation modeling equation for this experiment. Measure the voltage reading across the entire circle (with the red CBL voltage lead positioned at the end point of the circle). Use this reading together with the direct variation modeling equation to predict the circle's circumference. Measure the diameter of the circle, then compute the circumference using the formula C = πd. How does this result compare with your prediction?

✂ Cut along dotted line



Teacher Notes

Activity 4: Pencil Me In



Math Concepts

- CBL™/CBL 2™
- Direct Variation
- Linear Function

Activity Notes

- The segment that is colored in with pencil must be very dark and consistent to get good data.
- Remind students to disconnect the battery leads as soon as they finish collecting data so that the battery does not become overheated. This will extend the life of the battery.

Sample Data

Segment Length (cm)	0	2	4	6	8	10	12
Voltage Reading (volts)	0	1.04	2.13	3.55	5.14	6.63	8.72

Analysis and Questions - Key

1. $k = 0.68$.
2. $a = 0.721$, $b = -0.436$. The *a*- and *k*-values are in close agreement; the *b*-value should equal zero.
3. Both fit the data well. The regression equation is not a direct variation equation since the *y*-intercept is non-zero.
4. Predicted = 3.40; actual = 2.93. They are in reasonably close agreement.

EXPLORATIONS

Activity 5

How do we measure temperatures? In almost all countries of the world, the Celsius scale (formerly called the centigrade scale) is used in everyday life and in science and industry. This scale sets the freezing temperature of water at **0** and the boiling temperature at **100**, with the distance between these two points divided into 100 equal intervals called *degrees*. The United States uses the Fahrenheit scale. This scale employs a smaller degree unit than the Celsius scale and its freezing point is set to a different temperature. For the temperatures we commonly use and observe, Celsius readings are lower than Fahrenheit values. You have probably noticed this if you have ever seen a thermometer that has both Celsius and Fahrenheit markings, or if you have ever driven by signs at banks and other businesses that display time and dual temperatures.

Introduction

In this experiment, you will use the TI InterActive!™ browser to access data relating to Celsius and Fahrenheit temperatures for a variety of cities all over the world. Based on the data collected, you will develop and test a mathematical relationship between these two temperature scales.

Equipment Required

- TI InterActive! software
- A working Internet connection

Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Two Hot, Two Cold* and add your name and the date. Click the Save button to save and name your document.
3. Click the Web Browser to open the TI InterActive! browser. Click on the Data Sites button. Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 5: Two Hot, Two Cold**.

4. Once the page has been loaded in the browser, scroll down the main page index and click on **WEATHER**.

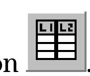
5. To find the forecasted temperature for today in Dallas, Texas, click on the down arrow next to the “choose a state” dialog box and click on Texas. Select Dallas from the options list and click **Go**. Record today’s high and low temperatures in Celsius and Fahrenheit in the table below.

6. Press the back button twice to return to the state list. Repeat the procedure described above for the other United States cities listed in the table. For cities in other countries, click the back button, select a region from the “choose a world region” box, then navigate to the city as you did before. Record all information in the data table.

7. Use the last two rows of the data table to record today’s high and low temperatures for any city in the world that you choose. Be sure to record the name of the city in the first column.

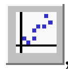
City	Celsius Temperature	Fahrenheit Temperature
Dallas, Texas (high)		
Dallas, Texas (low)		
Anchorage, Alaska (high)		
Anchorage, Alaska (low)		
New York, New York (high)		
New York, New York (low)		
Paris, France (high)		
Paris France (low)		

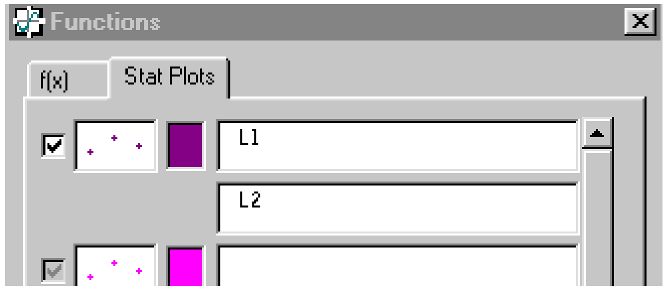
Recording the Data

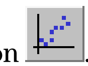
1. Close the browser window, then click the List button . Click the empty cell at the top of list **L1**. Enter the Celsius scale high temperature for Dallas, then press the down arrow key to move to the next cell. Enter the Celsius low temperature for Dallas, then press the down arrow key to move to the next cell. Continue entering the Celsius temperatures until you have entered all of them into **L1**.

2. Click the empty cell at the top of list **L2**. Enter the Fahrenheit scale high temperature for Dallas, then press the down arrow key to move to the next cell. Enter the Fahrenheit scale low temperature for Dallas, then press the down arrow key to move to the next cell. Continue entering the Fahrenheit temperatures until you have entered all of them into **L2**.

   When you are finished, check to be sure that **L1** and **L2** match your data table values exactly.

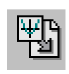
3. Click the Scatter Plot button , and then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key to move to the second text box, and type **L2** to specify the list containing the *y*-coordinates.



4. Press Enter, and then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

5. The plot of Fahrenheit temperatures versus Celsius temperatures should be linear and increasing. If you are not satisfied with your results, check to see that your temperatures are entered properly.

   If you are satisfied with your data, make a sketch of the Fahrenheit versus Celsius data that you collected on the axes on a blank grid in the Appendix. Label the horizontal and vertical axes on your sketch.

6. Click the Save to Document button  to save the graph in your TI InterActive! document.

Analysis and Questions

1. Click the Graph close box . Choose **Yes** when asked if you want to save changes. Click the Data Editor close box  to return to the main document.

   You can model the Fahrenheit versus Celsius data using a linear equation of the form $y = mx + b$. To start, you will need to compute the slope of the line using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   In this case, the *y*-values have been recorded in list **L2** and the *x*-values have been recorded in list **L1**. To find the slope, use the first two elements in each list. Click the Math Box button  and enter the slope expression exactly as shown below:

(L2[2] - L2[1])/(L1[2] - L1[1])

   Press Enter to compute the slope. Enter the resulting value, to two decimal places, in the space below:

$m =$

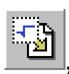
2. To find the y-intercept, b, in the linear equation $y = mx + b$, you will first need to choose any one of the data points you collected and identify the x-value (Celsius temperature) and corresponding y-value (Fahrenheit temperature) below:

$x =$ $y =$

Now substitute the number values for m, x, and y from above into the equation $y = mx + b$ and record the resulting equation below.

3. Your equation should contain only one unknown value, b. Use the space below to solve for b using basic algebra techniques, then record the b-value.

$b =$

TI InterActive!™ has a built-in feature that allows you to solve for an unknown. Click the Math Box button , and type the **solve** command exactly as shown below, using the same number values for x, y, and m that you used above:

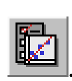
solve(y=m*x+b, b)

The only variable in the equation should be b. The b at the end of the **solve** command tells the computer to solve the equation for b. Press Enter to execute the command. How does the b-value found by the computer compare to the value you found using basic algebra techniques?

4. Double-click on the graph in the TI InterActive! document to refresh the Graph window. Click the f(x) tab in the upper left corner of the Functions window. Type **f(x):= m*x + b** in the uppermost text box of the f(x) tab, using the numerical value for m and b from above. Press Enter to superimpose the graph on the plotted data. How well does the linear model you found fit the data set?

5. TI InterActive! lets you check the value of m and b you found by calculating the line of best fit.

a. Click the Graph close box  to return to the Data Editor.

b. Click Statistical Regressions .

c. Click the down arrow next to Calculation Type, scroll down the list and click on **Linear Regression (ax + b).** In the text box labeled **X List**, type **L1**. In the box labeled **Y List**, type **L2**. Click **Calculate** to find the regression equation, $y = ax + b$, and its variables. Record the regression equation values of a and b below:

$y = ax + b$
$a =$
$b =$

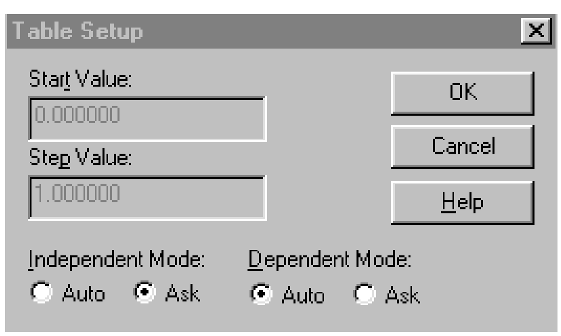
Click **Save Results**. TI InterActive!™ stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

How does the value of a in the linear regression equation compare with the m-value and b-value you found using algebraic techniques?

6. Double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq(x)** and then press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions Result together with the linear model you entered earlier.

Which equation seems to fit the data better?

7. You can use the equations you found in this activity to predict a Fahrenheit temperature for any given Celsius temperature. Click **Table** in the upper-right corner of the Graph window to display the table screen. Click Table Setup . Set the Independent Mode to **Ask** and the Dependent Mode to **Auto**, as shown below, then click **OK**.



An empty table is displayed. As you enter values for the independent variable (x), values for the dependent variable (y), will be generated using the equations in the function editor.

To start, type the number **100** (corresponding to 100 degrees Celsius, the boiling point for water) into the first *x*-cell, then click on the corresponding *y*-cell to display corresponding Fahrenheit temperature. Repeat for $x = 22$ (corresponding to 22 degrees Celsius, room temperature), and record your values in the second column of the table below.

Celsius temperature	Fahrenheit temperature (model)	Fahrenheit temperature (regression)
100		
22		

How do the results compare with known Fahrenheit temperatures for boiling water and room temperature?

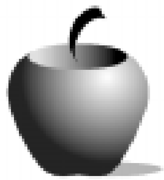
8. Save and print your TI InterActive!™ document.

Extensions

- Use the data you collected together with the TI InterActive! tools used in this activity to build a mathematical model that converts Fahrenheit temperatures to Celsius temperatures.
- Find a physical science or physics textbook that gives a conversion formula from Celsius temperatures to Fahrenheit temperatures. How does this conversion formula compare to the linear model you found in this activity?

Teacher Notes

Activity 5: Two Hot, Two Cold



Math Concepts

- Internet Data Collection
- Tables
- Linear Function

Activity Notes

- Start by asking students to describe the differences between Celsius and Fahrenheit temperatures. Can they predict what a graph of Fahrenheit versus Celsius would look like?
- If you do not have access to the Internet, you can use temperature listings in *USA TODAY*® for data.

Sample Data

City	Celsius Temperature	Fahrenheit Temperature
Dallas, Texas (high)	22	72
Dallas, Texas (low)	9	48
Anchorage, Alaska (high)	-13	8
Anchorage, Alaska (low)	-21	-7
New York, New York (high)	12	55
New York, New York (low)	1	35
Paris, France (high)	3	39
Paris, France (low)	-1	30

Analysis and Questions - Key

1. $m = 1.83$.
2. $x = 22$, $y = 72$; $72=1.83(72) + b$; $b = 31.76$.
3. $b = 31.76$. They have the same value.
4. The line fits the data reasonably well.
5. $a = 1.845$, $b = 32.232$. They match closely.
6. The regression equation seems to fit the data slightly better.
7. For C = 100, F(model) = 214.76 and F(regression) = 216.76. For C = 22, F(model) = 72.02 and F (regression) = 72.83. These values agree with the actual Fahrenheit values for boiling water (212 degrees F) and room temperature (about 70 degrees F).

EXPLORATIONS

Activity 6

The populations of the United States and the world have grown rapidly during recent history. Many different factors can affect the rate at which a population changes, including the climate, technology, and the economy. It is important in a number of areas to be able to predict future populations. Different mathematical models are appropriate over different lengths of time.

Introduction

In this activity, you will collect population data for the world on ten different days. You will then find a model for this data set assuming that over a short period of time the data can be considered to be linear. Finally, the model will be evaluated over a much longer period to determine the validity of the linear model.

Equipment Required

- Computer
- TI InterActive!™ software
- A working Internet connection
- Adobe® Acrobat® Reader software

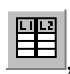
Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your document *Small World*, and add your name and the date. Click the Save button to save and name your document.
3. Click the Web Browser to open the TI InterActive! browser. Click on the Data Sites button. Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 6: It's a Small World**.
4. Once the page has been loaded in the browser, click **Population Clocks**. On the POPClocks page you should see the current estimated population of the United States and of the world.

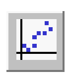
5. Record the date, day number, and population of the world in the table below. Use the number of days since the year 2000 began in column two, allowing January 1, 2000 to be day one. You will need ten different days on which to collect this data. Although taking readings at the same time each day might be the best technique, differences of a few hours either way will not have a significant effect on the model.

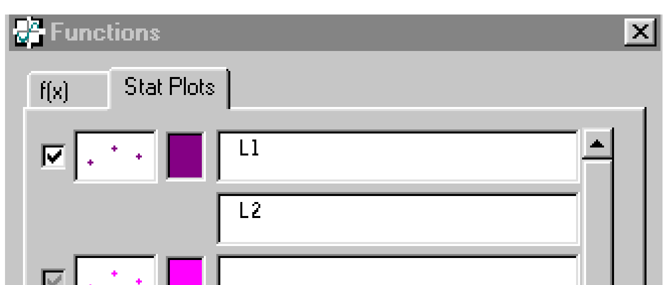
Date	Day Number	Population of the World

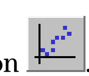
Recording the Data

1. Click the List button , then click the empty cell at the top of list **L1**. Type the initial value and then press the down arrow key on the keyboard to move to the next cell. Continue entering the number of days since January 1, 2000 until you have entered all of the day values into **L1**.

2. Click the empty cell at the top of list **L2**. Enter the corresponding population values for each day that you recorded in **L1**. You may need to resize **L2** so you can read the data. To do this, click and drag the bar on the right side of the cell labeled **L2**.

3. Click the Scatter Plot button  and then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key and move to the second text box. Type **L2** to specify the list containing the *y* coordinates.



4. Press Enter, and then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

5. The plot of number of days versus the population of the world should appear to be linear in nature. Click on the Save to Document button to copy this plot into your TI InterActive!™ document.

Analysis and Questions

1. Click on Trace and move along the plot of your data using the arrow keys. Choose two points that would appear to lie on a line of best fit and make a note of the day numbers of these points below.

   Day # Day #

2. Record the day # and population for the two days you chose above as ordered pairs in the space below. Refer to the table or Data Editor for the exact values of the coordinates.

3. Click on Save to Document to return to your TI InterActive! document. Click on Math Box and use the two points you selected above and the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of the line of best fit. Record your answer below.

4. A simple way to describe slope is change in *y* over change in *x*. What is the real-world interpretation of the slope that you found in the question above?

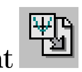
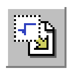
5. To find the *y*-intercept of your linear model, you will need to solve the equation $y = mx + b$ for *b*. To do this, click on Math Box and enter **solve(y=mx+b,b)** substituting the value of the slope you found in step **3** for *m* and either of the points you selected in step **2** for *x* and *y*. Press Enter to solve for *b*, and record the solution below.

   $b =$

6. A simple way to describe the *y*-intercept is to say it is the point where the graph crosses the *y*-axis. In this model, what is the real-world meaning of the *y*-intercept?

7. Use the values of *m* and *b* that you found to write a linear model for your data in the form $y = mx + b$.

8. Double-click on the saved graph in your document to return to the Graph window. Click on the f(x) tab of the Functions dialog box and enter the equation you found in step **7**. Press Enter to turn on the function and graph it. How well does it fit the data you collected?

9. You can use the equation you just found to predict future or estimate past population values for the world. To simplify this, you can define a function with the values you found above. Click on Save to Document  to return to your TI InterActive! document. Click on Math Box  and type **pop(x):=mx+b** using the values you found for *m* and *b* above and press Enter. You can now find any population value for day *x* by typing **pop(x)** in a Math Box and pressing Enter. To start, predict the population of the world one year from today by adding 365 to today's day number. (For example, if today was the tenth day, **pop(375)** would give the population one year from today.) Record this value in the space provided.

10. How reliable do you feel that your prediction above is? Discuss your reasoning.

11. The United States Census bureau originally predicted that the world would attain a population of six billion people on July 19, 1999. Use the Math Box and **pop(x)** to find the number of people your model estimates that there were on that date and record your answer on the space provided. (An easy way to determine the day number is to find what day of the year July 19, 1999 was and add that to –365.)

12. Did your model produce a reasonable value for this date? Explain.

13. On January 1, 2000 over six billion people celebrated the new year. According to your model, how many people will celebrate the new year on January 1, 3000? Use the Math Box and **pop(x)** to find this value and record it below.

14. Does the answer you got in the problem above seem reasonable? Discuss why or why not.

15. Click on the Math Box. Use TI InterActive!™ to determine when your model predicts that the population of the world was zero. To do this, type **solve(0=mx+b,x)** substituting your values for *m* and *b* and press Enter. Remember to convert your answer from days to years. Is this answer reasonable? Explain.

16. Based on the questions above, when do you think a linear model would be appropriate to use in predicting population values? When would it be inappropriate? Explain your answer.

17. Save and print your TI InterActive! document.

Extensions

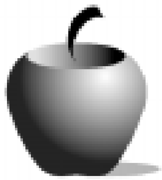
- Repeat this activity using the population of the United States rather than the world.
- Choose a date during the last week of the school year and use your model to predict the population of the world on that day. When the day you chose arrives, check the population clock and compare your prediction to the actual value.
- Collect population values from over the course of history and set up a plot of the data. Try to find an appropriate mathematical model for population growth over long periods.

Teacher Notes

Activity 6: It's a Small World

Math Concepts

- Internet Data Collection
- Linear Function



Activity Notes

- Begin collecting data a couple of weeks in advance of the date you plan to do this activity. Daily commitment is only a couple of minutes and will allow different students to experience using the web.
- Students need to be careful when determining the day number. Remind them to take into account the different lengths of various months.

Sample Data

Date	Day Number	World Pop.
1/06/00	6	6,036,284,662
1/07/00	7	6,036,524,192
1/08/00	8	6,036,703,934
1/09/00	9	6,036,955,290
1/10/00	10	6,037,177,346
1/11/00	11	6,037,366,570
1/12/00	12	6,037,547,288
1/13/00	13	6,037,736,380
1/14/00	14	6,037,964,912
1/15/00	15	6,038,165,017

Analysis and Questions - Key

1. Day #7, day #14
2. (7, 6036524192) (14, 6037964912)
3. The slope is approximately 205817.
4. The slope represents the increase in world population per day.
5. The value of b is 6035083474.
6. The y-intercept represents the population of the world at the beginning of the year 2000.
7. $y = 205817x + 6035083474$
8. The model fits the data very well.
9. The world population one year from day 375 will be 6,112,267,849.

10. The prediction should be close, as there should not be any major changes in the rate at which the population is growing over the next year.

11. July 19th is the 200th day of the year. With respect to Jan 1, 2000, this would be day -165. The population on day -165 was 6,001,126,669.

12. The value is reasonable. The percentage error is very small.

13. There are approximately 365,000 days in a millennium. (**Note**: Some students may wish to be more precise here.) The predicted population is 81,158,291,474.

14. The answer is too large. At some point, population growth will begin to level off due to food supplies and overcrowding.

15. The model says the population of the world was zero on day -29,333 or 80 years ago. This is obviously not true; there were many people in 1920.

16. A linear model is only appropriate for short-term population growth. Different models such as exponential or logistic would be better for long term models.

Activity 7

When a ball is dropped on a flat surface, it will rebound to a certain percentage of the original drop height. As the ball continues to bounce up and down, each successive bounce after the first will continue to rebound to approximately this same percentage of the previous bounce's height. You can model this behavior mathematically with an exponential function. Exponential functions are characterized by the property that equal changes in the independent variable produce equal percentage changes in the dependent variable.

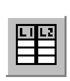
Introduction

In this activity, you will collect motion data for a bouncing ball using TI InterActive!™ and a CBL™ or CBL 2™ unit. You will then analyze this data and attempt to find the exponential relationship between the bounce number and the maximum height that the bounce reaches.

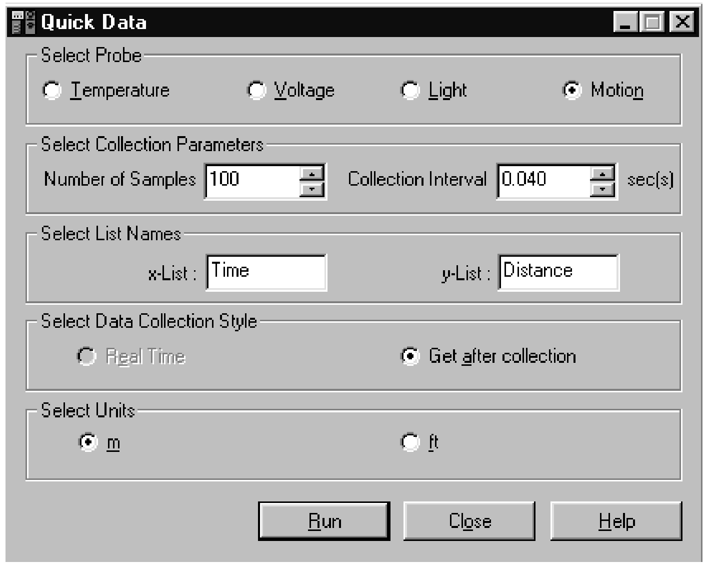
Equipment Required

- Computer
- TI InterActive! software
- CBL/CBL 2
- Motion detector
- TI-GRAPH LINK™ cable
- Ball

Instructions

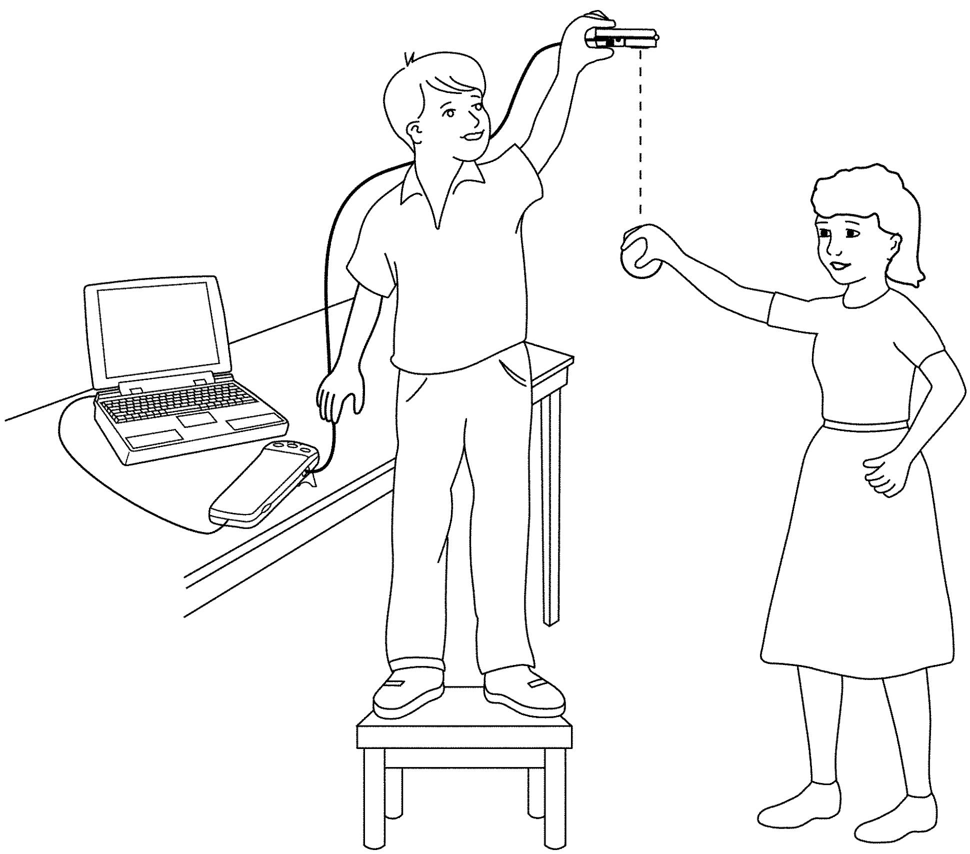
1. Plug the TI-GRAPH LINK cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL/CBL 2 unit.
3. If you are using a CBL, plug the motion detector into the **Sonic** port. If you are using a CBL 2, plug the motion detector in to the **DIG/SONIC** port.
4. Start TI InterActive! The software opens to a new, blank document.
5. Title your document *Rebound* and add your name and the date. Click the Save button  to save and name your document.
6. Click the List button  to open the Data Editor.

7. From the **Data** menu, choose **Quick Data**.
8. Adjust the Quick Data dialog box so the settings match the ones shown below.



Collecting the Data

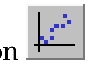
1. Have a student hold the motion detector approximately 1.5 to 2 meters above the floor and parallel to it, as shown in the illustration below.



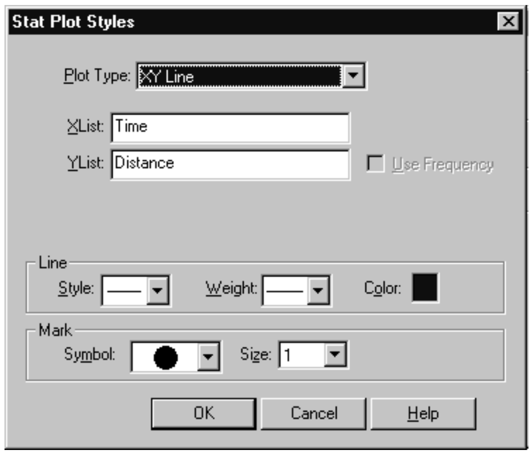
2. A second student should hold the ball approximately .5 meters below the detector and prepare to release it when the unit begins to click. Be careful that the hands are immediately moved out from under the detector as soon as the ball is released.

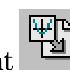
3. If you are using a CBL™, turn it on by pressing the red ON/HALT button.

4. Click on the **Run** box on the Quick Data screen to begin data collection.

5. When data collection is done, click the Zoom Statistics button  in the Graph window. The viewing boundaries adjust automatically to show all the plotted data.

6. If the scatter plot looks confusing, click the Stat Plots tab of the Functions dialog box, then click the box to the right of the check box. This will bring up the Stat Plot Styles dialog box shown below. Change the Plot Type to **XY Line** and click **OK**.

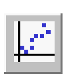


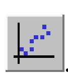
7. Your plot should appear to be a number of parabolas opening upward and rising from left to right. If you are satisfied with your data, click on Save to Document  to save your graph in your TI InterActive!™ document. If you are not satisfied, close the Graph window and return to step **2** to collect a new data set.

Recording the Data

1. The plot you recorded is a distance versus time graph. Convert the recorded distances to heights by subtracting each value from the distance to the floor. The distance to the floor will be the maximum distance point that you collected.

2. Double-click on listname **L1**, enter **time** in the **Formula** box, and click **OK**. Double-click on listname **L2**, enter **max(distance)-distance** in the **Formula** box, and click **OK**. This will convert your distance readings to heights.

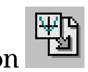
3. Click the Graph button  to open the Graph window and Functions dialog box. Click the Stat Plots tab to display **L1** and **L2** and click Enter to turn on the plot.

4. Click the Zoom Statistics button . The viewing boundaries adjust automatically to show the plot of height versus time.

5. Press 

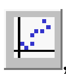
and move along the data plot. Record your initial drop height as bounce zero. Continue to record the maximum heights of each successive bounce in the table on the next page.

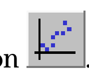
Bounce #	Height (m)
0	
1	
2	
3	
4	
5	
6	

6. Close the Trace Value dialog box and click the Save to Document button  to return to the Data Editor.

7. Click the empty cell at the top of list **L3**. Type the bounce number and then press the down-arrow key on the keyboard or press Enter to move to the next cell. Continue until you have entered all of the bounce numbers from the table above into **L3**.

8. Click the empty cell at the top of list **L4**. Type the corresponding height for the bounce in list **L3** and then press the down-arrow key or press Enter to move to the next cell. Continue until you have entered all of the height readings from the table above into **L4**.

9. Click the Graph button , and then click the Stat Plots tab. In the uppermost text box, type **L3** to specify it as the list containing the *x*-coordinates. Press the Tab key and move to the second text box. Type **L4** to specify the list containing the *y*-coordinates.

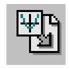
10. Press Enter, and then click the Zoom Statistics button . The viewing boundaries are adjusted automatically to show your plot of bounce number versus height.

11. Your bounce number versus height plot should appear to be a graph of an exponential decay. Click on Save to Document  to copy this plot to your TI InterActive! document.

Analysis and Questions

1. The theoretical model for the bouncing ball data recorded on the axes above is exponential. You will attempt to fit this data set with a curve of the form: $y = AB^x$ where A is the initial height of the ball and B is the ratio of the successive maximum heights of each bounce. Use the information you recorded in the table above to find the initial height:

$A =$

2. Click Save to Document  to return to the Data Editor. Highlight the values you entered in **L3** and **L4**, click Copy , and then click on the spreadsheet tab at the bottom of the Data Editor. The cursor will be in cell A1. Click Paste . This pastes the contents of lists 3 and 4 into columns A and B.

The variable B in the equation $y = AB^x$ represents the ratio of the maximum height of each bounce in relation to the preceding bounce. To find the ratio of the first bounce to the original drop height, move to cell C1, type **=B2/B1**, then press Enter. To find the remaining ratios, click on cell C1. Move the cursor to the lower right hand corner of the cell, then click and drag until all cells adjacent to the filled cells in column B except for the last are selected. Release the click and all the ratios will be calculated. Record these ratios in the table below.

3. Move the cursor to an empty cell and find the average of these ratios. Type **=average(c1,cx)** where x is the number of the last cell containing a ratio, and press Enter. Record the average in the space below.

$B =$

4. Use the values you found for A and B to write an equation modeling bounce height as a function of bounce number in the form $y = AB^x$.

$y =$

5. Close the Data Editor and double-click on the graph of bounce number versus height in the TI InterActive! document to refresh the Graph window. Click the f(x) tab in the Functions dialog box and enter the formula that you found in step **5** above. Press Enter to turn on the function. How well does your formula fit the data?

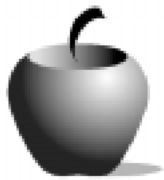
6. Use your model to predict the height of the ball on its 8th bounce.
7. According to your model, how many bounces will take place before the ball no longer reaches a maximum height of 5 centimeters?
8. Save and print your TI InterActive!™ document.

Extensions

- Find the total distance the ball would travel over the course of 8 bounces.
- Have TI InterActive! find a model for this activity by performing an exponential regression on the data. How does this model compare to the one you found in this activity?
- Repeat this activity using different balls, drop heights, and surfaces, and observe how each affects the values of *A* and *B* in your model.
- Connect the TI-GRAPH LINK™ cable to your calculator and use TI InterActive! to send the data you collected from the computer to the calculator. Analyze the data on your calculator using the same methods applied in this activity.

Teacher Notes

Activity 7: On the Rebound



Math Concepts

- CBL™/CBL 2™
- Spreadsheets
- List Manipulation
- Ratios
- Exponential Function

Activity Notes

- If there is not enough room near the computer to collect this data, it can be collected using the calculator with a CBL or CBL 2. The data can then be transferred to a TI InterActive!™ document.
- Caution students to be sure that nothing blocks the path between the motion detector and the ball. Hands and cords are common obstructions.

Sample Data

***Note**: Due to the large number of data points collected, the sample data contains only the values of the maximum heights of the bounces recorded in the activity.*

Bounce #	Height (m)
0	.840
1	.683
2	.535
3	.418
4	.340
5	.266
6	.209

Analysis and Questions - Key

1. The initial height of the ball was .84 meters.

2.

Cell C
0.813
0.783
0.781
0.813
0.782
0.785

3. The average of the ratios in the table is .799.

4. The equation is $y = .84(.799)^x$.

5. The model is an almost perfect fit.
6. The predicted height on the 8th bounce is .14 meters.
7. On the 13th bounce, the ball will fail to reach .05 meters.

EXPLORATIONS

Activity 8

Cooling Off

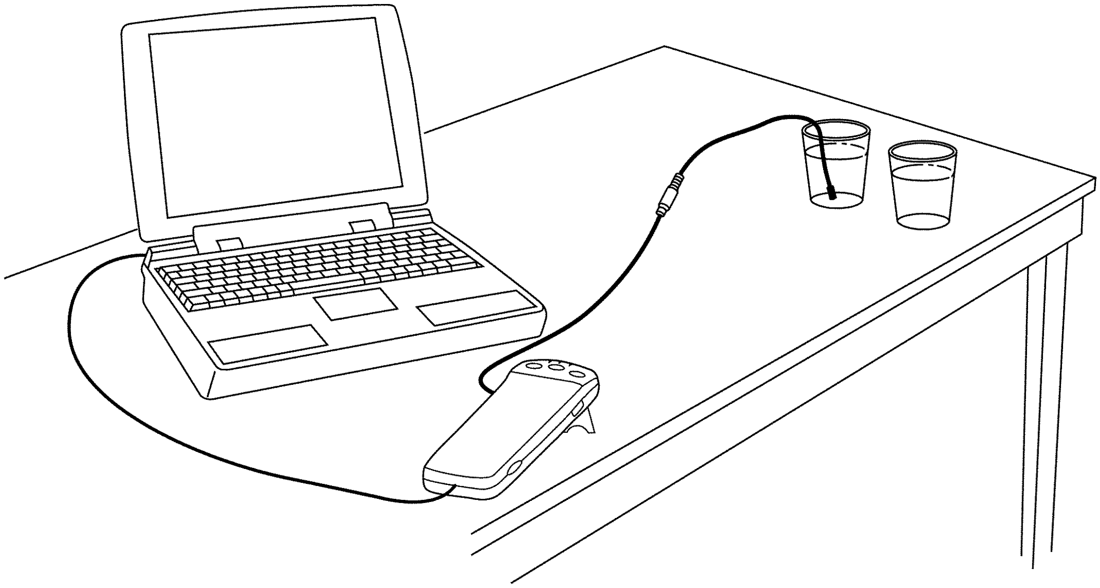
If you pour yourself a hot cup of coffee and place it on the counter, it immediately starts to cool off. The cooling process is quick at first, and then slowly levels off as the beverage approaches room temperature. In this situation, the rate at which the temperature of the drink changes is approximately proportional to the difference between its temperature and the temperature of its surroundings. This rule is commonly called *Newton's Law of Cooling*.

Introduction

In this activity, you will use a CBL™ or CBL 2™ together with TI InterActive!™ software and a temperature sensor to collect data that will simulate the temperature variations that occur as a liquid is cooling. You will then use the mathematical techniques and the statistical features of TI InterActive! to build a mathematical model for the resulting data set.

Equipment Required

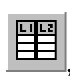
- Computer
- TI InterActive! software
- CBL/CBL 2
- TI temperature probe
- TI-GRAPH LINK™ cable
- Cup of hot water
- Cup of ice water

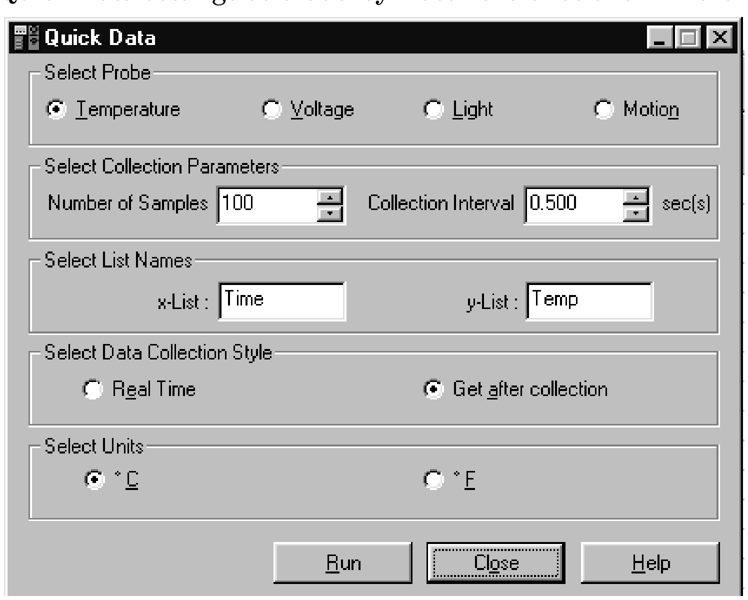


Setup

1. Plug the TI-GRAPH LINK™ cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL™/CBL 2™.
3. Connect the TI temperature probe into the Channel 1 (**CH1**) port on the CBL/CBL 2.
4. If you are using a CBL, turn it on by pressing the red [ON/HALT] button.

Collecting the Data

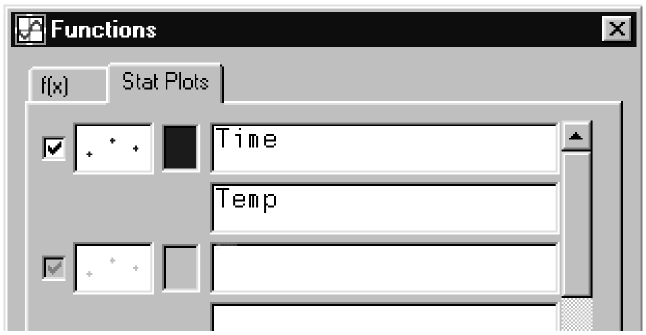
1. Start TI InterActive!™ The software opens to a new, blank document.
2. Title your new document *Cooling Off* and add your name and the date. Click the Save button  to save and name your document.
3. Click the List button , then click the Quick Data button  on the menu bar to open the data collection setup window.
4. Adjust the Quick Data settings so that they match the ones shown here:

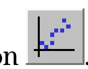


5. Place the temperature probe in the cup of hot water for about a minute. You will simulate the action of a cooling liquid by recording temperature data as the probe cools in a cup of ice water.

6. When you are ready to start collecting data, remove the temperature probe from the hot water, plunge it into the ice water and, *at the same time*, click **Run** in the Quick Data box. It will take about 50 seconds to collect the data. While the data is being collected, stir the temperature probe in the ice water.

7. As soon as the CBL™/CBL 2™ is finished collecting the temperature data, it will be transferred to the computer and a Graph window will open on the screen. The plot setup box shown below will appear.

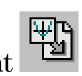


8. Press Enter, then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

   The plot of Celsius temperatures versus time should be curved and decreasing.

   If you are not satisfied with your results, start again and collect a new set of data.

   If you are satisfied with your data, you can make a sketch of the temperature versus time data that you collected on a blank grid in the Appendix. Label the horizontal and vertical axes on your sketch.

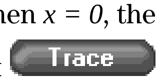
9. Click the Save to Document  button to save the graph to your TI InterActive!™ document.

Analysis and Questions

1. According to Newton's Law of Cooling, the temperature difference between a hot object (in this case, the temperature probe) and its surroundings (in this case, the ice water) decreases exponentially with time. Since the temperature of the ice water is 0 degrees Celsius, we can model this data set with an equation of the form:

   $$y = A\,B^x$$

   where *x* is time and *y* is temperature.

   When $0 < B < 1$, *y* is said to *decay exponentially* with *x*. The constant *B* is called the *base*. Notice that when $x = 0$, the *y* value is equal to *A*. Double-click the graph to open the Graph window. Click 

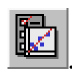
 and note the *y*-coordinate shown in the Trace Value dialog box that corresponds to $x = 0$. Click the Copy button. Close the Trace Value box, the Functions dialog box, and the Graph window. In a blank spot in your TI InterActive! document, Click on Edit, Paste, to past the Trace Value result in the document.

2. In order to find an exponential model for data you collected, you will need to find an appropriate value for *B*. We will use the guess-and-check method. Double-click the graph to open it, and click the f(x) tab in the upper left corner of the Functions dialog box. For exponential decay, the model demands that $0 < B < 1$, so start with an initial guess of $B = 0.5$. Type **f(x):= A*.5^x** in the uppermost text box of the f(x) tab, using the numerical value of *A* from above. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of B produced a model that matched the data closely. Double-click the graph to open it, click in the text box of the f(x) tab again and edit the exponential equation, replacing the old value, B = 0.5, with your new guess for B. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a B-value that models the data well. Press the Copy button. Close the Trace Value dialog box, Functions dialog box, and the Graph window. In a blank spot in your TI InterActive!™ document, click on Edit, Paste, to paste the Trace Value result in the document.

3. TI InterActive! lets you check the values of *A* and *B* you just found by calculating the exponential curve of best fit. Close the Function dialog box, and the Graph window to return to your TI InterActive! document.

   a. Double-click on the list to open the Data Editor.

   b. Click Statistical Regressions .

   c. Click the down arrow  next to Calculation Type, scroll down the list and click on **Exponential Regression**.

   d. In the text box labeled **X List**, type **time**; in the box labeled **Y List**, type **temp**. Click **Calculate** to find the regression equation, $y = a\,b^x$, and its variables.

   e. Click the checkbox next to *a* and *b* to select these values. Click the **Save Results** button. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

   How do the values of *a* and *b* in the exponential regression equation compare with the *A*-value and *B*-value you found by guess-and-check?

4. Double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq(x)** and then press Enter. TI InterActive! graphs the original data set, the equation that was created as the Statistical Regressions result, and your modeling curve. How well does each equation fit the data?

5. Describe how the value of B affects the shape of the temperature versus time graph, $y = A\,B^x$.

6. Why must the value of B be less than 1 for this modeling activity? What shape does the graph have if B is greater than 1?

7. For the experiment you performed in this activity, does it make a difference whether the temperature readings are taken in Celsius degrees or Fahrenheit degrees? Explain.

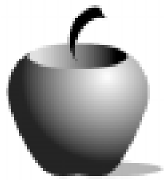
8. Save and print your TI InterActive!™ document.

Extensions

- Repeat the experiment, but this time allow the temperature sensor to cool off in the air rather than in ice water. In this case, the temperatures will level off at a non-zero value. This means that the modeling equation will be shifted: $y = A\,B^x + C$. Can you find values for A, B, and C so that this equation provides a good fit for the data you collected? What variable (or variables) represents the starting temperature? What is the physical meaning of the constant C in the modeling equation? Is it possible to use the TI InterActive! exponential regression feature to model this set of data? Why or why not?
- What would a temperature versus time data set look like if the sensor *started* in ice water and then was submerged in hot water? Sketch your prediction for the resulting heating curve, then perform an experiment to check your prediction. Is it possible to find a mathematical equation that describes this type of phenomenon?

Teacher Notes

Activity 8: Cooling Off



Math Concepts

- CBL/CBL 2
- Exponential Function

Activity Notes

- The water used in this activity should be very hot, but does not need to be boiling. Students should work quickly to collect the data before the water cools off too much.
- You might want to discuss the concept of asymptotic behavior with your students since the temperatures in this experiment approach the surrounding temperatures, in this case 0°C.

Sample Data

***Note**: This data set is based on readings every 2 seconds, not every 0.5 seconds as prescribed in the activity.*

Time (sec)	Temp (F)	Time(sec)	Temp (F)	Time (sec)	Temp (F)
0	41.59	18	11.86	36	5.22
2	36.99	20	10.85	38	3.35
4	30.49	22	10.07	40	2.69
6	25.99	24	10.07	42	2.36
8	24.69	26	7.53	44	2.24
10	19.84	28	8.19	46	1.7
12	19	30	7.53	48	2.58
14	15.26	32	4.89	50	0.47
16	13.89	34	4.78		

Analysis and Questions - Key

1. $A = 41.59$.
2. $B = 0.93$.
3. $a = 43.987$, $b = 0.933$; they match closely.
4. Both equations fit the data well.
5. The value of B changes the steepness of the curve.
6. It must be less than 1 so that the curve shows decay; for B-values greater than 1, the curve is increasing, not decreasing.
7. The data would have the same shape but the level-off value would be non-zero and, as a result, the modeling equation would be $y = AB^x + C$, where C represents 32°F.

EXPLORATIONS

Activity 9

You will most likely need to buy a car in the future. Due to the high prices of new cars and the large number of cars coming back from new car leases, used cars have become a very popular alternative for cost-conscious consumers and people looking for their first car. New cars *depreciate*, or lose value, as soon as they are purchased, but used cars have already had that depreciation built into the price. New cars have sticker prices posted on their windows and there are many consumer magazines and buying guides for new cars, but how are used car prices determined? Various companies publish guides to used-car values for dealers and consumers to use.

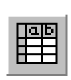
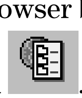
Introduction

In this activity, you will use the TI InterActive!™ browser to gather data from used car web sites and determine appropriate mathematical models for the price of used cars with respect to the age of the car. In the activity, you will discover what happens to the value of a car over time.

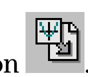
Equipment Required

- TI InterActive! software
- A working Internet connection

Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Used Cars* and add your name and the date. Click the Save button  to save and name your document.
3. Click the Spreadsheet button  to open the spreadsheet.
4. Click the Web Browser button  to open the TI InterActive! browser. Click the Data Sites button . Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 9: Used Cars**.

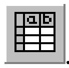
5. Once the page has been loaded in the browser, scroll down to the **And/Or** section and check the box for the Ford Mustang. Scroll down to the bottom of the page and click on the **CONTINUE** button. You will use the Mustang convertible for your first investigation, so check the box for the convertible. (Just the plain convertible, nothing fancy.) Scroll down and click on the **GO SEARCH** button. You should find lots of data for the values of Mustang convertibles.
6. When the page of Mustang data appears, scroll down to the data and drag the cursor over it to select. Click 

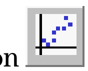
 to import the data into your TI InterActive!™ spreadsheet.
7. Before you proceed, clean up the data in the spreadsheet.
   a. Click on Edit, Replace.
   b. In the **Find What** text box, type **Convertible** and in the **Replace With** box, type a space.
   c. Click Replace All. This removes the word convertible from all the cells in your spreadsheet.
   d. With the Replace dialog box still open, type **$** in the **Find What** text box. (Leave the space in the **Replace With** box.)
   e. Click Replace All.
   f. Click Close.
   g. Click on the **B** at the top of the second column. Press and hold the shift key, then click on columns **C** and **D**.
   h. Click on Edit, Delete. (In the Delete dialog box, *Entire Column* will be selected.) Click **OK**.
   i. Columns **F** and **G** become the new columns **C** and **D**. Using the directions in steps **g** and **h**, delete these new columns **C** and **D**.
8. Click on the Save to Document button .

Investigating the Data

1. Look through the data. You may need to drag the bottom handle of the spreadsheet frame to display all of the data. In your TI InterActive! document, record any observations you may have about the values of Mustang convertibles with respect to the year they were produced.
2. In looking at the data, is it clear that the value of the Mustang convertible is a function of its age? In other words, its current value is dependent upon how old the car is. Many people consider old Mustangs to be classic cars. What do you think it means to be a "classic car?" Look at the data. At what time does the car change from being a classic car to being just another used car? Explain your answer in your TI InterActive! document.

3. Your first investigation will focus on the years 1983 through 1998 when the Mustang convertible is losing value, or depreciating in value. The car's value depends on its condition, or in other words, it depends on how well the previous owners took care of it. For this exercise, assume that the car is in excellent condition.

a. Set up a new spreadsheet in TI InterActive!™ by clicking on the Spreadsheet button . The age of a 1983 Mustang is determined by subtracting 1983 from the current year. Column **A** will represent the number of years since the current year, and column **B** will represent the value of the Mustang. The car's value depends on its condition. Assume the car is in excellent condition. Using the year 2000 as an example, in cell **A2**, type the age in years, **17** (2000 - 1983 = 17). Press the down arrow key to move to the next cell. Continue until you have entered all of the years since 2000 into column **A**. In column **B**, put the value according to Hemmings of the Mustang in excellent condition for each of the years.

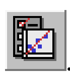
b. Highlight the data in columns **A** and **B**, and click the Scatter Plot button . A graph of the data, a StatPlot, appears. The viewing boundaries are adjusted automatically to show all the plotted data.

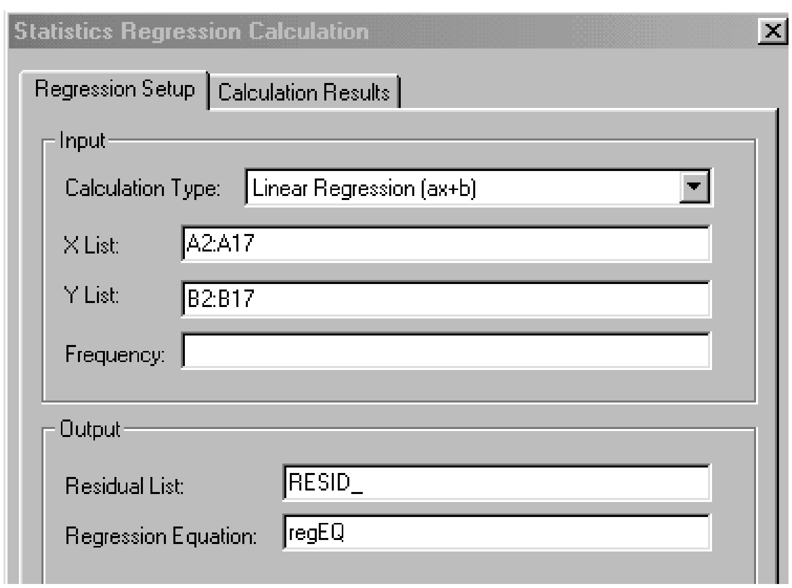
c. Click Save to Document to save the open graph in your TI InterActive! document.

d. Close the Data Editor window.

4. Look at the Scatter plot. Is the graph of the years since the current year versus value a linear function of the form $y = ax + b$?

5. To justify your answer, you need to try to fit a line to the data. TI InterActive! lets you determine values of a and b by calculating a regression line of best fit. Use a linear regression model on the data. To do the regression, double-click on the spreadsheet to open it again, then:

a. Highlight the same data in the spreadsheet, and on the toolbar, click Statistical Regressions . Note that Linear Regression ($ax + b$) is the default selection.



b. Click on the Calculate button at the bottom of the window. Click on Save Results. Close the Data Editor.

c. Double-click on the graph to open the Graph window. In the Functions dialog box, click on the f(x) tab. Type **regEQ(x)** in the first text box and click at the left to place a check mark in the checkbox. This adds the line to your graph.

Note: *The graph must be located below the Linear Regression results in your TI InterActive! document.*

d. Click on the Save to Document  button.

6. Does this line fit the data well? Explain your answer.

7. Click on the Save button  to save your TI InterActive!™ document.

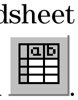
Continuing the Investigation

You can use the spreadsheet functionality of TI InterActive! to further the investigation. Linear functions increase or decrease at a constant rate. (See *Activity 2: Tight Rope*.) Look at the following spreadsheet of $y = 3x - 2$ on the domain $-5 < x < 5$. The spreadsheet on the right has all the rules to help you create your own linear function spreadsheet.

	A	B	C
1	**x**	**y**	**differences**
2	**-5**	**-17**	
3	**-4**	**-14**	**3**
4	**-3**	**-11**	**3**
5	**-2**	**-8**	**3**
6	**-1**	**-5**	**3**
7	**0**	**-2**	**3**
8	**1**	**1**	**3**
9	**2**	**4**	**3**
10	**3**	**7**	**3**
11	**4**	**10**	**3**
12	**5**	**13**	**3**

	A	B	C
1	x	y	differences
2	-5	=3*A2-2	
3	=A2+1	=3*A3-2	=B3-B2
4	=A3+1	=3*A4-2	=B4-B3
5	=A4+1	=3*A5-2	=B5-B4
6	=A5+1	=3*A6-2	=B6-B5
7	=A6+1	=3*A7-2	=B7-B6
8	=A7+1	=3*A8-2	=B8-B7
9	=A8+1	=3*A9-2	=B9-B8
10	=A9+1	=3*A10-2	=B10-B9
11	=A10+1	=3*A11-2	=B11-B10
12	=A11+1	=3*A12-2	=B12-B11

As the *x*-values increase by 1, what is the constant rate of increase in the *y*-values?

1. Set up a new spreadsheet in TI InterActive! by clicking on the Spreadsheet button . Create a linear function spreadsheet similar to the one above, but choose different values for the slope and *y*-intercept in the linear function model $y = ax + b$. Describe your observations. Share values of a and b with other members of your class. Make general conjectures about the differences of linear functions.

2. Close this spreadsheet and double-click on the Mustang data spreadsheet in the TI InterActive!™ document. Examine the differences in your Mustang values. Using the method shown above, create a column with the differences in values. Do they increase at something close to a constant rate? Explain.

3. If not, look at the ratio of successive values. For example: $\frac{B2}{B3}, \frac{B3}{B4}, \frac{B4}{B5}$, and so forth. Is the ratio relatively constant? When the *y* values of a function increase or decrease at a constant *ratio*, the function is *exponential*. Put the ratios in column C and use the average function on the spreadsheet, **=AVERAGE(C3:C17)** to calculate the average of these ratios. It will not be perfect, but is the ratio of successive values relatively constant?

   Record this average ratio in your TI InterActive! document, and also record any conjectures that you are willing to make about the Mustang data. This ratio can be thought of as the *depreciation ratio* for the Mustang. What do you think that means in the analysis of the data?

Analysis and Questions

When the relationship between the variables *x* and *y* is exponential, it can be expressed in the form $f(x) = ab^x$, where *b* is the constant ratio between the *y* values.

In order to find an $f(x) = ab^x$ model for the Mustang data, you will only need to find the value of *a*, the initial value of the car, since you know *b* = the average of the ratios. At this point your model is $f(x) = a^* (R^x)$ and you can use the guess-and-check method. When *x* is zero, the current year, $(R^\circ) = 1$ so *a* represents the value of the car in the current year.

1. Use TI InterActive! to find the value of *a*.

   a. Double-click on the graph of the Mustang data.

   b. Click the f(x) tab in the upper left corner of the Functions dialog box.

   c. Start with an initial guess of *a* = $18,000. Type **f(x):= 18000*R^x**, where R is your constant ratio, in the uppermost text box of the f(x) tab.

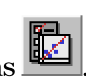
   d. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of *a* produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the exponential function, replacing the old value, *a* = 18,000, with your new guess for *a*. Press Enter to update the graph. Repeat the guess-and-check procedure until you find an *a*-value that models the data well, and record it in your TI InterActive! document.

2. With TI InterActive!, you can check the values of *a* and *b* you just found by calculating the exponential function, $y = ab^x$, of best fit.

   a. Click the Graph window close box .

   b. Double-click on the spreadsheet data.

   c. Highlight the data and click Statistical Regressions .

   d. Click the down arrow  next to Calculation Type, scroll down the list, and click on Exponential Regression.

e. Click Calculate.

f. Click the Save Results button. TI InterActive!™ stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

3. How does the value of a in the exponential regression equation compare with the a-value you found by guess-and-check? What does the value of a from the regression equation represent in this mathematical model?

4. What should the b-value from the regression equation represent in this mathematical model?

5. Double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regEQ** and press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions result. Which equation seems to fit the data better? Why?

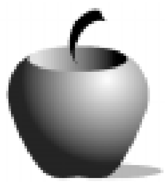
6. Save and print your TI InterActive! document.

Extensions

- Ford Mustangs built between 1965 and 1973 are considered classic cars. Instead of depreciating in value, the car's value is actually increasing or *appreciating* in value as the number of years since 2000 increases. Place the values for years since 2000 and value of the Mustang into your spreadsheet and use the differences and ratio technique to determine if this growth is linear or exponential. The years 1972 and 1973 seem to be *outliers* in the data set. You may want to disregard these two values. Use regression to determine an appropriate model and write up the investigation in a new document in TI InterActive! Carefully explain your model and justify your conjecture about linear growth versus exponential growth. Use appropriate graphs and/or tables of values to support your argument.
- Go back to the web site and repeat the investigation. This time use a Chevrolet Corvette instead of a Mustang. Corvettes built between 1956 and 1975 are considered classic cars, but thereafter are just used cars. Analyze the data from 1986 to 1998. Is the rate of depreciation and appreciation the same as for the Mustang? Why is it or is it not the same?
- Check out other Web sites to find values of used cars. Kelley's Blue Book at **http://www.kbb.com** is a good place to start. Do you find similar values for the Ford Mustangs that the Hemmings Motor News™ Web site gave? What factors determine the value of a used car?

Teacher Notes

Activity 9: Used Cars



Math Concepts

- Internet Data Collection
- Spreadsheets
- Ratios
- Linear Function
- Exponential Function

Activity Notes

- Most cars depreciate over time. This activity introduces an exponential decay model and compares the decay (growth) to the constant growth of linear functions.
- Using differences of the *y* values, students should determine that linear functions increase by the constant value of *m* as *x* increases by 1.
- Exponential models increase or decrease at a constant ratio (the growth or decay factor) as *x* increases by 1.
- The values of used cars are frequently changing. Check the Hemmings Motor News™ web site or use other appropriate web sites prior to starting this activity.
- The finite differences spreadsheet can be done by the teacher as a demo, or have all the student groups investigate and compare their results.

Investigating the Data - Key

1. Early Mustangs are appreciating in value, but Mustangs from 1983 forward are losing value.
2. Each year has only one value associated with it, so it is a function. The car changes from being a classic car to being just another used car after 1973, when it begins to depreciate in value.
3. No, it is not linear.
4. $a \approx -950$ $b \approx 16{,}500$ No, it does not fit the data well. For example, too many points are below the line, or look at the residuals.
5. The constant rate of increase in the *y*-values is 3.
6. The differences equal *m*, the slope.
7. No, because it is not a linear function.
8. Yes, the ratio is relatively constant. $R \approx .91$

Analysis and Questions - Key

1. $a \approx 19{,}000$ $f(x) = 19{,}000 * .907^x$
2. $a = 19{,}135$ $b = .91^x$
3. *a* represents the resale value of the current year Mustang Convertible.
4. *b* represents the decay ratio.
5. The regression equation should be a better fit, but allow students to justify their answer.

EXPLORATIONS

Activity 10

Economists often use math to analyze growth trends for a company. Based on past performance, a mathematical equation or formula can sometimes be developed to help make predictions about future results. As you will see, it is relatively easy to build a mathematical equation that fits a given set of data. The trick is finding a mathematical model that has predictive value—one that can help us plan for events in the real world.

Introduction

In this experiment, you will use the TI InterActive!™ browser to access data relating to the openings of new Starbucks Coffee Company shops in the last decade. What mathematical model best describes the growth of Starbucks shops with time? To what extent can a modeling equation be used to predict how many shops there will be in the future?

Equipment Required

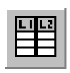
- A working Internet connection
- TI InterActive! software

Collecting the Data

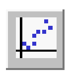
1. Start TI InterActive! The software opens to a new, blank document.
2. Title your new document *Coffee Break* and add your name and the date. Click the Save button  to save and name your document.
3. Click the Web Browser  to open the TI InterActive! browser. Click on the Data Sites button . Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 10: Coffee Break**.
4. Once the page has been loaded in the browser, scroll down to the heading *The Company* and click *News*. Then click *Company Overview*. Finally, click *Timeline*.
5. Scroll through the Starbucks timeline narrative and note the total number of coffee shop locations each year, starting with 1987 and ending with 1997. Record this data in the table provided.

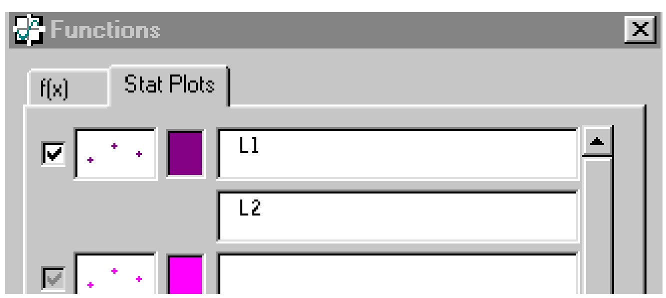
Year	Years since 1986	Total Number of Starbucks Locations
1986	0	1
1987	1	17
1988	2	
1989	3	
1990	4	
1991	5	
1992	6	
1993	7	
1994	8	
1995	9	
1996	10	
1997	11	

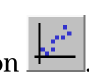
Recording the Data

1. Click the List button , and then click the empty cell at the top of list **L1**. Type the starting time value, **0**, and then press the down arrow key to move to the next cell. Continue entering the number of years since 1986 until you have entered all of the time values into **L1**.

2. Click the empty cell at the top of list **L2**. Type the total number of Starbucks locations for each corresponding time value in **L1**.

3. Click the Scatter Plot button  then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key to move to the second text box, and type **L2** to specify the list containing the *y*-coordinates.

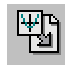


4. Press Enter, and then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

   The plot of number of coffee shops versus the number of years since 1986 should be curved upward.

If you are not satisfied with your results, check your data against the Web site again.

If you are satisfied with your data, you can make a sketch of the coffee shops versus the number of years since 1986 data that you collected on a blank grid in the Appendix. Label the horizontal and vertical axes on your sketch.

5. Click the Save to Document button  to save the graph in your TI InterActive!™ document.

Analysis and Questions

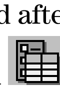
1. When the relationship can be expressed in the form $y = A\,B^x$, with $B > 1$, y is said to *grow exponentially* with x. The constant B is called the *base*. Notice that when $x = 0$, the y value is equal to A. Use your data table to determine the value of A and record it below:

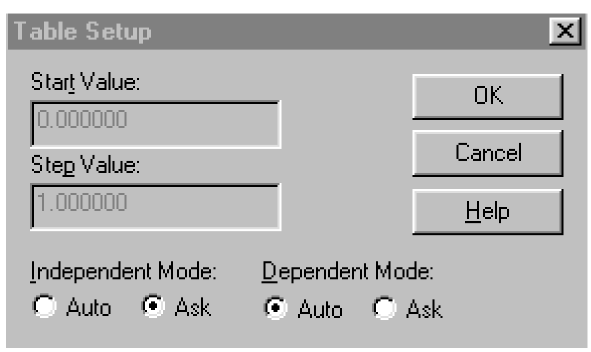
   $A =$

2. In order to find an exponential model for data you collected, you will need to find an appropriate value for B using the guess-and-check method. Click the f(x) tab in the upper left corner of the Functions dialog box. For exponential growth, the model demands that $B > 1$, so start with an initial guess of $B = 2$. Type **f(x):= A*2^x** in the uppermost text box of the f(x) tab, using the numerical value of A from above. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of B produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the exponential equation, replacing the old value, $B = 2$, with your new guess for B. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a B-value that models the data well and record it in the space below (give your answer to two decimal places):

   $B =$

3. Can the equation you just found be used to predict how many new Starbuck locations opened after 1997? Click  to display the table screen. Click the Table Setup button . Set the Independent Mode to **Ask** and the Dependent Mode to **Auto**, as shown on the next page, then click **OK**.



An empty table is displayed. As you enter values for the independent variable, *x*, values for the dependent variable, *y*, will be generated using the equation you entered in the Function dialog box. To start, type the number **12** (corresponding to 1998) into the first *x*-cell, then click on the corresponding *y*-cell to display the number of coffee stores your model predicts for that year. Repeat for $x = 13$ (corresponding to 1999), and record your values in the second column of the table below.

Year	**Predicted Number of Locations**	**Actual Number of Locations**
1998		
1999		

How do your predictions compare with the actual number of Starbucks locations that existed in 1998 and 1999? Follow steps **3** and **4** in the *Collecting the Data* section of this activity to return to the Starbucks web site and find out how many locations there were for these two years. Record this information in third column of the table above.

4. Why do you suppose there was such a big difference between your predicted values and the actual number of coffee shop locations? What real-world features of Starbucks growth were not accounted for in the exponential modeling equation?

5. The advanced statistical features of TI InterActive!™ allow you to create a modeling equation that more completely describes the Starbucks growth phenomenon you have been considering. The modeling equation is called a *logistics curve* and takes into account the fact that growth of this sort eventually levels out or reaches a *carrying capacity*. Because the logistics growth formula is very complicated, you will rely on the regression features of TI InterActive! to find the modeling equation for us.

  a. Click the Function Table close box (click **No** when asked if you want to insert a copy of the table into the document) and the Graph close box to return to the Data Editor.

  b. Click Statistical Regressions.

  c. Click the down arrow next to Calculation Type, scroll down the list and click on **Logistic Regression**.

  d. In the text box labeled **X List**, type **L1**; in the box labeled **Y List**, type **L2**.

  e. Click **Calculate** to find the regression equation and its variables. Click **Save Results**. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

6. Close the Data Editor. Make sure the graph is positioned below the equation in the TI InterActive! document, then double-click on the graph to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq** and then press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions Result together with your original guess-and-check exponential model. Which equation seems to fit the data better?

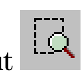
7. How well does the logistic model predict how many new Starbucks locations opened after 1997? Click 

again, then click the Table Setup button . Set the Independent Mode to **Ask** and the Dependent Mode to **Auto**, as before, then click **OK**. Type the number **12** (corresponding to 1998) into the first *x*-cell, then click on the corresponding *y*-cell to display the number of coffee stores your model predicts for that year. Repeat for $x = 13$ (corresponding to 1999), and record your values in the second column of the table below.

Year	Predicted number of locations	Actual number of locations
1998		
1999		

How do your predictions compare with the actual number of Starbucks locations that existed in 1998 and 1999? Refer to the comparison table you made earlier and record the actual values in the third column of the table above.

8. Did the logistic model provide reasonable predictions? Why is a logistic curve a better model for this data set than an exponential curve?

9. How does the logistic model account for the fact that the number of stores should eventually level off? To find out, click Zoom Out  several times to see how the curve behaves as the number of years increases. Estimate the level-off value for this function (called the *carrying capacity*). What does this value represent?

10. Save and print your TI InterActive!™ document.

Extensions

- Use the logistic modeling equation developed in this exercise to predict how many Starbucks shops there will be in the years 2000, 2005 and 2020. Describe any possible limitations the model might impose and under what circumstances it might fail completely.
- Can you think of some other real-world phenomena that might seem to behave exponentially but upon closer inspection actually grow (or decay) logistically? List as many examples as you can and, wherever possible, identify a reasonable level-off point or carrying capacity value.

Teacher Notes

Activity 10: Coffee Break

Math Concepts

- Internet Data Collection
- Tables
- Exponential Function
- Logistic Function

Activity Notes

- Note that this activity introduces logistic curves. Although the mechanics of the equation may be beyond the scope of your course, it is easy to explain how the shape of the curve is related to the way a company (or population) might grow.
- Most web sites are dynamic. Double-check the path to the company timeline before starting this activity with your class.

Sample Data

Year	Years Since 1986	Total Number of Starbucks Locations
1986	0	1
1987	1	17
1988	2	33
1989	3	55
1990	4	84
1991	5	116
1992	6	165
1993	7	272
1994	8	425
1995	9	676
1996	10	1015
1997	11	1412

Analysis and Questions - Key

1. $A = 1$.
2. $B = 2.05$.
3. For 1998, predicted = 5509, actual = 1886; for 1999, predicted = 11293, actual = 2200.
4. There is a big difference between the predicted values and the actual values because the modeling equation does not take into account that eventually, the number of stores will stop growing and start to level-off.
5. *regEQ(x)*=(3972.15)/(1 + 434.8*(e)^(-.498717 x)).
6. The logistic equation fits the data much better.

7. For 1998, predicted = 1896, actual = 1886. For 1999, predicted = 2386, actual = 2200.

8. The logistic model values match closely with the actual values. This model is more appropriate because it accounts for the fact that eventually the number of stores will reach a limiting value.

9. The level-off value is roughly 4000. This represents the expected value for the total number of Starbucks stores.

EXPLORATIONS

Activity 11

Curve Ball

When a ball bounces up and down, the height of the ball as a function of time can be modeled with a quadratic function. The resulting graph of this function would then be a parabola. This relationship is usually expressed in one of two ways. The form $y = ax^2+bx+c$, is commonly known as the standard form of a quadratic equation. It can also be written as $y = a(x-h)^2+k$, which is known as the vertex form of a quadratic function.

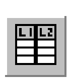
Introduction

In this experiment, you will record the motion of a bouncing ball using a CBL™/CBL 2™ and TI InterActive!™ software. You will then analyze the collected data and attempt to find a model for the height of the ball as a function of time.

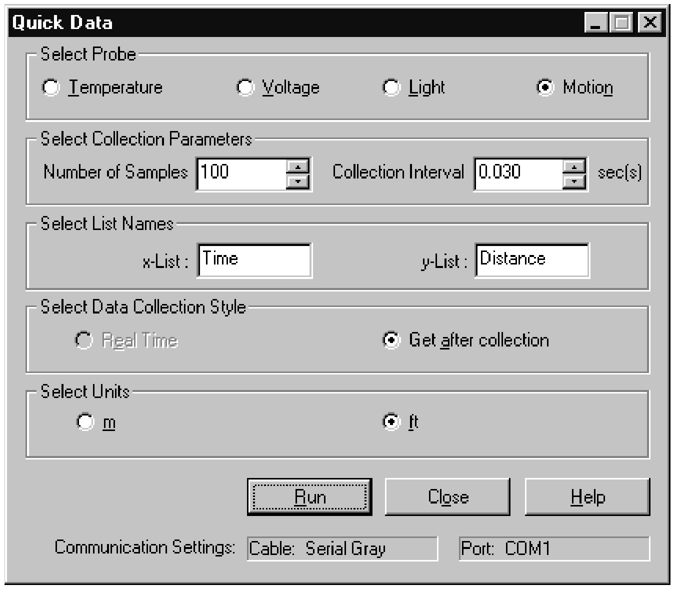
Equipment Required

- Computer
- TI InterActive! software
- CBL/CBL 2 (or a CBR™)
- TI-GRAPH LINK™ cable
- Motion detector
- Ball (racquetballs work well with this activity)

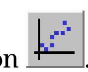
Instructions

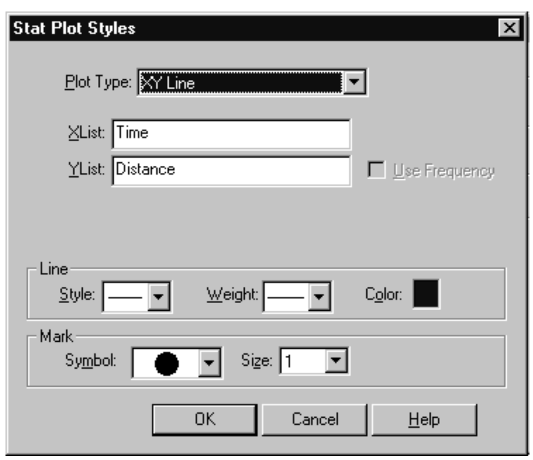
1. Plug the TI-GRAPH LINK cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL/CBL 2.
3. Plug the motion detector into the **CH1** port on the CBL/CBL 2.
4. Start TI InterActive! The software opens a new, blank document.
5. Title your new document *Curve Ball* and add your name and the date. Click the Save button  to save and name your document.
6. Click the List button  to open the Data Editor.
7. From the Data menu, choose Quick Data.

8. Adjust the Quick Data dialog box settings so that they match the ones shown below.



Collecting the Data

1. Be sure that the ball is bounced on a smooth, level surface. Do not allow anything to obstruct the path between the detector and the ball while the data is being collected.
2. Hold the motion detector about five or six feet above the floor and parallel to it.
3. To begin data collection, click Run on the Quick Data dialog box.
4. Release the ball, taking care that the hands of the ball dropper are moved away from the path of the motion detector upon release of the ball.
5. When data collection finishes, click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.



6. If the scatter plot looks confusing, click the Stat Plots tab of the Functions dialog box, then click the box to the right of the check box. This will bring up the Stat Plot Styles shown above. Change the Plot Type to **XY line** and click **OK**.

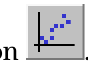
7. Your plot should appear to be a number of parabolas opening upward and rising from left to right. If you are satisfied with your data, click on Save to Document to save the graph in your TI InterActive!™ document. If you are not satisfied, close the Graph window and return to step **3** to collect a new data set.

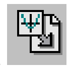
Recording the Data

1. The plot you recorded is a distance versus time graph. Convert the recorded distances to heights by subtracting each value from the distance to the floor. The distance to the floor will be the maximum distance point that you collected.
2. Double-click on listname **L1**, type **time** in the Formula box, and click OK. Double-click on listname **L2**, type **max(distance)-distance** in the Formula box, and click OK. This will convert your distance readings to heights.
3. Click the Graph button, and then click the Stat Plots tab of the Functions dialog box. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key and move to the second text box. Type **L2** to specify the list containing the *y* coordinates.
4. Press Enter, then click the Zoom Statistics button. The viewing boundaries adjust automatically to show all the plotted data.
5. For this activity, you will analyze only one bounce of the ball. To choose a particular bounce, press Trace and move to the first point on the left side of any bounce. Record the coordinates of this point in the table below. Continue to trace to the last point on the right side of the bounce and record its coordinates in the table below.

X	Y

6. Click the Save to Document button to close the graph and bring up the Data Editor.
7. Scroll downward, observing the values in **L1** until you find the *x*-value of the first point you entered in the table above. Place the cursor in this cell. Click and drag down and over so that all the values in lists **L1** and **L2** are highlighted from the first value in the table above to the last value in the table above.
8. Press Ctrl+C to copy the selected values from **L1** and **L2**.
9. Move the cursor to the first cell in **L3** and press Ctrl+V to paste the selected values into **L3** and **L4**.
10. Click the Graph button, and then click the Stat Plots tab. In the uppermost text box, type **L3** to specify it as the list containing the *x* coordinates. Press the Tab key and move to the second text box. Type **L4** to specify the list containing the *y*-coordinates.

11. Press Enter to turn the plot on. Click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

12. Click the Save to Document button  to save this graph in your TI InterActive!™ document.

13. Close the Data Editor.

Analysis and Questions

1. In this activity, the ball bounced straight up and down beneath the detector. The original plot, however, seemed to depict a ball moving sideways, rising, and upside down. Explain why this is so.

2. Click on the graph of the single parabola in your TI InterActive! document and press 

to move along the plot of your parabola. Estimate the *x*- and *y*-coordinates of the vertex, round these values to the nearest hundredth, and record them in the table below.

x-coordinate	y-coordinate

3. The theoretical model for an object in free fall is quadratic. We will attempt to fit the data with the vertex form of a quadratic equation $y = a(x-h)^2+k$. In this model the coordinates (h, k) represent the coordinates of the vertex and a is a constant. Substitute the values for *h* and *k* from the table above in this equation and record it below.

$y =$

4. Close the Trace Value dialog box and click on the f(x) tab of the Functions dialog box. Enter the equation above, but substitute a -1 for *a*. Click Enter to graph the function. How well does it fit the data?

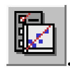
5. To obtain a good fit of the data you will need to adjust the value of *a*. Return to the Functions dialog box and try different values of a until you achieve a good model for the data. When you are satisfied with the fit, record your final values and equation below.

$a =$ $h =$ $k =$ $y =$

6. Describe how changing the value of *a* effects the shape of the parabola.

7. TI InterActive!™ can find its own model for the data by calculating the quadratic curve of best fit.

a. Click Save to Document  to close the Graph window.

b. Click on Stat Calculations Tool .

c. Click the down arrow  next to Calculation Type, scroll down the list and click on Quadratic Regression.

d. In the text box labeled X List, type **L3**; in the box labeled Y List, type **L4**.

e. Click Calculate to find the regression equation $y = ax^2 + bx + c$ and its variables. Record the regression equation values of *a*, *b*, and *c* below:

a	
b	
c	

8. In the space below, record TI InterActive!'s regression model using the values found above.

$y =$

9. Click Save Results to close the Statistical Regression Calculation dialog box and return to your TI InterActive! document.

Click the graph of your parabola saved in the TI InterActive! document and click on the f(x) tab on the Functions dialog box. Enter the model you recorded in step **8** in an available text box. Press Enter to see the graph of the model. How does it compare visually with the vertex form you had previously graphed?

10. Close the Graph window to return to your TI InterActive! document. Click on Math Box . Type **expand(a*(x-h)^2+k,x)**, substituting the values you found for *a*, *h*, and *k* in step **5** above, and press Enter. Record the results in the space below.

11. How does the standard form of the vertex form of the model compare with the standard form of the regression equation?

12. Save and print your TI InterActive!™ document.

Extensions

- Repeat this activity using different balls and different drop heights. What effect do these changes have on the value of *a* in either of your models?
- Choose another bounce to model from your original data set, other than the one you chose in this activity. Discuss the changes that result in the vertex form of your model. Can you come up with a quick way to find the equation of any bounce?
- Connect the TI-GRAPH LINK™ cable to your calculator and use TI InterActive! to send the data you collected from the computer to the calculator. Analyze the data on your calculator using the same methods applied in this activity.

Teacher Notes

Activity 11: Curve Ball

Math Concepts

- CBL/CBL 2
- List Manipulation
- Quadratic Function

Activity Notes

- If there is not enough room near the computer to collect this data, it can be collected using the calculator with a CBL™, CBL 2™, or CBR™. The data can then transferred to a TI InterActive!™ document.
- Caution students to be sure that nothing blocks the path between the motion detector and the ball. Hands and cords are common obstructions.

Sample Data

***Note**: Due to the large number of data points collected, the sample data contains only the points of the selected bounce.*

Time (sec)	Height(ft)	Time(sec)	Height(ft)	Time (sec)	Height(ft)
.66	0.12	.93	2.28	1.20	2.19
.69	0.38	.96	2.39	1.23	2.04
.72	0.72	.99	2.47	1.26	1.86
.75	1.03	1.02	2.52	1.29	1.65
.78	1.32	1.05	2.53	1.32	1.42
.81	1.57	1.08	2.52	1.35	1.15
.84	1.79	1.11	2.49	1.38	0.85
.87	1.98	1.14	2.41	1.41	0.52
.90	2.15	1.17	2.32	1.44	0.15

Analysis and Questions - Key

1. The data represents the distance from the motion detector to the ball with respect to time. The plot moves to the right because time is increasing. The data appears to be rising because the distance to the ball is increasing as the maximum heights of the ball decrease.
2. The vertex lies at approximately (1.05, 2.53).
3. $y = a(x - 1.05)^2 + 2.53$
4. The vertex is correct, but the graph is upside down and very wide.
5. $a = -16$, $h = 1.055$, $k = 2.53$, $y = -16(x - 1.055)^2 + 2.53$

6. Positive values for *a* cause the parabola to open upward, while negative values make it open downward. As the absolute value of *a* increases, the parabola appears to become thinner as a result of the vertical stretch taking place. As the absolute values of *a* decrease, the parabola appears to become wider.
7. a = -15.9167, b = 33.5786, c = -15.1804
8. $y = -15.9167x^2 + 33.5786x - 15.1804$
9. The models are virtually identical.
10. $y = -16x^2 + 33.76x - 15.2784$
11. The models are very close.

EXPLORATIONS

Appendix

Blank Grids

The grids on the next pages may be duplicated and used with the activities in this book if your students do not have access to printers to print their TI InterActive! documents, or if you prefer to have students draw the graphs on paper after they have completed the activity using TI InterActive!

Name ______________________

Date ______________________

Activity ________ **Question** ______

Activity _______ **Question** _____

Name ______________________

Date ______________________

Activity ________ **Question** ______

Activity _______ **Question** _____

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